sate in the second order in g is equivalent to obtaining it by minimizing the ground state energy to the same approximation in g. One may suppose that this equivalence will be true also in higher orders in g.

The authors take this opportunity to express their gratitude to N. N. Bogoliubov for discussing the work.

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RADIATIVE CORRECTIONS TO COMPTON SCATTERING TAKING INTO ACCOUNT POLARIZATION OF THE SURROUNDING MEDIUM

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A general method for taking into account polarization of the medium in the calculation of radiative corrections in phenomenological quantum mechanics is developed. The effect of a nonconducting medium on radiative corrections to Compton scattering is taken into account for an arbitrary dependence of the dielectric constant of the medium on frequency. It is shown that in some cases, account of the medium substantially changes the cross section in the region of small scattering angles.

1. INTRODUCTION

THE influence of the medium in the calculation of higher approximations in perturbation theory must, in general, be taken into account, because the integrations over the 4-momenta of virtual photons include a region of long-wave photons for which it is impossible to ignore the presence of neighboring atoms of the medium. This situation was first indicated by Landau and Pomeranchuk,[1] who noted that multiple scattering by the atoms of the medium should lead to a change in radiative corrections in those cases in which infrared catastrophes occur, i.e., where the region of soft quanta is essential. Ter-Mikaelian[2] noted that the difference of the dielectric constant of the medium from unity for soft quanta should strongly influence the radiative corrections.

A method of taking into account the multiple scattering by atoms of the medium was developed by Migdal.[3] In the following, we consider the influence of the medium on radiative corrections, connected with the difference of the dielectric constant and magnetic permeability of the medium, ϵ and μ, from unity in the region of soft quanta; we shall not take account of multiple scattering.

In order to develop a general method for taking into account the polarization of the medium in higher orders of perturbation theory, it is convenient to use a generalization by the author[4] of the Feynman-Dyson covariant perturbation theory to the case of phenomenological quantum electrodynamics in media. The general method obtained in this way will be applied to the Compton scattering, in order to obtain the cross section of sixth power in e, with account of the polarization of the surrounding medium. The notation of Feynman[5] will be used.

2. COVARIANT PERTURBATION THEORY

In the formulation of the Heisenberg representation[4] of phenomenological quantum electrodynamics in media, it is easy to see that the only difference of the theory from electrodynamics in vacuo is the equation for the potential of the electromagnetic field and the commutation relations for the operators of the noninteracting electromagnetic field. The supplementary relation can be put in the same form as in electrodynamics in vacuo by using a covariant method of separation of the longitudinal and scalar components of the potential, analogous to that employed in Ref. 6. Therefore, in the formulation of the perturbation theory, the usual Green's function for the photon will be replaced everywhere by the Green's function

$$G_{\lambda\nu}(x, x') = (2\pi)^{-2}\int d^4k g_{i\lambda}\{k_\rho^2 + \varkappa\,(u_\rho k_\rho)^2\}^{-1} g_{i\nu}\exp ik(x - x'), \tag{1}$$

where $\kappa = \epsilon\mu - 1$, u_ν is the 4-velocity of the medium, and $e_{i\nu}$ are the unit 4-vector directions of polarization of the photon, such that

$$e_{i\lambda}e_{i\nu} = \delta_{\lambda\nu};\quad e_{i\lambda}e_{j\lambda} = \delta_{ij}; \tag{2}$$

$$g_{i\nu} = \mu^{1/2}e_{i\sigma}(\delta_{\nu\sigma} - u_\nu u_\sigma[1 - (1+\varkappa)^{-1/2}]). \tag{3}$$

In the following we shall take the imaginary parts of ϵ and μ to be small, limiting the consideration to dispersion in the region of transparency. It should be noted that the usual choice of sign of the imaginary parts of ϵ and μ does not lead to a causal Green's function, but to a retarded one, as can be seen from the fact that in going over to the vacuum, the way of going around the pole is not the same as that of Feynman. (This is connected with the fact that only retarded potentials were employed in the derivation of the expressions for ϵ and μ.) Therefore, the prescription for going around the pole in Eq. (1) should be obtained from the requirement that (1) be the causal Green's function.[7] The prescription obtained from this can be, for example, given for positive frequencies in the form of the condition that there be an infinitesimal absorption, and for negative frequencies in the form of the requirement of symmetry of the theory with respect to past and future.

As is well known, in a dispersive medium, ϵ and μ are functions of that invariant variable, which, in the system of the medium, becomes the frequency. The only variable of this type is the scalar uk, and ϵ, μ, and κ are functions of it.

Thus, κ and μ are, in general, some complicated operators, and this leads to additional difficulties in formulating the theory. In order to get rid of these difficulties, it is possible from the very beginning to consider the theory in the momentum representation; however, one can obtain the same results by employing the usual simple method, introducing the dependence of κ and μ on uk only after going to the momentum representation.

Just as in the case of the vacuum, all divergences in the region of large momenta can be eliminated by renormalization of mass and charge. Consideration of possible types of diagrams shows that all conclusions about number and behavior of primitive divergences obtained for electrodynamics in vacuo[8] remain valid for electrodynamics in media. This follows from the fact that in the region of large momenta, ϵ and μ tend to unity and the matrix element for an arbitrary process in media in the region of large momenta coincides with the expression for the matrix element of the same process in vacuo.

The general rule for eliminating divergences from the scattering matrix element, analogous to that obtained by Dyson[8] for electrodynamics in vacuo, consists in subtracting several terms in an expansion of the divergent matrix element in $\hat{k}$ or $(\hat{p} - m)$, where one must set $\epsilon = \mu = 1$ in the subtracted matrix elements, i.e., these terms must be defined from the corresponding matrix element for the process in vacuo. The latter condition comes from the fact that the subtracted terms correspond to unobservable effects. The number of subtracted terms should be the minimum number for convergence of the remainder, which is that value matrix-element having physical significance.

Thus, the finite part of the self energy of an electron in media will differ from that in vacuo, leading to a difference in the mass of a free electron in media from that of a free electron in vacuo. From this it follows that after renormalization of the Green's function for an electron in media is carried out, it coincides with the Green's function for an electron in vacuo only in the zero order approximation of perturbation theory.

Taking account of the subsequent terms in perturbation theory leads to the appearance of additions to the mass if the electron moves in media, but not if it is in vacuo.

This leads to an essential difference in the Green's functions for a free electron in media and in vacuo, in spite of the fact that the equation for the operators of the electron-positron field in media has the same form as that in vacuo. The finite additions to the electron mass in the Green's function are essential in the infrared region and lead to the fact that the so called infrared catastrophe never arises in media. This effect, in essence, is a result of the fact that, strictly speaking, the electron moving in the potential field of the atoms of the medium cannot be considered as free, and therefore $p^2 \neq m^2$.

From this it follows that in order to eliminate the infrared catastrophe for electrodynamics in media, it is sufficient to take into account the corrections to the electron Green's function from the emission and absorption of a single virtual quantum, i.e., the change in mass connected with calculation of the self-energy diagram of second order. From the result in the same order, it is possible to obtain the expression for the Green's function of the electron in media in the form

$$i\,(2\pi)^{-2}\int d^4p\,(\hat{p}-m-\Delta)^{-1}\exp ik(x-x'), \tag{4}$$

where Δ is the difference in the change in mass for a free electron in media and in vacuo. We note that Δ depends on the invariants m and up, i.e., on the energy of the electron relative to the medium.

In the future, we consider only the experimentally observed cross section of Compton scattering, in which the possibility of production of additional soft quanta, not registered by the apparatus, is taken into account. Then the cross section in the region of soft quanta drops out of consideration, the infrared catastrophe does not arise, and it is not necessary to take account of Δ in Eq. (4).

3. COMPTON SCATTERING

In considering the influence of the medium on Compton scattering, it is of greatest interest to represent the scattering of photons sufficiently hard in the system of the medium so that ϵ and μ can be considered equal to unity for the frequencies of both the incident and scattered photons. The influence of the medium in this case will show up only in the virtual quanta and, consequently, the matrix element of second order in e for Compton scattering will not depend on polarization of the medium. (The corresponding diagrams do not include virtual photons.)

The matrix element of fourth order in e includes virtual photons and, consequently, will depend on the presence of the medium. We shall calculate the radiative corrections by the method of Feynman.[5]

Since the radiative corrections for Compton scattering were calculated in the work of Feynman and Brown,[9] it is then necessary for us to find the difference between radiative corrections in media and in vacuo. In view of the fact that we restrict the calculation to only terms of sixth order in e in the cross section, the matrix element of fourth order enters into the cross section linearly, so that we only need to calculate the difference of the matrix elements of fourth order for Compton scattering in media and in vacuo. This significantly simplifies the necessary calculations, since in the region of large momenta the difference of matrix elements which interests us goes to zero.

Denoting the 4-momenta of the incident and scattered photons by k_1 and k_2, respectively, and describing the initial and final states of the electron by the 4-momenta p_1 and p_2 and the spinors v_1 and v_2, it is easy to obtain the well known expression for the matrix element of second order

$$W^0_{st} = \hat{e}_s(\hat{p}_1+\hat{k}_1-m)^{-1}\hat{e}_t + \hat{e}_t\,(\hat{p}_1-\hat{k}_2-m)^{-1}\hat{e}_s, \tag{5}$$

in which the 4-vector polarizations of the incident and emitted photons are denoted by e_s and e_t.

The diagrams of fourth order for Compton scattering are given in Refs. 10, 5, and 9. The difference of fourth-order matrix elements which interests us does not contain divergences in the ultra violet region; therefore, for simplicity, we shall not explicitly introduce the Feynman cut off factor.

We write the matrix element of fourth order for Compton scattering in vacuo in the form

$$W^{(1)}_{st}(0,\lambda) = \int d^4q\,(q^2-\lambda^2)^{-1}F_1(q), \tag{6}$$

where $F_1(q)$ is a matrix function of momenta and polarization of the electron and the photons, λ is a fictitious mass of the photon, which is introduced in electrodynamics in vacuo in order to eliminate the infrared catastrophe.

The explicit form of $F_1(q)$ was obtained by Brown and Feynman.[9] As noted above, in media the infrared catastrophe does not arise; however, in the following it is also convenient to introduce λ into the matrix element of fourth order for Compton scattering in media, denoting it by $W^1_{st}(\kappa,\lambda)$:

$$W^{(1)}_{st}(\varkappa,\lambda) = \int \frac{\mu d^4q}{q^2+\varkappa(uq)^2-\lambda^2}\left\{F_1(q)-\frac{\varkappa}{1+\varkappa}F_2(q)\right\}, \tag{7}$$

where $F_2(q)$ differs from $F_1(q)$ only by the replacement of the 4-vector polarizations of the virtual photon by the 4-velocity of the medium, u_ν.

Calculating the integral in Eq. (7)

$$\int d^4q\,(q^2-a^2-\lambda^2)^{-1}F_1(q),$$

in which a^2 is determined from the asymptotic form of κ for large uq:

$$\varkappa(uq) \approx -(a/uq)^2; \quad a^2 = 4\pi ZNe^2/m, \tag{8}$$

it is easy to transform Eq. (7) to the form

$$W_{st}^{(1)}(\varkappa, \lambda) = W_{st}^{(1)}(0, \sqrt{a^2+\lambda^2}) + \int \frac{d^4q}{q^2-a^2-\lambda^2}\left\{\frac{(\mu-1)q^2-\mu a^2-\varkappa(uq)^2}{q^2+(uq)^2\varkappa-\lambda^2}\right\}F_1(q) - \int \frac{d^4q}{q^2+(uq)^2\varkappa-\lambda^2}\,\frac{\mu\varkappa}{1+\varkappa}F_2(q). \tag{9}$$

The second term in Eq. (9) differs from the integral in Eq. (6) only by the presence in the integrand of the additional factor

$$\left(\frac{q^2-\lambda^2}{q^2-a^2-\lambda^2}\right)\left(\frac{(\mu-1)q^2-\mu a^2-\varkappa(uq)^2}{q^2+(uq)^2\varkappa-\lambda^2}\right),$$

which is small for $q^2 \gg a^2$, ω_i^2 or $(uq)^2 \gg a^2$, ω_i^2. This means that the region of q essential for the integration is bounded, in the system of coordinates connected with the medium, so that the values of all components of the 4-momentum of the virtual photon q_ν are small compared with the momentum transfer $\xi > a$.

If terms of order a/ξ and ω_i/ξ (where ω_i is the proper frequency) are neglected henceforth, then the calculation of the integral considered is limited to the first nonvanishing term in the expansion of $F_1(q)$ in powers of q.

Analogous considerations can be made also for the third term in Eq. (9), since $F_2(q)$ has the same structure as $F_1(q)$.

Employing an expansion in powers of q/m, it is easy to find that the principal contribution in the expression of interest comes from the diagrams denoted in Ref. 9 as J, M′, and M″. In the approximation indicated, it is possible to obtain the expression

$$W_{st}^{(1)}(\varkappa, \lambda) = W_{st}^{(1)}(0, \sqrt{a^2+\lambda^2}) + \frac{4e^2}{\pi i}\int \frac{d^4q\,[(\mu-1)q^2-\mu a^2-\varkappa(uq)^2]}{(q^2-a^2-\lambda^2)(q^2+(uq)^2\varkappa-\lambda^2)}\,W_{st}^0\left\{\frac{e_l p_1}{(p_1+q)^2-(m+\Delta)^2} - \frac{e_l p_2}{(p_2+q)^2-(m+\Delta)^2}\right\}^2 - \frac{4e^2}{\pi i}\int \frac{d^4q}{q^2+(uq)^2\varkappa-\lambda^2}\,\frac{\mu\varkappa}{1+\varkappa}\,W_{st}^0 \times \left\{\frac{up_1}{(p_1+q)^2-(m+\Delta)^2} - \frac{up_2}{(p_2+q)^2-(m+\Delta)^2}\right\}^2, \tag{10}$$

in which W_{st}^0 and Δ are defined in Eqs. (5) and (4). The first term in Eq. (10) can be obtained from the result of Brown and Feynman by replacing λ^2 by $a^2+\lambda^2$. It is easy to see that Eq. (10) remains finite for $\lambda = 0$, if the Δ in the denominator is not neglected, and the infrared catastrophe does not arise. Therefore, there is no necessity of adding the cross section for double Compton scattering, integrated over the momentum of an additional small quantum, to that obtained from Eq. (10), as is done in the case of electrodynamics in vacuo to eliminate the infrared divergence. However, such an addition is desirable to carry out in order to take into account experimental conditions in which it is impossible to discriminate between single Compton scattering and double Compton scattering with emission of an additional quantum sufficiently small so that it is not registered by the experimental apparatus. It is natural for the experimentally-observed cross section to depend on the threshold of the apparatus, i.e., on the maximum energy of the photon ω_m for which the photon will not be measured.

Since in the following we will be interested only in the experimentally-observed cross section for Compton scattering, it is possible to neglect the quantity Δ in Eq. (10), disregarding λ for convenience of calculation.

4. EXPERIMENTALLY-OBSERVED CROSS SECTION

Denoting the cross section for Compton scattering with account of terms of sixth order in e in vacuo and in media by $d\sigma_K(0, \lambda)$ and $d\sigma_K(\kappa, \lambda)$, respectively, we easily obtain from Eq. (10)

$$d\sigma_K(\varkappa, \lambda) = d\sigma_K(0, \sqrt{a^2+\lambda^2}) - \frac{4e^2}{\pi i}d\sigma_0 \times \int \frac{d^4q\,[(\mu-1)q^2-\mu a^2-\varkappa(uq)^2]}{(q^2-a^2-\lambda^2)(q^2+(uq)^2\varkappa-\lambda^2)}\left\{\frac{e_l p_1}{q^2+2p_1q} - \frac{e_l p_2}{q^2+2p_2q}\right\}^2 + \frac{4e^2}{\pi i}d\sigma_0\int \frac{d^4q}{(q^2+(uq)^2\varkappa-\lambda^2)}\,\frac{\mu\varkappa}{1+\varkappa}\left\{\frac{up_1}{q^2+2p_1q} - \frac{up_2}{q^2+2p_2q}\right\}^2, \tag{11}$$

in which the usual cross section for the Compton effect without radiative corrections is denoted by $d\sigma_0$, where

$$d\sigma_0 = \left(\frac{e^2}{mc^2}\right)^2\left(\frac{\omega_2^2}{2\omega_1^2}\right)\left(\frac{\omega_1}{\omega_2} + \frac{\omega_2}{\omega_1} - \sin^2\vartheta\right)d\Omega_2. \tag{12}$$

It is important to note that the integral over q in Eq. (11) was obtained for arbitrary ϵ and μ; it was derived only by assuming absence of absorption and by assuming the asymptotic behavior of κ for large uq, Eq. (8). Therefore, it is desirable to calculate the integrals without introducing additional assumptions about the specific form of ϵ and μ. This is easy to do if, instead of integrating over components q_μ, one integrates over components Q_μ and over s, connected with q_μ by the relations

$$s = uq; \quad Q_\mu = q_\mu - u_\mu u_\rho q_\rho; \quad q_\mu = Q_\mu + u_\mu s. \tag{13}$$

The indicated change of variables makes it possible to resolve the difficulty which arises in consideration of those forms of the dependence of ϵ on (uq), which do not lead to a single-valued inverse function $uq(\epsilon)$, i.e., to the situation where, in the system of the media, several frequencies of the particle correspond to one and the same momentum of the real photon. This difficulty arises in case one tries to carry out the integration in Eq. (11) just as in electrodynamics in vacuo, i.e., first over the fourth component of the 4-momentum of the virtual photon. The change of variables, Eq. (13), makes it possible to first carry out the integration over Q_ν and then over $s = uq$, so that the fact that $uq(\epsilon)$ is not single valued does not show up in the integration. In this way, the transformation, Eq. (13), makes it possible to generalize the usual methods of quantum electrodynamics to such types of dependency of ϵ on uq which occur in practice.

The integration over Q_ν can be conveniently carried out as in Ref. 7. For example, for an integral analogous to those in Eq. (11), one easily obtains

$$\int \frac{d^4k\,(1; k_\sigma)\,F(uk)}{(k^2-2pk-A(uk)+i\eta)^3} \tag{14}$$

$$= \int_{-\infty}^{\infty} ds F(s) \int \frac{d^4Q\,\delta(uQ)\,(1; Q_\sigma + u_\sigma s)}{[Q^2-2pQ+s^2-2ups+A(s)+i\eta]^3}$$

$$= \frac{1}{16i} \int_{-\infty}^{\infty} \frac{ds F(s)\,(1; p_\sigma + u_\sigma(s-up))}{[s^2-2sup-A(s)+(up)^2-p^2+i\eta]^{3/2}}.$$

To obtain the experimentally observed cross section, it is necessary to put (11) together with the cross section for double Compton scattering, integrated over small values of the additional quantum.

It is convenient to write this cross section as a 4-dimensional integral over the 4-momentum of the additional small quantum, employing a method analogous to that proposed by Abrikosov[11] for quantum electrodynamics in vacuo

$$d\sigma_D(\varkappa, \lambda) = d\sigma_0 \frac{e^2}{\pi i} \int_{(uq)^2 \leqslant \omega_m^2} \frac{4d^4q}{q^2+\varkappa(uq)^2-\lambda^2}$$

$$\times \left(\frac{g_l p_1}{(p_1+q)^2-(m+\Delta)^2} - \frac{g_l p_2}{(p_2+q)^2-(m+\Delta)^2}\right)^2, \tag{15}$$

where the integration is carried out over a region bounded by the condition that the frequency of the additional quantum in the system of the medium, uq, does not exceed the threshold of the experimental apparatus.

Denoting by $d\sigma_D(0, \lambda)$ the cross section for the double Compton effect in vacuo [integrated over the momentum of the small quantum, as in Eq. (15)], it is easy to bring Eq. (15) to a form analogous to Eq. (11).

$$d\sigma_D(\varkappa, \lambda) = d\sigma_D(0, \sqrt{a^2+\lambda^2})$$

$$+ \frac{4e^2}{\pi i} d\sigma_0 \int_{(uq)^2 \leqslant \omega_m^2} \frac{d^4q\,[(\mu-1)q^2-\mu a^2-(uq)^2\varkappa]}{(q^2+(uq)^2\varkappa-\lambda^2)(q^2-a^2-\lambda^2)}$$

$$\times \left\{\frac{e_l p_1}{q^2+2p_1q} - \frac{e_l p_2}{q^2+2p_2q}\right\}^2 - \frac{4e^2}{\pi i} d\sigma_0 \tag{16}$$

$$\times \int_{(uq)^2 \leqslant \omega_m^2} \frac{d^4q}{(q^2+(uq)^2\varkappa-\lambda^2)} \frac{\mu\varkappa}{(1+\varkappa)} \left\{\frac{up_1}{q^2+2p_1q} - \frac{up_2}{q^2+2p_2q}\right\}^2$$

[where, just as in Eq. (11), Δ is neglected].

Putting together Eqs. (11) and (16), one easily finds for the experimentally observable cross section for Compton scattering

$$d\sigma = d\sigma_{\text{vac}} - \frac{4e^2}{\pi i} d\sigma_0 \int_{(uq)^2 \geqslant \omega_m^2} \frac{d^4q\,[(\mu-1)q^2-\mu a^2-(uq)^2\varkappa]}{(q^2+(uq)^2\varkappa-\lambda^2)(q^2-a^2-\lambda^2)}$$

$$\times \left\{\frac{e_l p_1}{q^2+2p_1q} - \frac{e_l p_2}{q^2+2p_2q}\right\}^2 + \frac{4e^2}{\pi i} d\sigma_0 \tag{17}$$

$$\times \int_{(uq)^2 \geqslant \omega_m^2} \frac{d^4q}{q^2+(uq)^2\varkappa-\lambda^2} \frac{\mu\varkappa}{1+\varkappa} \left\{\frac{up_1}{q^2+2p_1q} - \frac{up_2}{q^2+2p_2q}\right\}^2,$$

in which the first term $d\sigma_{\text{vac}}$ does not depend on the properties of the medium, since in putting together $d\sigma_K(0, \lambda)$ and $d\sigma_D(0, \lambda)$, the terms depending on λ cancel[9] and, consequently, $d\sigma_{\text{vac}}$ is the experimentally observable cross section for the Compton effect in vacuo. The second and third terms are corrections, connected with the polarization of the medium. The region of integration in them is bounded by the condition $(uq)^2 \geq \omega_m^2$, so that the integrals remain finite in the infrared region. This makes it possible to set $\lambda = 0$ in the future.

It should be noted that in the expression obtained there are terms with different powers of q in the denominators of the integrands. This is explained by the fact that in the form given, it was more convenient to employ a technique of covariant integration after the substitution (13). After the integration over Q, terms of higher order in s/m naturally had to be thrown away, since terms of these orders were dropped earlier.

We cannot consider values of ω_m very close to the proper frequencies of the atoms of the medium, since absorption in the medium has not been taken into account. Therefore, in our case the threshold of sensitivity of the apparatus cannot be less than the atomic frequencies. In this case μ can be considered equal to unity, which makes it possible to obtain from Eq. (17) the expression

$$\frac{d\sigma - d\sigma_{\text{vac}}}{d\sigma_0} = \frac{8e^2}{\pi i} \int_{\omega_m}^{\infty} ds\,(a^2+\varkappa s^2)$$

$$\times \int \frac{d^4Q\,\delta(uQ)}{(Q^2+s^2(1+\varkappa))(Q^2+s^2-a^2)}$$

$$\times\left[\frac{e_l p_1}{Q^2+2p_1Q+2sup_1+s^2}-\frac{e_l p_2}{Q^2+2p_2Q+2sup_2+s^2}\right]^2$$

$$+\frac{8e^2}{\pi i}\int\limits_{\omega_m}^{\infty}\frac{\varkappa ds}{1+\varkappa}\int\frac{d^4Q\,\delta(uQ)}{Q^2+s^2(1+\varkappa)} \tag{18}$$

$$\times\left[\frac{up_1}{Q^2+2p_1Q+2sup_1+s^2}-\frac{up_2}{Q^2+2p_2Q+2sup_2+s^2}\right]^2$$

In the following, we consider the special case in which the electron is at rest in the medium before the collision. In this case it is possible to obtain the final formulas in an especially simple form. The momentum transfer will be equal to the momentum of the electron after the collision. Since a macroscopic description of properties of media is not possible for collisions with a large momentum transfer, we limit the calculation to only the first terms in an expansion in powers of the ratio of the momentum transfer to the mass. In this approximation, the second integral in Eqs. (17) and (18) is most important, containing the factor $\kappa/(1+\kappa)$ in the integrand. The final result can be conveniently written in the form of the ratio of the difference between the experimentally observable cross section of sixth power in e for Compton scattering in media and in vacuo to the cross section of fourth order in e for Compton scattering. In the first nonvanishing approximation in powers of the ratio ξ/m and q/ξ, where ξ is the momentum transfer and q is the 4-momentum of the virtual photon, one easily finds that

$$\frac{d\sigma-d\sigma_{\text{vac}}}{d\sigma_0}=\frac{e^2m}{2\pi\sqrt{2m}}\int\limits_{\omega_m}^{\infty}\frac{ds}{s^{3/2}}\frac{\varkappa(s)}{1+\varkappa(s)}\left\{1+y\left(1-2\ln\frac{2}{y}\right)\right\}, \tag{19}$$

where it is assumed that $y^2=\dfrac{2ms}{\xi^2}<1$.

5. DISCUSSION OF RESULTS

The applicability of the formulas obtained is determined by the following considerations. It is well known (see, for example, Ref. 12) that a phenomenological description of the surrounding medium is possible for those collisions in which the longitudinal momentum transfer is sufficiently small so that its inverse is larger than the interatomic distances in the medium. Thus, Eq. (19) remains valid in the region of sufficiently small photon-scattering angles satisfying the condition

$$2\sin\frac{\vartheta}{2}<N^{1/3}\sqrt{\frac{m}{\omega_1(\omega_1+m)}}=\left(\frac{\pi}{2}Ze^2\frac{(m+\omega_1)\,\omega_1^3}{m^2a^2}\right)^{-1/6}, \tag{20}$$

which relates to the case in which the electron is at rest before scattering. The notation $a^2 = 4\pi NZe^2/m$, where N is the number of atoms per unit volume and Z is the atomic charge, has been used.

This condition is of a general type, connected with a phenomenological description of media; besides this, the region of angles in which Eq. (19) is valid, is limited by the supplementary conditions under which (19) was derived. Since, for small scattering angles and for an initial photon energy comparable with rest mass energy of the electron, it is possible to consider $\xi^2 \approx \omega_1^2\vartheta^2$, then instead of Eq. (20), the region of angles in which Eq. (19) is applicable, is limited by the condition

$$\vartheta>\frac{m}{\omega_1}\sqrt{\frac{\omega_m}{m}}, \tag{20'}$$

which, for example, for liquid hydrogen at a density ~ 0.07 g-cm^{-3}, $\omega_m \sim 30$ ev, $\omega_1 \sim 0.5$ m, gives angles of the order of 1°.

To estimate the magnitude of the corrections connected with the presence of the medium, we consider a simple form of dependence $\kappa(s)$, obtained under the assumption that in the medium there is only one proper frequency.

$$\varkappa(s)=a^2(b^2-s^2)^{-1}. \tag{21}$$

Using the explicit form $\kappa(s)$ of Eq. (21), it is easy to obtain from Eq. (19)

$$\frac{d\sigma-d\sigma_{\text{vac}}}{d\sigma_0}=\frac{e^2}{2\pi\sqrt{2}}\,\frac{a^2}{a^2+b^2}\left(\frac{m}{\omega_m}\right)^{1/2}\Phi\left(\sqrt{\frac{\omega_m}{\sqrt{a^2+b^2}}}\right), \tag{22}$$

where $\Phi(z)$ is defined by

$$\Phi(z)=2-z\left[\tan^{-1}\frac{1}{z}+\frac{1}{2}\ln\left|\frac{1+z}{1-z}\right|\right].$$

In the case that ω_m is substantially larger than the atomic frequencies, the formula simplifies considerably:

$$\frac{d\sigma-d\sigma_{\text{vac}}}{d\sigma_0}=\frac{-e^2}{5\pi\sqrt{2}}\left(\frac{a^2}{\omega_m^2}\right)\sqrt{\frac{m}{\omega_m}}. \tag{23}$$

The result shows that in this case, for liquid hydrogen of density ~ 0.07 g-cm^{-3}, $a \sim 7$ ev, $b \sim 10$ ev, for $\omega_m \sim 30$ ev and $\xi > 5$ kev, we have from Eq. (23) $(d\sigma - d\sigma_{\text{vac}})/d\sigma_0 = -0.3\times10^{-2}$, i.e., the medium corrections are comparable with the radiative corrections. Particularly large corrections are obtained, as can be seen from Eq. (19), when ω_m is near to a zero of the function $\epsilon(\omega)$. When ω_m exactly coincides with a zero of $\epsilon(\omega)$ [in the particular case (21), when $\omega_m = (a^2+b^2)^{1/2}$] the expression $(d\sigma - d\sigma_{\text{vac}})/d\sigma_0$ becomes infinite. This divergence has no physical significance and is connected with the fact that in our approximation of infinitesimal absorption it is not possible to consider frequencies close to zeros and to poles of $\epsilon(\omega)$. However, this circumstance can be an indication that for ω_m close to zero the real part of

$\epsilon(\omega)$ and, for small absorption, the corrections connected with the medium, will be anomalously large.

In the experimental study of radiative corrections, it must be kept in mind that the accidental coincidence of ω_m with a zero of the real part of $\epsilon(\omega)$ can substantially distort the results for small scattering angles.

With diminishing momentum transfer ξ, i.e., with decreasing angle of scattering, the ratio $(d\sigma - d\sigma_{vac})/d\sigma_0$ decreases to zero, from which it follows that corrections of large magnitude coming from the medium can be expected only at certain scattering angles.

Thus, in the experimental measurement of the cross section for Compton scattering at small angles, it is necessary to take into account the possibility of a significant change in the differential cross section as a result of the influence of the medium.

In conclusion, I would like to use this opportunity to express deep gratitude to E. L. Feinberg for suggesting this problem and constant interest in this work, and M. L. Ter-Mikaelian for valuable discussion.

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ON THE THEORY OF THE THERMAL CONDUCTIVITY AND ABSORPTION OF SOUND IN FERROMAGNETIC DIELECTRICS

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The temperature dependence of the thermal conductivity and the coefficient of absorption of sound in ferromagnetic dielectrics is determined. It is shown that spin waves play the principal role in these processes at low temperatures.

1. As is well known, the kinetic properties of ordinary dielectrics are determined by the phonon spectrum. In ferromagnetic dielectrics the elementary excitations consist of spin waves in addition to phonons. It is therefore of interest to ascertain the role of the spin waves in thermal conduction and sound absorption in these substances.

We will show that at low temperatures the thermal conductivity in an unbounded ferromagnetic dielectric which contains no impurities is deter-