ANGULAR DISTRIBUTION OF INELASTICALLY SCATTERED DEUTERONS

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Inelastic scattering of deuterons on Mg^{24} and C^{12} nuclei is considered on the basis of the generalized nuclear model. The Mg^{24} nucleus is considered to be deformed, and the C^{12} nucleus to be spherical. Assuming that the rotational level is excited in the Mg^{24} nucleus upon collision with deuterons, and that a single phonon is excited in the C^{12} nucleus, the angular distributions of the scattered deuterons have been computed with account of the Coulomb interaction. The results are compared with experimental data.

1. Haffner[1] has carried out an experimental investigation of the angular distribution of inelastically scattered deuterons on some light nuclei. He compared the resultant distributions with the theoretical values.[2,3]

Huby and Newns[2] considered only the nuclear interaction between deuterons and the target nucleus; calculation of the angular distribution was carried out in a fashion similar to the calculation of the distribution of (dp) reactions in Ref. 4.

In the work of Mullin and Guth[3] on obtaining the angular distribution, only the electric interaction of the deuteron with the nucleus was taken into account. Comparison with experimental data, made by Haffner, shows that the theoretical computation of the angular distribution, taking into account only the nuclear interaction, gives a satisfactory agreement at large scattering angles, while the computation taking into account only the electric interaction, leads to satisfactory agreement for small scattering angles.

It is therefore of interest to consider the simultaneous effect of both interactions, with a view to obtaining better agreement of theory with experiment in all regions of angles. In the researches mentioned above, the wave function of the nucleus is considered unknown. As a consequence, within the framework of these researches, it is not possible to clarify the relative role of each of these two interactions.

Clearly, consideration of both interactions is possible only of the basis of a definite nuclear model, which allows us to establish excitation mechanism. As Rakavy has shown,[5] we can consider the lighter nuclei, with mass number 18 to 28, on the basis of a generalized nuclear model, in which the coupling of the subshell particles with the surface shell of O^{16} is strong. This means that one must consider these nuclei to be strongly deformed. Therefore, they must possess rotational levels, among which the levels of the even-even nuclei are known to possess the simplest property.

In the present research, we consider the inelastic scattering of deuterons on certain even-even nuclei on the basis of the generalized model of the nucleus.

We assume that in the collision of the deuteron with the nucleus, only collective degrees of freedom are excited as a consequence. In this situation, two cases are possible: (1) rotational and vibrational levels are excited, and (2) the excitation of the nucleus bears a phonon character. Existence of the first or second case depends on whether the original nucleus is deformed or not.

2. If we assume that the nucleon in the free state interacts with the surface of the nucleus in the same way as in the bound state, we can write for the interaction energy of the deuteron with the nucleus (in the case in which the initial state of the nucleus is deformed*)

$$H = \pm V_0 R_0 \delta(r_p' - R_0) \sum a_\nu Y_{2\nu}(\vartheta_p', \varphi_p')$$
$$\pm V_0 R_0 \delta(r_n' - R_0) \sum a_\nu Y_{2\nu}(\vartheta_n', \varphi_n') + V' + V(r), \qquad (1)$$

in the case in which the nucleus is initially undeformed,

*The presence of two signs in Eqs. (1) and (2) takes into account the possibility that the interaction of the free nucleon with the surface vibrations of the nucleus can have both an attractive and a repulsive character.

$$H = \pm V_0 R_0 \delta(r_p - R_0) \sum_\mu \alpha_\mu Y_{2\mu}(\vartheta_p, \varphi_p)$$

$$\pm V_0 R_0 \delta(r_n - R_0) \sum_\mu \alpha_\mu Y_{2\mu}(\vartheta_n, \varphi_n) + V' + V(r), \qquad (2)$$

where V_0 is the depth of the potential well, R_0 is the equilibrium nuclear radius, $(r_p, \vartheta_p, \varphi_p)$ and $(r_n, \vartheta_n, \varphi_n)$ are the coordinates of the components of the deuteron (the proton and the neutron), while the same coordinates, marked with primes in Eq. (1), denote the fact that they refer to a system of coordinates connected to the principal axes of the deformed nucleus. The quantities

$$a_2 = a_{-2} = \frac{\beta}{\sqrt{2}} \sin\gamma, \quad a_0 = \beta\cos\gamma \, (a_1 = a_{-1} = 0)$$

denote the parameters which determine the shape of the deformed nucleus possessing rotational and vibrational levels. The parameters α_μ must be regarded as the creation operators of the phonons. V' is the operator of electric interaction of the proton with the nucleus, which is equal to[3]

$$V' = 0 \qquad \text{for } r_p \leqslant r_0,$$

$$V' = \sum_{k=1}^{Z} e^2/|\mathbf{r}_p - \mathbf{R}_k| \qquad \text{for } r_p \geqslant r_0, \qquad (3)$$

where R_k are the radius vectors of the protons of the target nucleus, $V(r)$ is the radially symmetric part of the nuclear potential.

We expand the potential V' in multipoles:

$$V' = \sum V'_l, \qquad (4)$$

where

$$V'_l = \sum_{k=1}^{Z} \frac{e^2}{r_p}\left(\frac{R_k}{r_p}\right)^l P_l[\cos(\mathbf{r}_p, \mathbf{R}_k)]$$

$$= \frac{e^2}{r_p^{l+1}} \frac{4\pi}{2l+1} \sum_m Y_{lm}(\vartheta_p, \varphi_p) \sum_{k=1}^{Z} R_k^l Y^*_{lm}(\vartheta''_k, \varphi''_k).$$

We make use of the well-known relations between the parameters of collective motion in the nucleus nnd the variables of the individual particles[6]

$$\alpha_{lm} = \frac{4\pi}{3A} \sum_{k=1}^{A} \left(\frac{R_k}{R_0}\right)^l Y_{lm}(\vartheta''_k, \varphi''_k).$$

If we assume that the neutrons and protons produce identical deformation of the nucleus, then we can write:

$$V'_l = \frac{3Ze^2}{r_p^{l+1}} \frac{R_0^l}{2l+1} \sum \alpha_{lm} Y_{lm}(\vartheta_p, \varphi_p). \qquad (5)$$

In the nuclear interaction of nucleon (1) with (2), consideration is generally limited to surface deformations of second order. Therefore, in the electric interaction (5) also, one should limit oneself to the parameter $\alpha_{2m} \equiv \alpha_m$. Taking this into account and transforming to the coordinates connected with the deformed nucleus, we shall obtain

$$V'_2 = \frac{3Ze^2R_0^2}{5r_p^3} \sum_\mu \alpha_\mu Y_{2\mu}(\vartheta_p, \varphi_p)$$

$$= \frac{3Ze^2R_0^2}{5r_p^3} \sum_\nu a_\nu Y_{2\nu}(\vartheta'_p, \varphi'_p), \qquad (6)$$

where

$$\alpha_\mu = \sum_\nu a_\nu D^2_{\mu\nu}(\vartheta_i),$$

while $D^2_{\mu\nu}$ is the transformation matrix of the spherical harmonics $Y_{2\mu}$ and $\vartheta_i = (\vartheta_1, \vartheta_2, \vartheta_3)$ are the Eulerian angles.

Since the ground state of even-even nuclei is the state 0^+, while the interaction operators (1), (2), and (6) give transitions only with a change in moment by two units, then, after the process, the nucleus is in the state 2^+ which is the first excited level of even-even nuclei.

3. The matrix element of the process under consideration, corresponding to the interaction operators (1) and (6), has the form

$$H_{if} = \frac{1}{(2\pi\hbar)^{3/2}\sqrt{v}} \sum_\nu \Big\{ \pm V_0 R_0 \Big[\int \exp\left\{-i\frac{\mathbf{k}'}{2}(\mathbf{r}_p + \mathbf{r}_n)\right\}$$

$$\times \Phi^*(|\mathbf{r}_p - \mathbf{r}_n|)\, \psi_f^*(x)\, \delta(r'_p - R_0)\, a\, Y_{2\nu}(\vartheta'_p, \varphi'_p)$$

$$\times \exp\left\{i\frac{\mathbf{k}}{2}(\mathbf{r}_p + \mathbf{r}_n)\right\} \Phi(|\mathbf{r}_p - \mathbf{r}_n|)\, \psi_i(x)\, d\mathbf{r}_p d\mathbf{r}_n dx$$

$$+ \int \exp\left\{-\frac{i\mathbf{k}'}{2}(\mathbf{r}_p + \mathbf{r}_n)\right\}$$

$$\times \Phi^*(|\mathbf{r}_p - \mathbf{r}_n|)\, \psi_f^*(x)\, \delta(r'_n - R_0)\, a_\nu Y_{2\nu}(\vartheta'_n, \varphi'_n)$$

$$\times \exp\left\{i\frac{\mathbf{k}}{2}(\mathbf{r}_p + \mathbf{r}_n)\right\} \Phi(|\mathbf{r}_p - \mathbf{r}_n|)\, \psi_i(x)\, d\mathbf{r}_p d\mathbf{r}_n dx \Big]$$

$$+ \frac{3Ze^2R_0^2}{5} \int \exp\left\{-i\frac{\mathbf{k}'}{2}(\mathbf{r}_p + \mathbf{r}_n)\right\}$$

$$\times \Phi^*(|\mathbf{r}_p - \mathbf{r}_n|)\, \psi_f^*(x)\, a_\nu Y_{2\nu}(\vartheta'_p, \varphi'_p)$$

$$\times \exp\left\{i\frac{\mathbf{k}}{2}(\mathbf{r}_p + \mathbf{r}_n)\right\} \Phi(|\mathbf{r}_p - \mathbf{r}_n|)\, \psi_i(x)\, \frac{dr_p}{r_p^3}\, d\mathbf{r}_n dx \Big\}. \qquad (7)$$

In the case of interaction operators (2) and (6), we have for the matrix element of the transition

$$H_{if}=\frac{1}{(2\pi\hbar)^{3/2}V\overline{v}}\sum_{\mu}(0\,|\,\alpha_\mu\,|\,1)$$

$$\times\Big\{\pm V_0R_0\Big[\int\exp\Big\{-i\frac{\mathbf{k}'}{2}(\mathbf{r}_p+\mathbf{r}_n)\Big\}\Phi^*(|\mathbf{r}_p-\mathbf{r}_n|)$$

$$\times\delta(r_p-R_0)\,Y_{2\mu}(\vartheta_p,\varphi_p)$$

$$\times\exp\Big\{i\frac{\mathbf{k}}{2}(\mathbf{r}_p+\mathbf{r}_n)\Big\}\Phi(|\mathbf{r}_p-\mathbf{r}_n|)\,d\mathbf{r}_p d\mathbf{r}_n$$

$$+\int\exp\Big\{-\frac{i\,\mathbf{k}'}{2}(\mathbf{r}_p+\mathbf{r}_n)\Big\}\Phi^*(|\mathbf{r}_p-\mathbf{r}_n|)$$

$$\times\delta(r_n-R_0)\,Y_{2\mu}(\vartheta_n,\varphi_n)$$

$$\times\exp\Big\{i\frac{\mathbf{k}}{2}(\mathbf{r}_p+\mathbf{r}_n)\Big\}\Phi(|\mathbf{r}_p-\mathbf{r}_n|)\,d\mathbf{r}_p d\mathbf{r}_n\Big]$$

$$+\frac{3Ze^2R_0^2}{5}\int\exp\Big\{-i\frac{\mathbf{k}'}{2}(\mathbf{r}_p+\mathbf{r}_n)\Big\}$$

$$\times\Phi^*(|\mathbf{r}_p-\mathbf{r}_n|)\,Y_{2\mu}(\vartheta_p,\varphi_p)\exp\Big\{i\frac{\mathbf{k}}{2}(\mathbf{r}_p+\mathbf{r}_n)\Big\}\frac{d\mathbf{r}_p}{r_p^3}d\mathbf{r}_n\Big\},\qquad(8)$$

where $\Phi(r)=\sqrt{\alpha/2\pi}\;e^{-\alpha r}/r$ is the wave function of the interior state of the deuteron, **k** and **k′** are the wave vectors of the center of mass of the deuteron before and after scattering, $\psi_f(x)$ and $\psi_i(x)$ are the wave functions of the deformed nucleus in the excited and ground states, respectively. These functions have the form

$$\psi_f=\frac{1}{\sqrt{2\pi}}Y_{IM}(\vartheta,\varphi)\cdot\varphi_{n'_\beta n'_\gamma}(\beta,\gamma),$$

$$\psi_i=\frac{1}{\sqrt{2\pi}}Y_{00}(\vartheta,\varphi)\,\varphi_{n\,n_\gamma}(\beta,\gamma),\qquad(9)$$

where I is the spin of the nucleus in the excited state, M is its projection on a fixed axis, and φ is the wave function of the vibrational state of the surface of the nucleus.

The quantity $(0|\alpha_\mu|1)$ in Eq. (8) represents the matrix element of the creation of a single phonon. It is equal to[6]

$$(0\,|\,\alpha_\mu\,|\,1)=\sqrt{\hbar\omega/2C},\qquad(10)$$

where C is a coefficient characterizing the deformability of the nucleus, $\hbar\omega$ is the energy of the phonon.

Expanding the plane waves in (8) in the form of spherical waves in the fixed system, and in (7) in a system connected with the principal axes of deformation of the nucleus, we get for the matrix elements (after integration):

$$H_{if}=-\frac{4V_0R_0^3\sqrt{5\pi}}{\sqrt{\overline{v}}\,(2\pi\hbar)^{3/2}}\sqrt{\frac{\hbar\omega}{2C}}\Big[\frac{4\alpha}{q}\tan^{-1}\frac{q}{4\alpha}\Big]$$

$$\times\Big[\pm J_2(qR_0)+\frac{0.3Ze^2}{V_0R_0}\frac{J_1(qr_0)}{qr_0}\Big]\qquad(11)$$

and

$$H_{if}=-\frac{4V_0R_0^3\sqrt{5\pi}}{\sqrt{\overline{v}}\,(2\pi\hbar)^{3/2}}\Big[\frac{4\alpha}{q}\tan^{-1}\frac{q}{4\alpha}\Big]\Big[\pm J_2(qR_0)$$

$$+\frac{0.3Ze^2}{R_0V_0}\frac{J_1(qr_0)}{qr_0}\Big]\sum\int\psi_f^*a_\nu D_{\nu0}^2(\vartheta\varphi\psi)\,\psi_i d\tau,\qquad(12)$$

where J_1 and J_2 are the spherical Bessel functions, $q=|\mathbf{k}-\mathbf{k}'|$. Making use of the known expression for $D^2_{\nu0}$, we have

$$A=\sum_\nu\int\psi_f^*a_\nu D_{\nu0}^2\psi_i d\tau=(n_\beta n_\gamma\,|\,a_M\,|\,n'_\beta n'_\gamma)\,\delta_{I2}/\sqrt{5}.\qquad(13)$$

If we consider the case in which only rotational levels are excited in the deformed nucleus, then we must set $n'_\beta=n'_\beta$ and $n'_\gamma=n'_\gamma$ in Eq. (13). Furthermore, we must expect that the mean values of the parameters a_M will differ slightly from their equilibrium values. Therefore, for the sake of simplicity in our case, we can in Eq. (13) take $\beta=\beta_1$ and $\gamma=0,\pi$, where β_1 is the equilibrium value of β. As a result, we obtain

$$A=\pm\beta_1\delta_{I2}\delta_{M0}/\sqrt{5},\qquad(14)$$

since $a_2=a_{-2}=0$ and $a_0=\pm\beta$ here.

4. For the differential cross section of the process under study, we get (for the case in which the initial nucleus is assumed to be deformed):

$$\Big(\frac{d\sigma}{d\Omega}\Big)_1=\frac{20\mu^2(V_0R_0^2)^2}{\pi\hbar^4}R_0^2\sqrt{1+\frac{Q}{E_0'}}\,\frac{\beta_1^2}{5}\Big[\frac{4\alpha}{q}\tan^{-1}\frac{q}{4\alpha}\Big]^2$$

$$\times\Big[\pm J_2(qR_0)+\frac{0.3Ze^2}{R_0V_0}\frac{J_1(qr_0)}{qr_0}\Big]^2;\qquad(15)$$

in the case in which the deformation of the initial nucleus is neglected,

$$\Big(\frac{d\sigma}{d\Omega}\Big)_2=\frac{5\hbar\omega}{2C\beta_1^2}\Big(\frac{d\sigma}{d\Omega}\Big)_1,\qquad(16)$$

where μ is the reduced mass of the system, and E_0 is the energy of the incident deuteron. The sign $\pm$ in front of J_2 is determined by the sign in the expression for the interaction (1) and (2).

5. We compare these theoretical distributions with the experimental data for the Mg^{24} and C^{12} nuclei. We first consider the Mg^{24} nucleus. In accord with Ref. 5, this nucleus must be regarded as deformed. Since its energy of creation is comparatively small (1.37 Mev), it is natural to assume that in collisions with deuterons, only rotational states are excited in them. This allows us to apply Eq. (15) to the Mg^{24} nucleus. Before carcying out a comparison of the angular distribution with experimental data, we shall show in what fashion it is possible to find the values of the parameters β_1, R_0, r_0, and V_0 which enter into Eq. (15).

In the expression for the energy of rotation of even-even nuclei,[6]

$$E_I = (\hbar^2/2J)\, I\,(I+1), \tag{17}$$

where $J = 3B\beta_1^2$ is the effective moment of inertia, $I = 2$, while $B = (3/8\,\pi)\,MAR_0^2$ (M is the mass of the nucleon and A the mass number), we replace $E_I = E_2$ by the experimentally observed value of the excitation energy 1.37 Mev. At the same time we find the connection between the parameters β_1 and R_0. For a determination of each of these parameters separately, we must also have a second relation between them. As a second relation we can take the express which defines the quadrupole moment of the nucleus[6]

$$Q_0 = \pm (3/\sqrt{5\pi})\, ZR_0^2\beta_1. \tag{18}$$

Making use of the experimental value of the quadrupole moment of the Mg^{24} nucleus ($Q_0 = 0.66 \times 10^{-24}\,cm^2$), and taking it into account that Eq. (18) usually gives values approximately twice as great as the experimental, we can in a rough way determine R_0 and β_1 from the two relations given above. As a result, we get $R_0 = 4 \times 10^{-13}$ cm and $\beta_1 = 0.77$. The value of the parameter r_0 can be estimated in the following way. Inasmuch as the choice of the electrical interaction in the form (3) denotes that it is essentially small in the region occupied by the nucleus, then it is natural to set $r_0 \approx R_0 + \delta R$, where δR is the maximum change in the linear dimensions of the nucleus, produced by the deformations. According to Bohr and Mottelson,[6] it is equal to

$$\delta R = \sqrt{5/4\pi}\,\beta_1 R_0 \tag{19}$$

in our case ($\gamma = 0, \pi$). As a result, we get $r_0 = 5.9 \times 10^{-13}$ cm for the parameter r_0 for the Mg^{24} nucleus. In so far as the depth V_0 of the potential well is concerned, we can determine it from the condition that the value of the principal maximum in the angular distribution is determined only by the nuclear interaction. If we employ this assumption and make use of the experimental value of the principal maximum, we obtain $V_0 = 1.84$ Mev. It is easy to see that in our choice of the parameters, the electric interaction plays a role comparable with the nucleus interaction.

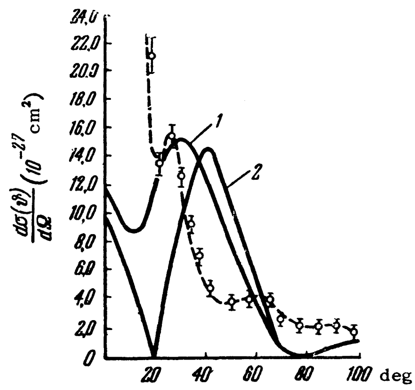

FIG. 1

The angular distribution obtained on the basis of Eq. (15) is shown in Fig. 1, where Curve 2 corresponds to the minus sign in (15) and Curve 1 to the plus sign. The rapid fall-off of Curve 2 at small angles is caused by the electrical term in the interaction, while the principal maximum and the further development of the curve is due to the nuclear term. The principal maximum on Curve 2 is displaced by about 10° relative to the position of the experimental maximum.

Curve 2 possesses a minimum at small angles, while at this same value of the angle, the experimental curve also has a minimum; however, in contrast to the experimental case, the theoretical value of the minimum is zero.

Better agreement with experiment is obtained when we take the plus sign in (15). In this case, the position of the principal maximum coincides with the one observed experimentally. Moreover, in agreement with experiment, the minimum of the theoretical curve does not lie on the axis, although it is displaced somewhat in the direction of smaller angles in comparison with the position of the experimental minimum.

We can apply Eq. (16) to the C^{12} nucleus which, before the reaction, is not deformed. Inasmuch as the excitation energy is comparatively small (4.43 Mev), we consider that a single phonon excitation takes place in the collision with the deuteron. For an estimate of the parameter C entering into Eq. (16), we make use of the well-known formula[6]

$$C = 4R_0^2 S - 3Z^2e^2/10\pi R_0, \tag{20}$$

where the surface tension of the nucleus S is determined from the relation[7]

$$4\pi S R_0^2 = 15.4 A^{2/3}\,\text{Mev}. \tag{21}$$

It should be noted that this way of determining the parameter C is very crude in its application to the C^{12} nucleus, since the formulas employed hold

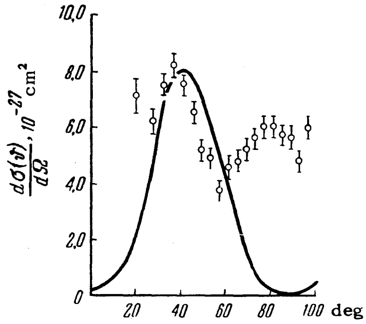

FIG. 2

for heavy nuclei. Inasmuch as the C^{12} nucleus is not considered to be deformed, we have no relation between r_0 and R_0, in contrast to the Mg^{24} case. Therefore we use the rough values $R_0 = 4 \times 10^{-13}$ cm and $r_0 = 6 \times 10^{-13}$ cm. In such a case, C = 24.4 Mev.

If we assume here too that the principal maximum is connected with the nuclear interaction, we obtain $V_0 = 5.41$ Mev. For the values of the parameters that we have chosen, it is seen that the electrical interaction plays almost no role in the angular distribution. The angular distribution obtained on the basis of Eq. (16) is shown by the solid curve in Fig. 2. As we see, the theoretical curve does not have a minimum and disagrees, in many respects, with the experimental data. It agrees with experiment only in relation to the presence of the principal maximum.

It is possible that this nonconformity is produced by our incorrect assumption that, in the process considered by us, a single phonon excitation arises in the C^{12} nucleus. Nor is it excluded that, in such light nuclei as C^{12},, the generalized model is generally non-applicable.

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DISPERSION OF LIGHT IN THE EXCITON ABSORPTION REGION OF CRYSTALS

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The theory of light waves in exciton-absorbing crystals, developed in Ref. 1 on the basis of a new relation between specific polarization and the electric field, is applied to cubic crystals. For each direction of propagation, the existence of three types of light waves is predicted in these crystals. One of these types is similar to ordinary waves, whereas the other two are essentially anomalous. The frequency dependence of the refractive indices of the three types is considered. Fresnel's formulas are generalized for light passing through the boundary between the crystal and a vacuum. New formulas are obtained for the coefficient of reflection from the crystal surface and for the transparency of a plane parallel plate. Methods are suggested for experimental testing of the theory and for obtaining a direct proof of the existence of second and third light in cubic crystals.

THE present article is an immediate continuation of Ref. 1, in which it was shown that in the region of exciton absorption of light the specific dipole moment of dielectric polarization and the electric field are related through a differential equation rather than by a simple linear algebraic expres-