[1] D. A. Kirzhnits, J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 115 (1957), Soviet Phys. JETP **5**, 60 (1957).

[2] D. A. Kirzhnits, Thesis, Physics Institute, Acad. Sci. (U.S.S.R.), 1956.

[3] S. Golden, Phys. Rev. **105**, 604 (1957).

[4] P. Gombas, Statistical Theory of the Atom (Russian Translation) I L, M., 1951.

[5] A. S. Kompaneets and E. S. Pavlovskii, J. Exptl. Theoret. Phys. (U.S.S.R.) **31**, 427 (1956), Soviet Phys. JETP **4**, 328 (1957).

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REMARKS ON A NOTE BY F. S. LOS' "PHASE OF A SCATTERED WAVE"[1]

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LOS' has considered the equation

$$\frac{d^2G}{d\rho^2} + \left[1 - \frac{l(l+1)}{\rho^2} - U(\rho)\right]G = 0 \qquad (1)$$

and asserts that "it is necessary to find a solution of Eq. (1) which assumes the following form as $\rho \to 0$

$$G = A_0\rho^{l+1} \qquad (2)$$

and the following asymptotic form at large values of ρ

$$G = \mathrm{const}\cdot\sin(\rho - \pi l/2 + \delta_l); \quad \delta_l = \mathrm{const}\ldots» \qquad (3)$$

It should be noted that under the condition $\int^{\infty} U(\rho)\,d\rho < C$, given in the author's note, when $\rho \to \infty$, $G(\rho)$ cannot assume the asymptotic form given in (3). The case $\int^{\infty} |U(\rho)|\,d\rho < C$ is considered in Courant and Hilbert[2] where the same system of equations is obtained, using the method employed by Los'. Equations (6) and (7) given by Los' correspond to Eq. (56) and (57) in Courant and Hilbert.

In Ref. 2, a proof is given for the asymptotic formula (3) which is more detailed than that given in the note by Los'; in this proof it is not shown that $A(\rho) \to$ const. when $\rho \to \infty$ in Eq. (4).

For $\rho \to 0$, Los' has obtained expressions for $\delta(\rho)$ and $A(\rho)$ to which the following remarks apply:

1. It is not meaningful to express $\delta(\rho)$ in terms of $\int_0^{\rho} \rho\gamma(\rho)\,d\rho$ since in the approximation which is used only the first terms of the series $\gamma(\rho) = \gamma_0 + \gamma_1\rho + \ldots$ should be retained
2. The constant coefficients in (2) and in the expression for $A(\rho)$ are denoted by the same symbol — A_0. These coefficients differ by a factor of $l + 1$.
3. Even if these errors are ignored, it should be noted that the title of the note does not reflect its contents.

The use of the supplementary condition (5) is valid for $\rho \to \infty$, when $A(\rho) \to$ const and $\delta(\rho) \to$ const. However, when $\rho \to 0$, in place of (5) any other supplementary condition can be used. The different supplementary conditions correspond to different phases $\delta(\rho)$. The problem becomes indeterminant. By the definition of the phase of a scattered wave it follows that the phase $\delta(\rho)$ found by means of such an additional condition is not the phase of the scattered wave.

Thus, in the form in which it has been published the note given by F. S. Los' is not useful for an analysis of scattering and can only introduce confusion on the part of the reader.

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[1] F. S. Los', J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 273 (1957).

[2] Courant and Hilbert, Methoden der Mathematischen Physik, (Russ. Transl.) 1933, Vol. I, p. 314.

Translated by H. Lashinsky
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SECOND MOMENT OF THE PARAMAGNETIC ABSORPTION CURVE WHEN THE SPIN MAGNETISM IS NOT PURE

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IN Refs. 1 — 5, equations were given for the second moment $\langle\nu^2\rangle$ of the curve $f(\nu)$ of paramagnetic absorption in the absence of a static magnetic H_0, and for the second moment $\langle(\Delta\nu)^2\rangle$ of the curve $\psi(\nu)$ of paramagnetic resonance (ν is the frequency of the variable magnetic field H_t directed along the t-axis). These equations are, however, valid only in the case of absorption in magneto-anisotropic crystals, where the g factor depends strongly on the direction of H_0. The reason for this phenomenon is that when the orbital motion of the electrons is not fully quenched with respect to the intercrystalline electrical field E, the latter changes strongly the character of the internal interactions in the paramagnetic.[6]

We have derived equations for $\langle\nu^2\rangle$ and $\langle(\Delta\nu)^2\rangle$ in magneto-anisotropic crystals which contain only magnetically equivalent ions. We have assumed that for $H_0 = 0$ the energy ground state of magnetic ions possesses a two-fold Kramers degeneracy and that only this doublet is populated. These conditions are, for instance, realized in double nitrates and ethylsulphates of the rare-earth elements at helium temperatures. In our calculations we have found the dependence of $\langle\nu^2\rangle$ and $\langle(\Delta\nu)^2\rangle$ on the direction of H_0 and H_t with respect to the crystal axis, on the spin temperature T, the anisotropic exchange interaction $\mathcal{H}_{\text{exch}}$, the dipole-dipole interaction $\mathcal{H}_{\text{d}}$, and the hyperfine structure interaction $\mathcal{H}_{\text{hfs}}$. Our final equations are expressed in terms of the coefficients of the spin Hamiltonian. This circumstance makes it much easier to evaluate the moments of the $\psi(\nu)$ curves and makes it possible to study the internal interactions in paramagnetics through the plentiful material accumulated recently by paramagnetic resonance methods.[7]

The following is the final expression for

$$\langle(\Delta\nu)^2\rangle = \langle(\Delta\nu)^2\rangle_{\text{d}} + \langle(\Delta\nu)^2\rangle_{\text{exch}} + \langle(\Delta\nu)^2\rangle_{\text{hfs}} + \langle(\Delta\nu)^2\rangle_{\text{d-exch}} \tag{1}$$

for the case where $H_0 \parallel c$ and $H_t \parallel a$, where a, b, and c are the principal axes of the g-tensor; the indices "d", "exch", "hfs", and "d-exch" indicate the contributions to $\langle(\Delta\nu)^2\rangle$ from $\mathcal{H}_{\text{d}}$, $\mathcal{H}_{\text{exch}}$, $\mathcal{H}_{\text{hfs}}$ and the dipole-exchange interactions, respectively:

$$\langle(\Delta\nu)^2\rangle_{\text{d}} = (18Ph^2)^{-1} \sum_{I(\neq j)} \{(B_{aa}^{Ij} + B_{bb}^{Ij})^2 K + B_{cc}^{Ij2} D - B_{cc}^{Ij}(B_{aa}^{Ij} + B_{bb}^{Ij}) L\}, \tag{2}$$

$$\langle(\Delta\nu)^2\rangle_{\text{exch}} = 2(Ph^2)^{-1} \sum_{I(\neq j)} \{(J_a^{Ij} + J_b^{Ij})^2 K + J_c^{Ij2} D - J_c^{Ij}(J_a^{Ij} + J_b^{Ij}) L\}, \tag{3}$$

$$\langle(\Delta\nu)^2\rangle_{\text{d-exch}} = \left(\frac{3}{2}Ph^2\right)^{-1} \sum_{I(\neq j)} \{(B_{aa}^{Ij} + B_{bb}^{Ij})(J_a^{Ij} + J_b^{Ij}) K + B_{cc}^{Ij} J_c^{Ij} D - 0{,}5\,[B_{cc}^{Ij}(J_a^{Ij} + J_b^{Ij}) + J_c^{Ij}(B_{aa}^{Ij} + B_{bb}^{Ij})]\, L\}, \tag{4}$$

$$\langle(\Delta\nu)^2\rangle_{\text{hfs}} = (3h^2)^{-1} I(I+1) \sum\nolimits_{\nu} A_{c\nu}^2, \tag{5}$$

where we have used the notation

$$P = 1 + e^m,\ K = e^m,\ L = 1 + 3e^m,\ m = -g_{cc}\beta H / kT,$$

$$D = 1{,}5 + 2e^m + 0.5\,e^{2m},$$

$$J_a^{Ij} = -J^{Ij}B^2, \quad J_b^{Ij} = -J^{Ij}(B^2\cos^2\varphi + F^2\sin^2\varphi),$$

$$J_c^{Ij} = -J^{Ij}(B^2\sin^2\varphi + F^2\cos^2\varphi),$$

$$B_{\alpha\alpha}^{Ij} = -3g_{\alpha\alpha}^2\beta^2 r^{Ij\,(-3)}\left\{\frac{3}{2}\gamma_\alpha^{Ij2} - \frac{1}{2}\right\},$$

$$B^2 = \langle + - | 2S_\xi^I S_\xi^j | - + \rangle, \quad F^2 = \langle + + | 2S_\varepsilon^I S_\varepsilon^j | + + \rangle.$$

The superscripts refer here to the particles and the subscripts to the coordinate axes, h· is Planck's constant, k Boltzmann's constant, β the Bohr magneton, $A_{c\nu}$ the components of the tensor of the hyperfinestructure spin interaction of the nucleus with the electrons inside the paramagnetic ion,[8] $g_{\alpha\alpha}$ the components of the g tensor along the α axis, r^{Ij} the radius vector from ion I to ion j, γ^{Ij} the cosine of the angle between r^{Ij} and the α axis (see Ref. 2), ν an index which takes on the values of the principal axes of the tensor $A_{c\nu}$. In deriving Eqs. (3) and (4) it was assumed that the field E has an axis of symmetry ϵ which is a principal axis of the g tensor, like η and ξ. In these equations γ^{Ij} are the usual exchange integrals,[2] $|+\rangle$ and $|-\rangle$ are the correct zeroth order wave functions for the Kramers doublet for the case $H \parallel \epsilon$, φ is the angle between ϵ and H_0, and S is the effective electron spin of the paramagnetic ion. If we put $T = \infty$, $g_{\alpha\alpha} = \text{const}$ in Eq. (2), we obtain the well-known Eq. (11) of Ref. 2 for spin $\frac{1}{2}$. From Eq. (5) we get for $\nu = \alpha$ Eq. (28) of Ref. 4. For $T = \infty$, $g_{\alpha\alpha} = \text{const}$, the contributions (3) and (4) of the exchange interactions vanish, which also agrees with previously obtained results.[2] The lattice sums, which take into account more than 99% of the contribution of all neighbors of the paramagnetic ion in the crystal lattice of the ethyl sulfates of the rare-earth elements, have the values (the symbol $\sum\limits_{I(\neq j)}$ is omitted),

$$B_{\varepsilon\varepsilon}^{Ij2} = 1201 g_{\varepsilon\varepsilon}^4 d, \quad B_{\eta\eta}^{Ij2} = 514 g_{\eta\eta}^4 d, \quad B_{\xi\xi}^{Ij2} = 508\, g_{\xi\xi}^4 d,$$

$$B_{\xi\xi}^{Ij} B_{\eta\eta}^{Ij} = 88 g_{\xi\xi}^2 g_{\eta\eta}^2 d, \quad B_{\eta\eta}^{Ij} B_{\varepsilon\varepsilon}^{Ij} = -603\, g_{\eta\eta}^2 g_{\varepsilon\varepsilon}^2 d,$$

$$B_{\xi\xi}^{Ij} B_{\varepsilon\varepsilon}^{Ij} = -597\, g_{\xi\xi}^2 g_{\varepsilon\varepsilon}^2 d,$$

where the η axis is directed to the nearest neighbor of the paramagnetic ion in the $\xi\eta$ plane which is perpendicular to ϵ, where $d = \beta^4 a^{-6}$, and where a is the largest direction in the elementary cell.[6] For double nitrates of the rare-earth elements[6] $B_{\epsilon\epsilon}^{Ij2} = 32 g_{\epsilon\epsilon}^4 d$ and for $H_0 \parallel \epsilon$ we have

$$\langle (\Delta\nu)^2 \rangle_d = (18\, Ph^2)^{-1} \sum_{I(\neq j)} \{ e^m (x^4 + 3x^2 + 2) + 0.5\, e^{2m} + 1.5 + x^2 \} B_{\varepsilon\varepsilon}^{Ij2}, \quad x = g_\perp / g_\parallel, \tag{6}$$

where $g_\perp$ and $g_\parallel$ are the factors of the spectroscopic splitting.

For ethyl sulfates of the rare-earth elements, the term arising from $\mathcal{H}_d$ has the form

$$\langle \nu^2 \rangle_d = 4a^{-6} c \{ 229\, g_\perp^4 + \cos^2\psi\, (g_\parallel^2 / g^2)(553\, g_\perp^4 + 147\, g_\perp^2 g_\parallel^2) + \sin^2\psi\, (g_\perp^2 / g^2)(101\, g_\perp^4 + 534\, g_\parallel^4 + 871\, g_\parallel^2 g_\perp^2) \},$$
$$c = \beta^4 / 16h^2, \quad g^2 = g_\parallel^2 \cos^2\psi + g_\perp^2 \sin^2\psi, \tag{7}$$

where ψ is the angle between H_t and ϵ.

In conclusion, the author expresses his thanks to Professor S. A. Al' tshuler for suggesting this topic and considering this paper.

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[1] L. J. F. Broer, Physica, **10**, 801 (1943).

[2] J. H. Van Vleck, Phys. Rev. **74**, 1168 (1948).

[3] K. Kambe and J. F. Ollom, J. Phys. Soc. Japan, **11**, 50 (1956).

[4] J. S. van Wieringen, Disc. Faraday Soc. **19**, 118 (1955).

[5] G. Ia. Glebashev, Уч. зап. Казанского гос. ун-та (Scient. Notes, Kazan' State Univ.) **116**, book 1, 121 (1956).

[6] J. M. Daniels, Proc. Phys. Soc. (London) **A66**, 673 (1953).

[7] K. D. Bowers and J. Owen, Repts. Progr. Phys. **18**, 304 (1955).

[8] B. Bleaney and M. C. M. O'Brien, Proc. Phys. Soc. (London) **B69**, 1216 (1956).

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THERMAL CONDUCTION OF SUPERCONDUCTORS

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THANKS to the presence of a gap in the energy spectrum of superconductors,[1] the number of electronic excitations and hence the electronic thermal conductivity is exponentially small $[\sim \exp(-T_k/T)]$ for $T \ll T_k$ (T_k is the temperature of the transition to the non-superconducting state). For $T \ll T_k$ the principal role is therefore played by the lattice thermal conduction, which is connected with the reflection of phonons from boundaries and the scattering of phonons by impurities and lattice defects (the phonon-electron interaction is inappreciable in view of the fact that the number of electronic excitations is very small for $T \ll T_k$), while it is well-known[2] that phonon-phonon interaction does not play a role for $T \ll \Theta_D$, where Θ_D is the Debye temperature).

However, for somewhat higher temperatures, but still appreciably less than T_k, the electronic heat conduction κ_e becomes comparable with the lattice heat conduction and can even exceed it for not very impure specimens. Clearly the largest contribution to κ_e is then given by the scattering of the electrons by impurities. Only for $T \lesssim T_k$ can the interaction of the electrons with the phonons and with one another also play an appreciable role for κ_e.

We consider the scattering of electrons by impurities. Let the Hamiltonian of the interaction of the electrons with the impurity atoms for the normal metal be of the form

$$H' = \sum_{\mathbf{k}} (a_{\mathbf{k},1/2}^{+} a_{\mathbf{k}',1/2} + a_{\mathbf{k},-1/2}^{+} a_{\mathbf{k}',-1/2}) V_{\mathbf{k},\mathbf{k}'}$$

($\frac{1}{2}$ and $-\frac{1}{2}$ are the spin coordinates, and $a_{\mathbf{k},\pm\frac{1}{2}}$ the amplitude in second quantization). According to Ref. 3 the electronic excitations in superconductors can be described by new amplitudes

$$\alpha_{\mathbf{k}0} = u_k a_{\mathbf{k},1/2} - v_k a_{-\mathbf{k},-1/2}^{+}; \quad \alpha_{\mathbf{k}1} = u_k a_{-\mathbf{k},-1/2} + v_k a_{\mathbf{k},1/2}^{+};$$

$$\left.\begin{matrix} u_k^2 \\ v_k^2 \end{matrix}\right\} = \frac{1}{2}\left(1 \pm \xi / \sqrt{\Delta^2(T) + \xi^2}\right); \tag{1}$$

$\xi = (p^2 - p_0^2)/2m \approx v_0(p - p_0)$ is the energy of a