SOUND EXCITATIONS IN FERMI SYSTEMS

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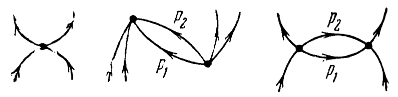
It is well-known that the propagation of sound waves in a degenerate perfect Fermi gas is impossible. The formally evaluated sound velocity ($c^2 = \partial P/\partial\rho = p_0^2/3m^2$) is less than the particle velocity at the Fermi surface which means the possibility of the decay of a sound quantum into a particle and a hole in the Fermi sea. This result remains the same for a Fermi gas with weak repulsions between the particles, since the spectrum in that case has practically the previous character.[1,2] The position changes completely in the case of a Fermi system with attractions. The production of bound pairs of particles on the Fermi surface leads to the appearance of a gap in the spectrum of the one-particle excitations.[3,4] Sound quanta with an energy not exceeding the value of the gap can therefore not decay. It is thus possible in Fermi systems with attractions to have sound excitations with small momenta.

To discuss these excitations it is convenient to use the Green function method. It is well known that sound waves arise from the excitation of particles in the condensed Bose system,[5,6] i.e., in our case from the motion of bound pairs. In that way sound excitations can be considered to be bound states of two elementary excitations with a total momentum different from zero. For our calculation we can thus use the method proposed in Gell-Mann and Low's paper[7] (see also Ref. 8). According to this method, the equation for the bound states is obtained by discarding from the equation for the Green function of two excitations the inhomogeneity (which does not have the frequencies corresponding to the bound states).

To take into account the reshuffling due to the production of a condensed Bose-system of bound pairs, we must transform the original Hamiltonian with direct interactions between the particles, using Bogoliubov's method.[4] In that way we get

$$H = E_0 + H_0 + H', \qquad H_0 = \sum_{\mathbf{p}} \varepsilon(p)\,(\alpha^+_{p0}\alpha_{p0} + \alpha^+_{p1}\alpha_{p1}),$$
$$\varepsilon(p) = {}^1/_2\sqrt{\Delta^2 + (p^2 - p_0^2)^2}\,. \tag{1}$$

Here p_0 is the limiting Fermi momentum, $\Delta = \overline{\omega}e^{-1/\rho}$ is the value of the energy gap, and H' is the Hamiltonian of the interaction between the excitations. Since the result does not depend on the form of the interaction, we took the interaction between the particles in the original Hamiltonian in the most convenient form, analogous to the interactions of electrons in a metal. The Green function of the interaction is constructed in the usual manner from the operators $\alpha_{\mathbf{p}0}$ and $\alpha_{\mathbf{p}1}$. For our purpose it is sufficient to take the zeroth approximation for these functions. The interaction Hamiltonian H' contains in first order only one interaction graph between excitations (graph a).



a b c

It is easily seen that nothing is added by a repetition of this graph, since the integration over momentum in the intermediate state, for the case of small total momenta of interest to us, yields $\ln(\overline{\omega}/\Delta)$, cancelling the smallness of the interaction constant. In the second order, graphs b and c also enter. It is easily seen that graph b is of the same order of magnitude as graph a. Indeed, the total momentum of excitations p_1 and p_2 is fixed and equal to the momentum of the excited state k. The integration over these momenta leads thus again to a logarithm that cancels the smallness of the interaction. In contradistinction, in graph c, in which the total momentum of the excitations p_1 and p_2 is not given, the large total momenta are significant and the compensating logarithm is absent. In this way the Green function of two excitations, K, is determined by an infinite sequence of graphs a and b. This sequence is similar to the set of graphs for the one-particle Green function of a Bose gas.[9] Introducing along with the function K the function $\overline{K}$,

$$\overline{K} = \langle T\,\{\alpha^+_{p_10}\alpha^+_{p_21}\alpha^+_{p_30}\alpha^+_{p_41}\}\rangle,$$

one can construct a set of equations for these functions, similar to the set (5.2) of Beliaev's paper.[9] Dropping the inhomogeneity we go over to the following set of equations for the functions

$$\chi_{\mathbf{k},\omega}(\mathbf{p}) = (\alpha_{p+k/2,0}\,\alpha_{p-k/2,1})_{0s}, \quad \varphi_{\mathbf{k},\omega}(\mathbf{p}) = (\alpha^+_{p-k/2,0}\,\alpha^+_{p+k/2,1})_{0s},$$

$$\chi_{\mathbf{k}\omega}(\mathbf{p}) = \frac{1}{\varepsilon(\mathbf{p}+\mathbf{k}/2) + \varepsilon(\mathbf{p}-\mathbf{k}/2) - \omega}\left\{\int d\mathbf{p}'\cdot\gamma_{11}(\mathbf{p},\mathbf{p}')\,\chi_{\mathbf{k}\omega}(\mathbf{p}') - \int d\mathbf{p}'\cdot\gamma_{12}(\mathbf{p},\mathbf{p}')\,\varphi_{\mathbf{k}\omega}(\mathbf{p}')\right\}, \tag{2}$$

$$\varphi_{\mathbf{k}\omega}(\mathbf{p}) = -\frac{1}{\varepsilon(\mathbf{p}+\mathbf{k}/2) + \varepsilon(\mathbf{p}-\mathbf{k}/2) + \omega}\left\{\int d\mathbf{p}'\cdot\gamma_{21}(\mathbf{p},\mathbf{p}')\,\chi_{\mathbf{k}\omega}(\mathbf{p}') - \int d\mathbf{p}'\cdot\gamma_{22}(\mathbf{p},\mathbf{p}')\,\varphi_{\mathbf{k}\omega}(\mathbf{p}')\right\},$$

where $\mathbf{k}$ and ω are the momentum and energy of an excitation, and

$$\begin{aligned}\gamma_{11} = \gamma_{22} &= g^2 \theta(p)\, \theta(p')\, (u_{\mathbf{p}+\mathbf{k}/2} u_{\mathbf{p}'-\mathbf{k}/2} + v_{\mathbf{p}+\mathbf{k}/2} v_{\mathbf{p}'-\mathbf{k}/2}) \\ &\times (u_{\mathbf{p}-\mathbf{k}/2}\, u_{\mathbf{p}'+\mathbf{k}/2} + v_{\mathbf{p}-\mathbf{k}/2} v_{\mathbf{p}'+\mathbf{k}/2}), \\ \gamma_{12} = \gamma_{21} &= g^2 \theta(p)\, \theta(p')\, (u_{\mathbf{p}+\mathbf{k}/2} v_{\mathbf{p}'-\mathbf{k}/2} - v_{\mathbf{p}+\mathbf{k}/2} u_{\mathbf{p}'-\mathbf{k}/2}) \\ &\times (u_{\mathbf{p}-\mathbf{k}/2}\, v_{\mathbf{p}'+\mathbf{k}/2} - v_{\mathbf{p}-\mathbf{k}/2} u_{\mathbf{p}'+\mathbf{k}/2}).\end{aligned} \tag{3}$$

Thanks to the degeneracy of the nuclei (3), the set of integral equations becomes an algebraic system, and the condition that this can be solved gives us an equation for $\omega(\mathrm{k})$. The integrals occurring in the dispersion relation are evaluated for small values of k and ω. The result is $\omega^2 = c^2k^2$, $c^2 = p_0^2/3m^2$.

It is necessary to emphasize that our result cannot be applied to a system of charged particles. In that case, by virtue of the Coulomb interaction, the sound vibrations go over into plasma waves of high frequency ($\omega^2 = 4\pi e^2 n/m$).

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THE CROSS SECTION OF THE PION–NUCLEON INTERACTION IN THE HIGHER ENERGY REGION

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IN a previous paper,[1] it was shown by the author that both for the model of a nucleus with homogeneous density and sharp boundaries, and also for the nuclear model in which the decrease in the density begins at the center of the nucleus, we must renounce the possibility of choosing a value of r_0 in the expression $R = r_0 A^{1/3} 10^{-13}$ cm (on the basis of these models) that will be the same for all nuclei investigated (R = nuclear radius, A = atomic weight). Also excluded is the William's density distribution[2] because of the great extension of such a nucleus.

Investigation of the cross section of the interaction of π^- and π^+ mesons of different energies with heavy nuclei[3] has shown that, with an accuracy to within 3%, the radial distribution of the protons and neutrons is identical. On this basis, it can be assumed that the distribution of nucleons in the nucleus coincides with the distribution of protons, which is determined in experiments on electron scattering. Application of the homogeneous, smooth model of the nucleus, obtained from experiments on the scattering of electrons for the analysis of cross sections of nuclear interactions of protons with energies from 0.9 to 34 Bev with nuclei of lead and graphite has given satisfactory results.[1]

In the present work, on the basis of experimental data relative to cross sections of inelastic collision of pions with graphite and lead nuclei[4,5] at the energies mentioned, we have carried out calculations of the cross section of inelastic interaction and the opacity of nuclei, making use of a homogeneous smooth model of the nucleus for this purpose. If we assume for the cross section of the interaction of pions with nucleons $\overline{\sigma}(\pi) = 33$ mbn, then the computed values of the cross section of the interaction coincide with the experimental for values of the radial parameter of the smooth distribution $c = (1.14 \pm 0.04) \times 10^{-13} A^{1/3}$ cm. With consideration of experimental errors, $\overline{\sigma}(\pi) = 33 \pm 4$ mbn. Here it has been assumed that the range of