to the relation

$$m/M_B = N_d + \frac{0.15\,(I_0/I-4)\,n_s}{1-2I_s/I},$$

where I_0 and I are the exchange integrals between the s and d electrons for one lattice site and between its neighboring sites respectively, I_S is the transfer integral of the s electron.[4] According to Eqs. (2) and (4), $\epsilon\,(\epsilon_0) = 0.4$, (-0.26) for Ni, 0.73 (-0.195) for Co, and 0.18 (-0.347) for Fe. Substituting these values in (3) we find that the computed and experimental values of m are in excellent agreement.

Quantities analogous to (2) also enter into the relation for alloys. For example, for the alloys Fe, Co, Ni (component A) with Cr and V (component B),

$$m_B/M_B = \ldots \mp [1+0.642\sum_i n_i(r_1-R_B)\lambda_B - 0.642\sum_i n_i(r_i-R_{AB})\lambda_A\lambda_B], \quad (5)$$

where i = 1, 2 for lattices A2 and A3, and 1 for lattice A1, while the upper sign applies for Fe — Cr and Fe — V, the lower, for Ni — Cr and Co —Cr. The computed points lie on straight lines which cross the ordinates (moments) -1, 1 and 1, $-1\,M_B$, and abscissas (concentrations) 42 at % Cr, 22.78 cm % Cr, 13.5 at % Cr and 20 at % V, respectively, for Fe — Cr, Ni — Cr, Co — Cr and Fe — V. This is in agreement with experiment.[5] For the observed concentrations, a = R_{AB} for Fe —Cr and Fe — V; above, deviations from linearity are observed. The change of moment (at 100 at % B) is almost the same as in experiments with weak solid solutions.

*It is shown that, with an accuracy to within 1%, R = R_s + R_d, where R_s and R_d are the "radii" of the s and d shells of isolated atoms, computed by Slater.[1] The quantity $r_1 - R = r_1 - (R_s + R_d)$ recalls the difference considered in the theory of ionic crystals between the equilibrium minimun interionic distance and the sum of the radii of the neighboring ions of the lattice which characterizes their collision. We note that the numerical values in Eq. (1) are also encounted in Ref. 2 on the ionic structure of spinel; thus, for example, the number 235/60 given the factor u which characterizes the departure of the structure of spinel from the ideal (for the latter case, u = 0.375); the number 13.75 is Madelung's constant, which corresponds to u = 0.385, etc.

†In each of the last three alloys there is one transitional metal; therefore, R for the particular metal is used in place of R_{AB}.

‡The quantity in square brackets in (1) is equal to R/0.13 = 7.7 R. The latter number, divided by 12 (the number of nearest neighbors in a metal with lattice type Al), is equal to 0.642 R. The coefficient 0.642/A obtained in such fashion enters in Eq. (2).

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RELATIVISTIC MOTION OF AN ELECTRON IN AN AXIALLY SYMMETRIC FIELD WHICH MOVES ALONG THE AXIS OF SYMMETRY

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1. Since the problem was first studied by Wiederöe[1] a detailed investigation of the relativistic motion of an electron in a varying axially symmetric field has been made in only two cases: motion in a magnetic field which is uniform and parallel to the axis of symmetry[2] and motion in a barrel-shaped magnetic field.[3,4] Below we study a new version of this problem in which a magnetic field which falls off in the direction of the axis of symmetry (bottle-shaped field) is displaced along this axis with variable or fixed velocity. Just as in the earlier cases, the new version of this problem can be used as the theoretical basis for a new type of accelerator — a linear induction accelerator or, as it might be called, a linear betatron.

2. Following Refs. 2 — 4, the equations of motion of the electron are determined from the Lagrangian function

$$L = -m_0c^2\sqrt{1-v^2/c^2} + (e/c)\,r\dot{\varphi}A$$

and have the form

$$\frac{d}{dt}(m\dot{r}) = \frac{e}{c}r\dot{\varphi}\frac{\partial A}{\partial r} = \frac{e}{c}r\dot{\varphi}\left[H_z - \frac{A}{r}\right],$$
$$\frac{d}{dt}(m\dot{z}) = \frac{e}{c}r\dot{\varphi}\frac{\partial A}{\partial z} = -\frac{e}{c}r\dot{\varphi}H_r, \quad (1)$$
$$mr\dot{\varphi} = -\frac{e}{c}A = -\frac{e}{c}\left(\frac{r\bar{H}}{2} + \frac{b}{r}\right),\ \frac{d}{dt}(mc^2) = -\frac{e}{c}r\dot{\varphi}\frac{\partial A}{\partial t},$$

where $m = m_0(1-v^2/c^2)^{-1/2}$, A is the A_φ component of the vector potential, b is a constant of

integration, $\overline{H}$ is the mean value of the z component of the magnetic field in a circle of radius r and the remaining notation is obvious. The third equation in (1) can also be written in the form:

$$m^2(\dot{r}^2 + \dot{z}^2 - c^2) + (e/c)^2A^2 + m_0^2c^2 = 0. \quad (2)$$

3. In the quasi-stationary approximation the following solution of the field equation applies in the region of the line $r = R$, $\zeta = z - z(t) = 0$:

$$A = r\left[\frac{H_z^0}{2} - \frac{a}{2}R^2 + \frac{ar^2}{4} - \frac{H_r^0}{R}\zeta - \frac{a\zeta^2}{1-\dot{z}^2/c^2}\right], \quad (3)$$

where H_r^0 and H_z^0 are the components of the field at $r = R$, $\zeta = 0$, $z(t)$ is an arbitrary function of the time which satisfies the condition $z = c^2 \times (\partial H_z/\partial r)_0/H_r^0$ and a is a constant. In a field of this kind, when $a = -2H_z^0/R^2$, Eq. (1) has the particular solutions $r = R$ and $z = z(t)$, i.e., the electron moves in a helix of constant radius if the function $z(t)$ is given by the equation

$$ct = \sqrt{z(t)}\sqrt{\alpha + z(t)} + \alpha \ln(\sqrt{z(t)} + \sqrt{\alpha + z(t)}), \quad (4)$$
$$\alpha = m_0^2c^4[1 + (eRH_z^0/m_0c^2)^2]/2e^2RH_z^0H_r^0,$$

where the particle energy is

$$E = mc^2 = \sqrt{2e^2H_z^0H_r^0(\alpha + z)}. \quad (5)$$

In the case of relativistic initial energies and $z \gg RH_z/2H_r$, from Eqs. (4) and (5) we have

$$z = ct, \quad E = E_0\sqrt{2zH_r/RH_z}. \quad (6)$$

4. If the field is displaced along the z axis with constant velocity u, i.e., $A = A(r, z-ut)$, from Eq. (1) we find

$$m(\dot{z} - c^2/u) = -Mc^2/u, \quad M = m_0(1 - \dot{r}_0^2\dot{\varphi}_0^2/c^2)^{-1/2}, \quad (7)$$

where M is the mass of the particle at $\dot{z} = 0$ and $\dot{r} = 0$.

According to Eqs. (2) and (7), an electron which originally moves in the wide section of the "magnetic bottle," where $H_z = H_0$ and $A = A_0$, falls into the "neck" of the bottle (where $H_z = H$) and then again is forced into the wide part, acquiring the following energy in the process

$$E/E_0 = 2[(m_0/M)^2 + (eA/Mc^2)^2] - 1. \quad (8)$$

If it is assumed that the motion is such that the adiabatic invariant $H_zr^2 = \text{const}$ is conserved, with a relativistic initial energy we have $A = r_0\sqrt{H_0H}$ and Eq. (8) can be written in the form:

$$E/E_0 + 1 = 2(A/A_0)^2 \approx 2H/H_0 \approx L/2l, \quad (9)$$

where L is the length of path over which acceleration takes place, and l is the length of the segment over which the field changes from H_0 to H.

5. Equations (8) and (9) do not hold if $u = c$. In this case the field equation yields

$$A = {}^1/_2rH(z - ct) + b/r, \quad (10)$$

and from Eqs. (1), (2) and (7), for the case $H_zr^2 = \text{const}$, we have

$$E - E_0 = (e^2/2E_0)(H_0r_0^2)[H - H_0],$$
$$z - z_0 = -\frac{e^2(Hr^2)}{2M^2c^4}\int_{z_0}^{z-ct}(H - H_0)\,d\zeta. \quad (11)$$

For relativistic initial conditions $Mc^2 = -eA_0$ and since $A_0 \approx H_0r_0$, we have

$$E/E_0 - 1 \approx \tfrac{1}{2}(H/H_0 - 1) = L/l. \quad (12)$$

6. According to Eqs. (9) and (12), in a linear accelerator in which the field moves with constant velocity u, we have: $E/E_0 \approx 2H/H_0$ or $E/E_0 \approx H/2H_0$, i.e., the situation is analogous to that in the usual betatron. However, in contrast to the betatron the strong field H can be concentrated in a very small region since $He^2 = \text{const}$.

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DETERMINATION OF THE VELOCITY OF IONIZING PARTICLES USING A HIGH-FREQUENCY ELECTRIC FIELD FOR TRACK MARKING

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AN interesting method of making direct determinations of the velocities of ionizing particles in a Wilson cloud chamber has recently been proposed by Gabor and Hampton.[1] In this scheme, the tracks are "marked" by a high-frequency (rf) electric field by using the difference in intensity of electron