In practice, the most convenient detector would consist of a "sandwich" of several photographic and luminiscent layers in close contact with each other. Superposition of light marks in several layers increases the total density and excludes errors due to stray light spots.

A similar detector can be used as indicator of electron-nuclear showers produced by high-energy particles in a dense medium. The method may be of value for measurements of the number and distribution of shower particles, having a higher resolving power — for large densities — than ionization methods of detection; it can be used in conjunction with nuclear emulsions (in analogy with the method proposed by Grigorov for ionization chambers[3]) for determination of shower position (more accurately than with chambers). We shall mention also the (limited) possibility of timing the events by the use of moving film.

The author would like to thank N. L. Grigorov for valuable advice.

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POLYATOMIC DISTANCES IN FERROMAGNETICS

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In the present paper we discuss the analogy noted by us between the dependence of the atomic magnetic moments of ferromagnetic metals and alloys on the concentration of electrons (total number of s- and d-electrons) in an atom and the same dependence of some quantity (of the dimensions of length) which, in the case of pure metals, is equal to the difference between the distance of nearest neighbors of the first coordination sphere of the crystalline lattice r_1 and some constant of the metal R. In the case of alloys, this quantity is expressed by an analogous difference.

Let us consider the transition elements with atomic numbers Z from 21 to 29 (see Fig. 1).

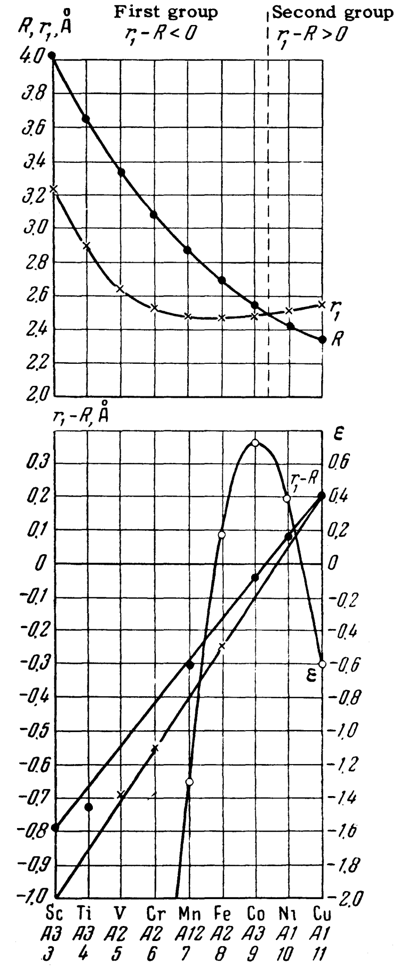

FIG. 1

We find that for these elements,

$$R = 0.13\,[(Z/2)^2 - (13.75 + l)\,Z + 26\,(l - 1) + 235.525], \qquad (1)$$

where $l = 0$ for $Z \leq 26$, $l = 1$ for $Z \geq 26$.*

In Fig. 1, we have plotted the values of r_1, R, $r_1 - R$, the electronic concentration C, and also the types of metallic lattices. It can be seen from the drawing that the metals separate into two groups: the first includes Co, Fe, etc, for which $r_1 - R < 0$; the second includes Ni and others, for which $r_1 - R > 0$. The value of $r_1 - R$, as a rule, depends linearly on C, in which case the points for the metals with lattices of type A2 (such as Fe) lie on the lower line, and those of the type A1 (like Ni) and A3 (like Co), on the upper line.

The points for binary disordered alloys lie on the lines 1 — 3 of Fig. 2.

Let us consider the quantity $r_1 - R_{AB}$, where r_1 is the distance between the closest atoms of the transition metals in the lattice of the alloy (it depends on its concentration), $R_{AB} = \lambda_A R_A + \lambda_B R_B$ (λ_A and λ_B are the atomic concentrations of the components of the alloy A and B, respectively, R_A and R_B are computed from Eq. (1).

It is shown in Fig. 2 that the alloys divide into two groups: the first is composed of those for which $r_1 - R_{AB} < 0$, the second, those for which $r_1 - R_{AB} > 0$. The alloys Ni — Fe, Fe — Co, Co — Cu, Fe — Cr, Ni — Cu, FeAl, CoAl, NiAl,[†] CuZn and others are similar to pure metals: $r_1 - R_{AB}$ and $a - R_{AB}$ (a = parameter of the lattice) depend linearly on the mean electronic concentration per atom of alloy $\bar{C} = \lambda_A C_A + \lambda_B C_B$, where C_A and C_B are the concentrations of electrons of components A and B, respectively. The lines 1 and 3 correspond to these alloys for lattices of the type A2 (CsCl), and the line 2 for lattices of the type A1 (NaCl) and A3. The alloys Co — Mn, Ni — Mn, Fe — V depart from being straight lines.

It is known that atomic magnetic moments of ferromagnetic metals and alloys behave in similar fashion. In particular, the curves 1 — 2 of Fig. 2 are analogous to the Slater-Pauling curve for these moments; the same alloys Co — Mn, Ni — Mn, Fe — V and others depart from these latter lines.[3] It then follows that these moments depend essentially on $r_1 - R$ in the case of pure metals and on $r_1 - R_{AB}$ in the case of alloys. To be more convincing, we shall show, in an example of pure ferromagnetic metals, that the difference $r_1 - R$ enters into the empirical relation assumed by us for the moments m under examination. With this aim, we shall consider the quantity[‡]

$$\varepsilon = 1 \pm 0{,}642\,[n_1(r_1 - R) + n_2(r_2 - R)], \tag{2}$$

where r_2 = distance of nearest neighbors of the second coordination sphere of the lattice, n_1 and n_2 are the numbers of nearest neighbors of the first and second coordination spheres of the lattice, respectively, the upper sign applying to the first group of metals, the lower, to the second. It was shown in Fig. 1 that $\epsilon > 0$ only for pure ferromagnetic metals. For the latter we get

$$m / M_B = N_d + 0{,}15\,(\varepsilon_0/\varepsilon - 4)\, n_s, \tag{3}$$

where M_B is the Bohr magneton, N_d = number of unpaired d electrons in the atom, n_s = number of s electrons in the atom,

$$\varepsilon_0 = -0{,}13 N_d (N_d - n_s + \beta)/(N_d - 1). \tag{4}$$

Here $\beta = 0$ for the first group of metals, $\beta = 1$ for the second group. The relation (3) is similar

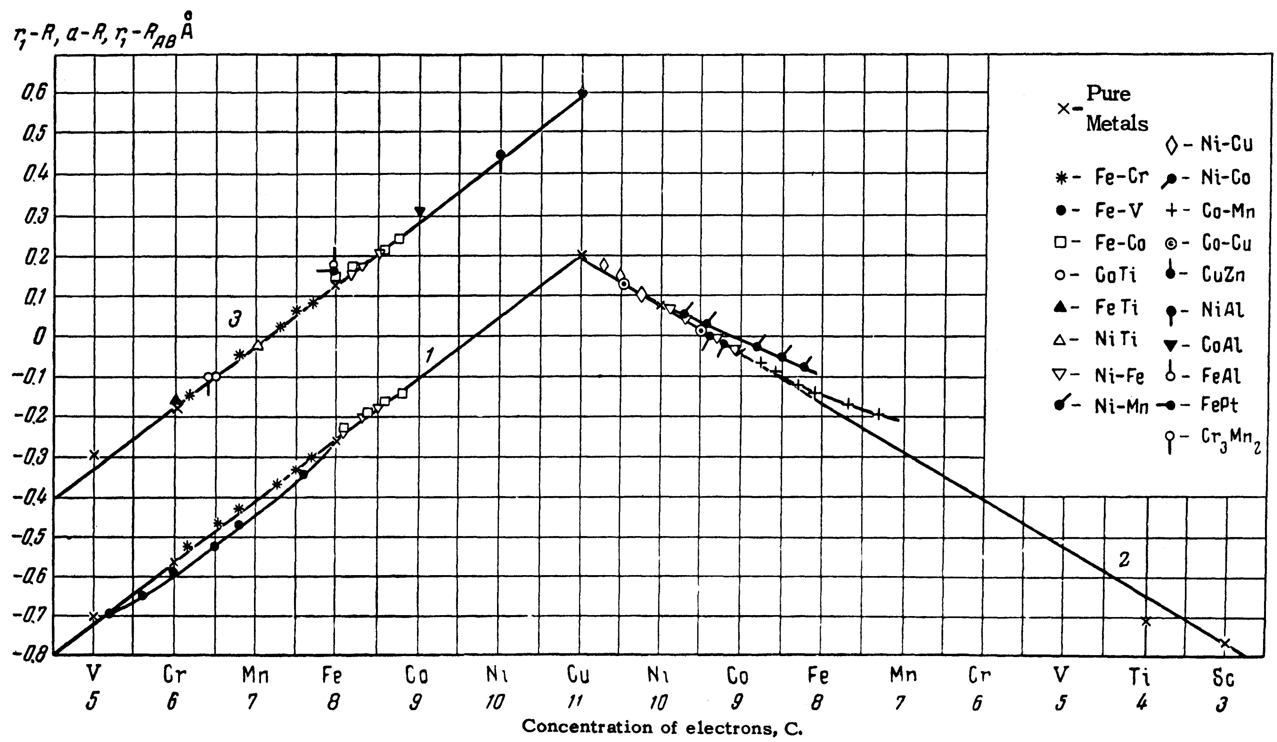

FIG. 2

to the relation

$$m/M_B = N_d + \frac{0.15\,(I_0/I - 4)\,n_s}{1 - 2I_s/I},$$

where I_0 and I are the exchange integrals between the s and d electrons for one lattice site and between its neighboring sites respectively, I_s is the transfer integral of the s electron.[4] According to Eqs. (2) and (4), $\epsilon\,(\epsilon_0) = 0.4,\ (-0.26)$ for Ni, $0.73\ (-0.195)$ for Co, and $0.18\ (-0.347)$ for Fe. Substituting these values in (3) we find that the computed and experimental values of m are in excellent agreement.

Quantities analogous to (2) also enter into the relation for alloys. For example, for the alloys Fe, Co, Ni (component A) with Cr and V (component B),

$$\begin{aligned} m_B/M_B = {} & \ldots \mp [1 + 0.642 \sum_i n_i (r_1 - R_B)\lambda_B \\ & - 0.642 \sum_i n_i (r_i - R_{AB})\lambda_A \lambda_B], \end{aligned} \tag{5}$$

where i = 1, 2 for lattices A2 and A3, and 1 for lattice A1, while the upper sign applies for Fe — Cr and Fe — V, the lower, for Ni — Cr and Co — Cr. The computed points lie on straight lines which cross the ordinates (moments) −1, 1 and 1, −1 M_B, and abscissas (concentrations) 42 at % Cr, 22.78 cm % Cr, 13.5 at % Cr and 20 at % V, respectively, for Fe — Cr, Ni — Cr, Co — Cr and Fe - V. This is in agreement with experiment.[5] For the observed concentrations, $a = R_{AB}$ for Fe — Cr and Fe — V; above, deviations from linearity are observed. The change of moment (at 100 at % B) is almost the same as in experiments with weak solid solutions.

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*It is shown that, with an accuracy to within 1%, $R = R_s + R_d$, where R_s and R_d are the "radii" of the s and d shells of isolated atoms, computed by Slater.[1] The quantity $r_1 - R = r_1 - (R_s + R_d)$ recalls the difference considered in the theory of ionic crystals between the equilibrium minimun interionic distance and the sum of the radii of the neighboring ions of the lattice which characterizes their collision. We note that the numerical values in Eq. (1) are also encountered in Ref. 2 on the ionic structure of spinel; thus, for example, the number 235/60 given the factor u which characterizes the departure of the structure of spinel from the ideal (for the latter case, u = 0.375); the number 13.75 is Madelung's constant, which corresponds to u = 0.385, etc.

†In each of the last three alloys there is one transitional metal; therefore, R for the particular metal is used in place of R_{AB}.

‡The quantity in square brackets in (1) is equal to R/0.13 = 7.7 R. The latter number, divided by 12 (the number of nearest neighbors in a metal with lattice type A1), is equal to 0.642 R. The coefficient 0.642/A obtained in such fashion enters in Eq. (2).

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RELATIVISTIC MOTION OF AN ELECTRON IN AN AXIALLY SYMMETRIC FIELD WHICH MOVES ALONG THE AXIS OF SYMMETRY

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1. Since the problem was first studied by Wiederöe[1] a detailed investigation of the relativistic motion of an electron in a varying axially symmetric field has been made in only two cases: motion in a magnetic field which is uniform and parallel to the axis of symmetry[2] and motion in a barrel-shaped magnetic field.[3,4] Below we study a new version of this problem in which a magnetic field which falls off in the direction of the axis of symmetry (bottle-shaped field) is displaced along this axis with variable or fixed velocity. Just as in the earlier cases, the new version of this problem can be used as the theoretical basis for a new type of accelerator — a linear induction accelerator or, as it might be called, a linear betatron.

2. Following Refs. 2 — 4, the equations of motion of the electron are determined from the Lagrangian function

$$L = -m_0 c^2 \sqrt{1 - v^2/c^2} + (e/c)\, r\dot{\varphi} A$$

and have the form

$$\begin{gathered} \frac{d}{dt}(m\dot{r}) = \frac{e}{c} r\dot{\varphi} \frac{\partial A}{\partial r} = \frac{e}{c} r\dot{\varphi}\left[H_z - \frac{A}{r}\right], \\ \frac{d}{dt}(m\dot{z}) = \frac{e}{c} r\dot{\varphi} \frac{\partial A}{\partial z} = -\frac{e}{c} r\dot{\varphi} H_r, \\ mr\dot{\varphi} = -\frac{e}{c} A = -\frac{e}{c}\left(\frac{r\overline{H}}{2} + \frac{b}{r}\right), \quad \frac{d}{dt}(mc^2) = -\frac{e}{c} r\dot{\varphi} \frac{\partial A}{\partial t}, \end{gathered} \tag{1}$$

where $m = m_0(1 - v^2/c^2)^{-1/2}$, A is the A_φ component of the vector potential, b is a constant of up the crystals as infinitely long unidimensional or two-dimensional atom complexes, bound together by "small" forces of one nature, whereas in the complex itself the atoms are bound by "big" forces of another nature.

6. The difference between the typical molecular crystals (e.g., the CH_4 or C_6H_6 crystals) and the heteropolar molecular crystals (such as the NaCl, $HgCl_2$ or PbS crystals) lies: (1) in the degree of molecularity β; (2) in the nature of the forces in the molecules; (3) in the nature of intermolecular forces. The quantity β is defined as the ratio of the intramolecular energy $U^a \cong D$ (D is the energy of dissociation of the diatomic molecule into ions) to the intermolecular energy U^e per bond. For the substances for which β is given below, it is possible to take $U^e \approx 2S/l$. Example:

$\beta = 300(CH_4)$, $200(HCl)$, $22(HgCl_2)$, $10(NaCl)$ taking $l = 12$ in all four cases.

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ERRATA

Page	Line	Reads	Should Read
Volume 5			
1043	Eq. (4)		$W = y^2 a_{14}^2 \sin 2\phi/2\rho\,(a_{11} a_{44}$ $- \alpha_{14}^2 \sin^2 3\phi)$ The coefficient k_2 equals $0.185 \times 10^{-3}\,cm^{-1}$.
1044	3 from bottom (l.h.)	$\Delta y = 2.87 \times 10^{-3}$ cm	$\Delta y = 3.18 \times 10^{-3}$ cm
	4 from top (r.h.)	$\Delta\vartheta_\Sigma = 7.2 \times 10^{-5}$ radians	$\Delta\vartheta_\Sigma = 5.9 \times 10^{-5}$ radians
Volume 6			
1090	4 and 5 from top	2—(d, 3n); and of the I_{53}^{127} cross section, 3—(d, 2n); 4—(d, 3n)	2—(d, 3n) on I_{53}^{127} and 3—(d, 3n); 4—(d, 3n) on Bi_{83}^{209}
1091	6 from bottom expression for determinant C(y)	ρ, γp, h, $1/\rho$	ρy_2, $\gamma p y_2$, $h y_2$, y_2/ρ
1094	7 from bottom	For $\gamma = 5/3$, μ has . . .	Here μ has . . .
Volume 7			
55	16 from bottom	Correct submittal date is April 5, 1957	
169	17 from bottom	Delete "Joint Institute for Nuclear Research"	
215	Table	Add: Note. Columns 2—9 give the number of counts per 10^6 monitor counts	
215	Table, column headings	1, 2, 3, 4-7, 8	1, 2, 3, 4, 8-7
312	Eq. (8)	$\ldots (1 \pm \mu/2M)^2$	$\ldots (1 \mp \mu/2M)^2$
313	2, r.h. col.	$\alpha_{33} = 0.235$	$a_{33} = 0.235$
692	Eq. (5)	$m_B/M_B = \ldots \mp [1 + \ldots]$	$m_B/M_B = \mp [1 + \ldots]$
461	Title	. . . Elastically Conducting	. . . Electrically Conducting