INFLUENCE OF THE LATTICE VIBRATIONS OF A CRYSTAL ON THE PRODUCTION OF ELECTRON-HOLE PAIRS IN A STRONG ELECTRICAL FIELD

L. V. KELDYSH

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor November 11, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 962-968 (April, 1958)

We have evaluated the probability for the production of an electron-hole pair in a semiconductor in a strong electrical field, taking into account the electron-phonon interaction. We have obtained the temperature dependence of this effect which is very appreciable at low temperatures. We have considered qualitatively other processes which may influence the probability of the diffusion of a valence electron into the conduction band, such as the absorption of several phonons and collisions with a slow (non-ionizing) conduction electron or with an impurity. It is pointed out that the first of these processes can play a decisive role in relatively weak fields.

In a previous paper[1] we noted that interactions leading to a change in the quasi momentum of an electron (in particular, the electron-phonon interaction) can play a very essential role in the electron-hole pair production process in a semiconductor when a strong electrical field is present. In the same paper we derived a set of equations (1.12) which determined the probability of the corresponding processes (the 1 denotes here and henceforth the equations of Ref. 1; we use also everywhere, where it is not stated specifically, the notation introduced in that paper). Since their probability is always small, the direct diffusion of a valence electron into the conduction band, considered in Ref. 1, and the transition involving a phonon are to a first approximation independent of one another and can be considered separately. The present paper is devoted to an evaluation of the probability of the latter process.

The initial condition can now be written in the form

$$c_j([N_{\mathbf{k}}],\ \mathbf{p},\ 0) = \delta_{jVc}([N_{\mathbf{k}}]). \tag{1}$$

The integration of equations (1.12) gives then the following expression for the probability of diffusion for one period of oscillation (we assume again for the time being that the field $\mathbf{E}$ is directed along one of the principal axes of the simple cubic lattice)

$$\left|c_c\left([N_{\mathbf{k}}], \mathbf{p}_0, \frac{2\pi\hbar}{eEd}\right)\right|^2 = \frac{\nu}{VN}\frac{1}{(e\hbar E)^2}\sum_{\mathbf{k}'}\Big\{N_{\mathbf{k}'}\left|c([N_{\mathbf{k}}] - 1_{\mathbf{k}'})\right.$$
$$\times\left.\int_{-\frac{\pi\hbar}{d}}^{\frac{\pi\hbar}{d}} M_{Vc}(\mathbf{p}, \hbar\mathbf{k}')\, Q^{+}_{Vc}\left(\mathbf{p}, \mathbf{k}', \frac{p_\parallel}{eE}\right)dp_\parallel\right|^2$$
$$+ (N_{\mathbf{k}'}+1)\left|c([N_{\mathbf{k}}] + 1_{\mathbf{k}'})\right.$$
$$\times\left.\int_{-\pi\hbar/d}^{\pi\hbar/d} M_{Vc}(\mathbf{p}, -\hbar\mathbf{k}')\, Q^{-}_{Vc}\left(\mathbf{p}, \mathbf{k}', \frac{p_\parallel}{eE}\right)dp_\parallel\right|^2\Big\}. \tag{2}$$

In deriving Eq. (2) we took it into account that, by virtue of the presence of fast oscillating factors, the terms with different $\mathbf{k}'$ do not interfere with one another. The experimentally observed diffusion probability $D_0(\mathbf{p}_\perp)$ can now be obtained by summation over $[N_{\mathbf{k}}]$, which in the given case leads simply to the substitution

$$N_{\mathbf{k}} \to \overline{N}_{\mathbf{k}} \equiv [\exp\{\hbar\omega_{\mathbf{k}}/kT\} - 1]^{-1}. \tag{3}$$

Here k is Boltzmann's constant and T the absolute temperature. Hence

$$D_0(\mathbf{p}_\perp) = \frac{\nu}{VN}\frac{1}{(e\hbar E)^2}$$
$$\times\sum_{\mathbf{k}}\Big\{(\overline{N}_{\mathbf{k}}+1)\left|\int_{-\pi\hbar/d}^{\pi\hbar/d} M_{Vc}(\mathbf{p}, \hbar\mathbf{k})\, Q^{+}_{Vc}\left(\mathbf{p}, \mathbf{k}, \frac{p_\parallel}{eE}\right)dp_\parallel\right|^2$$
$$+\overline{N}_{\mathbf{k}}\left|\int_{-\pi\hbar/d}^{\pi\hbar/d} M_{Vc}(\mathbf{p}, -\hbar\mathbf{k})\, Q^{-}_{Vc}\left(\mathbf{p}, \mathbf{k}, \frac{p_\parallel}{eE}\right)dp_\parallel\right|^2\Big\}. \tag{4}$$

The presence of the fast oscillating factors $Q^{\pm}_{Vc}(\mathbf{p}, \mathbf{k}, p_\parallel/eE)$ in the expressions under the integral sign in Eq. (4) leads to a very fast decrease of the diffusion probablity with increasing minimum value which the quantity $\epsilon_c(\mathbf{p}) - \epsilon_V(\mathbf{p} \pm \hbar\mathbf{k})$ attains for given $\mathbf{p}_\perp$ and $\mathbf{k}$. The basic contribution to (4) is thus made by terms with values of $\mathbf{k}$ near $\mathbf{k}_0$. which is the difference between the values of the quasi momentum corresponding to the minimum value of the energy in the conduc-

tion band and the one corresponding to the maximum value in the valence band, divided by $\hbar$. As a rule, as we noted already in Ref. 1, $\mathbf{k}_0 \neq 0$. This leads to some changes in the evaluation of the integrals in (4). Indeed, the width Δ of the forbidden band is by definition less than ϵ_0, the minimum value of the function $\epsilon_c(\mathbf{p}) - \epsilon_V(\mathbf{p})$, and the difference $\epsilon_c(\mathbf{p}) - \epsilon_V(\mathbf{p} - \hbar\mathbf{k}_0)$ tends therefore to zero in the complex $\mathbf{p}_\parallel$ plane before any of the energies occurring in it reach a branching point [the latter coincide with the zeroes of the function $\epsilon_c(\mathbf{p}) - \epsilon_V(\mathbf{p})$]. As a result, for the values

$$p_\parallel = [2m^*_\parallel(\varepsilon_{c\min} - \varepsilon_{V\max} \pm \hbar\omega_{\mathbf{k}_0})]^{1/2} \tag{5}$$

the integrands in (4) have not poles, but saddle points. The quantity $m^*_\parallel$ which enters in (5) is the reduced effective mass of the electron and hole, corresponding to motion in the direction of the electrical field.

Evaluating the integrals in (4) by the saddle-point method, and integrating the result obtained with respect to $\mathbf{p}_\perp$, we arrive at the following expression for the number of electrons per unit volume, which get into the conduction band per unit time:

$$n = 2^{3/2}\pi^{7/2}N(2\pi\hbar)^{-6}\left[\prod_i m^*_{ic}m^*_{iV}\right]^{1/2}\Delta^3\Omega_0^2\frac{1}{\hbar\Delta}|M_{Vc}(\mathbf{p}_{\min}, \hbar\mathbf{k}_0)|^2$$
$$\times (e\hbar E/\sqrt{2m^*_\parallel}\Delta^{3/2})^{6/2}\exp\left\{\mathbf{n}\gamma_0 - \frac{4\sqrt{2m^*_\parallel}}{3e\hbar E}(\Delta - \hbar\omega_{\mathbf{k}_0})^{3/2}\right\}$$
$$\times\left[\overline{N}_{\mathbf{k}_0} + (1 + \overline{N}_{\mathbf{k}_0})\exp\left\{-\frac{4\sqrt{2m^*_\parallel\Delta}}{e\hbar E}\hbar\omega_{\mathbf{k}_0}\right\}\right]. \tag{6}$$

Here $\mathbf{p}_{\min}$ is the value of the quasi momentum corresponding to the lowest electron state in the conduction band, while m^*_{ic} and m^*_{iV} are the principal values of the effective mass tensors of the electrons and holes. Just as was done in Ref. 1, one can easily show that Eq. (6) is valid for lattices of all symmetries and for an arbitrary direction of the field.

Equation (6) contains, generally speaking, an unknown matrix elements $M_{Vc}(\mathbf{p}_{\min}, \hbar\mathbf{k}_0)$. To get a very rough estimate, we can use the fact that the free-flight time τ of an electron in the conduction band is determined by the matrix elements $M_{cc}(\mathbf{p}_{\min}, \hbar\mathbf{k})$ which for $k \approx k_0$ have the same order of magnitude as $M_{Vc}(\mathbf{p}_{\min}, \hbar\mathbf{k}_0)$.

$$\frac{1}{\tau} \sim \frac{\Omega_0}{(2\pi\hbar)^3}\frac{2\pi}{\hbar}$$
$$\times\int|M_{cc}(\mathbf{p}, \mathbf{p}' - \mathbf{p}|^2\delta[\varepsilon_c(\mathbf{p}) - \varepsilon_c(\mathbf{p}') \pm \hbar\omega_{p-p'}]\,d^3p'$$
$$\sim \frac{\Omega_0}{(2\pi\hbar)^3\hbar}\overline{|M_{cc}(\mathbf{p}, \hbar\mathbf{k})|^2}\int\delta[\varepsilon_c(\mathbf{p}) - \varepsilon_c(\mathbf{p}')]\,d^3p'$$
$$= \frac{\Omega_0}{(2\pi\hbar)^3\hbar}\overline{|M_{cc}(\mathbf{p}, \hbar\mathbf{k})|^2}\frac{\overline{S[\varepsilon_c(\mathbf{p})]}}{|\operatorname{grad}\varepsilon_c(\mathbf{p})|}, \tag{7}$$

where $S(\epsilon)$ is the area of the surface $\epsilon_c(\mathbf{p}) = \epsilon$, and the bar indicates averaging over that surface. Taking it into account that in a semiconductor the motion takes place at the bottom of the conduction band and that thus

$$S[\varepsilon_c(\mathbf{p})] \approx 8\pi m^*_c\varepsilon_c(\mathbf{p}),\ |\operatorname{grad}\varepsilon_c(\mathbf{p})| \sim (2\varepsilon_c(\mathbf{p})/m^*_c)^{1/2},$$

and substituting $\epsilon_c(\mathbf{p}) \sim kT$ we get

$$\frac{1}{\tau} \sim \frac{\overline{|M_{cc}(\mathbf{p}_{\min}, \hbar\mathbf{k})|^2}}{(\hbar/m^*_c)(\pi\hbar/d)^2}\frac{\sqrt{2m^*_c kT}}{\pi\hbar/d} \tag{8}$$

If the lowest state of the conduction band is degenerate, which apparently is the case for all cases of practical interest, then the transitions with large changes in quasi momentum, will make a considerable contribution to τ^{-1}, i.e., $k \sim k_0$. We then have for the quantity in which we are interested

$$|M_{Vc}(\mathbf{p}_{\min}, \hbar\mathbf{k}_0)|^2 \sim \overline{|M_{cc}(\mathbf{p}_{\min}, \hbar\mathbf{k})|^2}$$

and (6) can be rewritten

$$n \sim \frac{N\tau^{-1}}{(2\pi\hbar)^6}\left(\prod_i m^*_{ic}m^*_{iV}\right)^{1/2}\Delta^3\Omega_0^2\frac{1}{\Delta m^*_c}\left(\frac{\pi\hbar}{d}\right)^2\frac{\pi\hbar}{\sqrt{2m^*_c kT_D}\,d}$$
$$\times\left(\frac{c\hbar E}{\sqrt{2m^*_\parallel}\Delta^{3/2}}\right)^{6/2}\exp\left\{\mathbf{n}\gamma_0 - \frac{4\sqrt{2m^*_\parallel}}{3e\hbar E}(\Delta - \hbar\omega_{\mathbf{k}_0})^{3/2}\right\}$$
$$\times\left[\overline{N}_{\mathbf{k}_0} + (1 + \overline{N}_{\mathbf{k}_0})\exp\left\{-\frac{4\sqrt{2m^*_\parallel\Delta}}{e\hbar E}\hbar\omega_{\mathbf{k}_0}\right\}\right]. \tag{9}$$

In this equation we have introduced the Debye temperature T_D, since in estimating τ we had assumed $N_{\mathbf{k}} \sim 1$. One can easily verify that relation (9) holds also in the case where the lowest state is non-degenerate. Indeed, the order of magnitude of the quantity of interest to us is given by

$$|M_{Vc}(\mathbf{p}_{\min}, \hbar\mathbf{k}_0)|^2 \sim \overline{|M_{cc}(\mathbf{p}_{\min}, \hbar\mathbf{k})|^2}\,\hbar k_0/\sqrt{2m^*_c kT}.$$

However, if we substitute in Eq. (8) a temperature on the order of the Debye temperature, we must still introduce an additional factor

$$N_{\mathbf{k}} \sim kT_D/\hbar\omega_{\mathbf{k}} \sim kT_D/cp$$
$$\sim kT_D/c\sqrt{2m^*_c kT_D} \sim \hbar k_0/\sqrt{2m^*_c kT_D}, \tag{10}$$

where c is the sound velocity, while the connection between $|M_{Vc}(\mathbf{p}_{\min}, \hbar\mathbf{k}_0)|^2$ and $1/\tau$ remains the same.

The basic qualitative difference between (6) and (1.19) is the explicit temperature dependence which occurs in the last factor. At low temperatures

$$T \lesssim T_0 \approx e\hbar E/4k\sqrt{2m^*_\parallel\Delta} \sim (E/10^5\ \text{v/cm})\,10^\circ\text{K}$$

the number n of pairs produced is practically independent of the temperature. When the temperature is increased, the current begins to increase exponentially, and does so right up to $T \sim T_D$.

(We assume $\hbar\omega_{\mathbf{k}_0} \sim kT_D$.) For $T > T_D$ one should observe a weak linear dependence of n on the temperature. In the interval from T_0 to T_D the current can change at constant electrical field by a factor $\exp(T_D/T_0)$, i.e., by several orders of magnitude. Of the greatest practical interest is the dependence of the critical field on the temperature. By critical field is customarily meant the value of the field for which the current reaches some well-defined value. In the region of greatest interest, $T_0 \leq T \leq T_D$,

$$\frac{\Delta E}{E_c} = -\frac{\hbar\omega_{\mathbf{k}_0}}{kT}\frac{3e\hbar E_c}{4\sqrt{2m_{\parallel}^*}\Delta^{3/2}}\frac{\Delta T}{T}. \tag{11}$$

Outside this region there is practically no dependence. We must note, however, that Eq. (6) contains, as is well-known, also an implicit temperature dependence which is connected with the change in character of the crystal parameters (Δ, $m_{\parallel}^*$, and so on) during heating. The relative change in these quantities for a 1°K change in temperature can be of the same order as the coefficient of linear expansion, i.e., $\sim 10^{-5}$. For germanium and silicon a direct measurement gives[2] $\gamma \equiv d\Delta/dT \approx -4 \times 10^{-4}$ ev/degree. The relative shift of the critical field is then expressed by the following formula

$$\frac{\Delta E_c}{E_c} = \frac{3}{2}\frac{\gamma T}{\Delta}\frac{\Delta T}{T}. \tag{11a}$$

A direct comparison of (11) and (11a) shows that for $T \sim T_D$ the temperature dependence of E_c produced by virtue of the thermal expansion is several times larger than the one following from (11). When the temperature is lowered the reverse situation is produced, since $|dE_c/dT|$ increases sharply according to (11) while the coefficient of linear expansion decreases.[3] At temperatures $T \lesssim 200$°K the temperature dependence is determined on the whole by Eq. (11). For $T \gtrsim T_D$, on the other hand, the thermal expansion is practically the only source of a temperature dependence in Eq. (6).

In the case of a complicated lattice there are always some kinds of phonons with the same wave vector but different frequencies (acoustical and optical). In that case the temperature dependence of the transmission coefficient must clearly have the form of a "staircase" curve, which is a superposition of functions of the form (6). As a consequence, the region where this dependence is appreciable can be extended appreciably. Thus, for germanium $T_D = 360$°K and the frequencies of the optical phonons correspond to a temperature[4]

$$T_{opt} \equiv \hbar\omega_{opt}/k \approx 500°K$$

A direct comparison shows that the factor in front of the exponential in Eq. (9) differs from the corresponding factor in Eq. (1.20) by the quantity

$$\frac{\hbar}{\tau\Delta}\left(\frac{eEd}{kT_D}\right)^{1/2}\left[\frac{1}{\Delta m_{\parallel}^*}\left(\frac{\pi\hbar}{d}\right)^2\right]^{1/4} \sim 10^{-3}.$$

At the same time, a difference between ϵ_0 and Δ, even by a factor two ($\epsilon_0 \sim 2$ ev), makes the exponential factor in Eq. (9) some 10 to 15 orders of magnitude larger than the one in Eq. (1.20), assuming that the quantities $m_{\parallel}$ which enter in these two equations are approximately the same. [We recall that $m_{\parallel}^*$ in Eq. (9) is the effective mass of an electron and a hole, while $m_{\parallel}$ in Eq. (1.20) is some formal quantity which is determined by the expansion (1.18).] If, however, $m_{\parallel}^* \gg m_{\parallel}$, direct diffusion makes a larger contribution to pair-production probability than a process involving the absorption of a phonon $\mathbf{k}_0$. This would merely mean that apparently the transitions involving the absorption of phonons having some other wave vector $\mathbf{k}_{min}$ play the basic role. Indeed, one can see from the general equations (4) and (5) that the probability of pair production involving a phonon $\mathbf{k}$ is determined essentially by the quantity

$$F(\mathbf{E}, \mathbf{k}, \mathbf{p}_\perp) = \mathrm{Re}\left\{\frac{i}{e\hbar E}\int_{0}^{q(\mathbf{p}_\perp, \mathbf{k})} [\varepsilon_c(\mathbf{p}) - \varepsilon_V(\mathbf{p} - \hbar\mathbf{k}) - \hbar\omega_{\mathbf{k}}]\, dp_{\parallel}\right\}. \tag{12}$$

The point $q(\mathbf{p}_\perp, \mathbf{k})$ in the complex $p_{\parallel}$-plane is determined by the condition that the expression under the integral sign tends to zero. For some well-determined values $\mathbf{k} = \mathbf{k}_{min}$ and $\mathbf{p}_\perp = \mathbf{p}_{\perp min}$, which, generally speaking, depend on the direction of the field E, the function $F(\mathbf{E}, \mathbf{k}, \mathbf{p}_\perp)$ reaches a minimum. The neighborhood of this point gives also the main contribution to the pair-production probability. Only in an exceptional case can it be shown that $\mathbf{k}_{min} = 0$ and the decisive fact is then the direct diffusion of the valence electron into the conduction band. For given $\mathbf{p}_\perp = \mathbf{p}_{\perp min}$, the function $\epsilon_c(\mathbf{p}) - \epsilon_V(\mathbf{p} - \hbar\mathbf{k}_{min})$ reaches some minimum value in the neighborhood of which it can be written in the form

$$\varepsilon_c(\mathbf{p}_{\perp min}, p_{\parallel}) - \varepsilon_V(\mathbf{p}_{\perp min} - \hbar\mathbf{k}_{\perp min}, p_{\parallel} - \hbar k_{\parallel min}) = \Delta(\mathbf{p}_{\perp min}, \mathbf{k}_{min}) + \frac{(p_{\parallel} - p_{\parallel min})^2}{2m_{\parallel}(\mathbf{p}_{\perp min}, \mathbf{k}_{min})}. \tag{13}$$

From this it is already absolutely clear that we obtain for the number of electron-hole pairs produced an expression which is completely analogous to Eq. (6), but instead of $\mathbf{k}_0$ we have $\mathbf{k}_{min}$, and instead of Δ and $m_{\parallel}^*$ we have $\Delta(\mathbf{p}_{\perp min}, \mathbf{k}_{min})$ and $m_{\parallel}(\mathbf{p}_{\perp min}, \mathbf{k}_{min})$. Practically, this differ-

ence becomes apparent in the more complicated angular dependence which can no longer be evaluated without giving explicitly the form of the functions $\epsilon_c(\mathbf{p})$ and $\epsilon_v(\mathbf{p})$. Apart from that, as is apparent from what has been said above, an appreciable reduction of the critical field can take place if even one of the functions $\epsilon_c(\mathbf{p})$ and $\epsilon_v(\mathbf{p})$ has sections with anomalously large curvatures.

Finally we consider still one factor which can show an appreciable influence on the form of the function $n(\mathbf{E}, T)$ for relatively weak fields. As can be seen, for instance, from Eq. (6), the probability for a process in which a phonon transfers to an electron not only the necessary momentum, but also an energy ϵ, is essentially determined by the factor

$$\exp\left\{-\frac{4\sqrt{2m_{\parallel}^*}}{3e\hbar E}(\Delta-\varepsilon)^{3/2}\right\},$$

i.e., it increases steeply with increasing ϵ. Collisions in which an electron receives an energy larger than $\hbar\omega_{\mathbf{k}_0}$, even if they have a small probability, can thus influence the number of pairs produced. From the point of view of the framework used by us, such a process must be one in which several phonons are absorbed. It is well-known, however,[5] that the phonon concept itself cannot be used to describe transitions in which the lattice absorbs or emits an energy much larger than kT_D. To ascertain even the qualitative aspects of this influence, we make the apparently very natural assumption that the probability of receiving an energy ϵ from the lattice has the form $a(\epsilon)\exp(-\epsilon/kT)$, where $a(\epsilon)$ is a slowly-varying function compared with the exponent. The total number of pairs produced will then be

$$n=\int_0^\infty b(\varepsilon)\exp\left\{-\frac{\varepsilon}{kT}-\frac{4\sqrt{2m_{\parallel}^*}}{3e\hbar E}(\Delta-\varepsilon)^{3/2}\right\}d\varepsilon. \quad (14)$$

Since the exponent contains a very large number, we can use the saddle-point method to evaluate the integral, whence we get

$$n=b(\varepsilon_m)\frac{e\hbar E}{2\sqrt{m_{\parallel}^*kT}}\exp\left\{-\frac{\Delta}{kT}+\frac{1}{24m_{\parallel}^*}\frac{(e\hbar E)^2}{(kT)^3}\right\}, \quad (15)$$

where

$$\varepsilon_m=\Delta-(e\hbar E/kT)^2/8m_{\parallel}^*. \quad (16)$$

Expression (15) is valid provided $\epsilon_m \gg 0$. For strong fields, Eq. (6) remains correct. Thus, right up to fields determined by the condition

$$eEd/kT\sim\sqrt{2m^*\Delta d^2}/\hbar\sim 1, \quad (17)$$

"multiphonon" processes make the main contribution to the probability of transmission of a valence electron into the conduction band, which leads to a completely new dependence of this probability both on the field and the temperature.

As can be seen from the criterion (17), the region of applicability of (15) reaches already at room temperature the experimentally observed range of fields (several times 10^5 v/cm).

The considerations given here can also be fully applied to another mechanism, whereby a valance electron receives an energy ϵ not from the lattice, but from another electron which is already in the conduction band. If the average energy of the conduction electrons is less than the ionization potential, only a very small number of them can produce impact ionization. However, the principal mass also takes part in the ejection of valence electrons, thanks to the process pointed out above, which is a combination of impact ionization and Zener diffusion. A qualitative equation, which can easily be obtained also for this mechanism, will be completely analogous to Eq. (15), only instead of T it will contain an effective conduction-electron temperature T_{eff} which in strong fields is considerably larger than the true temperature.[6] Its region of applicability will, however, be limited by the condition $kT_{eff} \ll \Delta$, i.e., to rather weak fields.

Finally, similar discussions show that collisions with impurities, during which, in general, no energy is transferred, can influence the probability for electron-hole pair production only at very low temperatures, $T \lesssim T_0$.

The most reliable method to distinguish these two mechanisms from one another experimentally is, clearly, the observation of the temperature dependence of the critical field. In the case of an electron-phonon interaction, $dE_c/dT < 0$ according to Eq. (11), while for an electron-electron interaction we must clearly have $dE_c/dT > 0$. Experiments performed on germanium[7,8] give us reasons to assume that electron-electron collisions play the dominant part in not too narrow p—n-junctions. There is an indication[9] that the opposite situation holds for narrow silicon p—n-junctions.

In conclusion, I should like to use the opportunity to express by gratitude to Professor V. L. Ginzburg for discussing the results of this paper.

[1] L. V. Keldysh, J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 994 (1957), Soviet Phys. JETP **6**, 763 (1958).

[2] H. Y. Fan, Repts. Progr. Phys. **19**, 107 (1956).

[3] L. D. Landau and E. M. Lifshitz, Статистическая физика (Statistical Physics), M., 1951, § 64.

[4] J. de Launay, Solid State Phys. **2**, 291-303 (1956).

[5] E. I. Adirovich, Некоторые вопросы теории люминесценции кристаллов (Some Problems in the The-

ory of Luminescence of Crystals) M., 1951, pp 37-38

[6] V. L. Ginzburg and V. P. Shabanskii, Dokl. Akad. Nauk. SSSR, **100**, 445 (1955); V. P. Shabanskii, J. Exptl. Theoret. Phys. (U.S.S.R.) **31**, 657 (1956), Soviet Phys. JETP **4**, 497 (1957).

[7] A. P. Shotov, J. Tech. Phys. (U.S.S.R.) **26**, 1634 (1956), Soviet Phys. JTP **1**, 1591 (1957).

[8] S. L. Miller, Phys. Rev. **99**, 1234 (1955).

[9] A. G. Chynoweth and K. G. McKay, Phys. Rev. **106**, 418 (1957).

Translated by D. ter Haar
190

SOVIET PHYSICS JETP VOLUME 34 (7), NUMBER 4 OCTOBER, 1958

QUANTUM THEORY OF THE HIGH FREQUENCY CONDUCTIVITY OF METALS

M. Ia. AZBEL'

Institute for Technical Physics, Academy of Sciences, Ukrainian S.S.R.

Submitted to JETP editor November 26, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 969-983 (April, 1958)

A quantum theory of conductivity is developed for metals placed in a high frequency electromagnetic field and a constant magnetic field. The dispersion law of the conduction electrons and the manner in which they are reflected from the surface are assumed to be arbitrary. It is shown that the amplitude of the quantum oscillations in the high frequency case is in general considerably larger than in the static case. The quantum oscillations considered here have not yet been observed experimentally.

1. INTRODUCTION

As is well known, in developing an electronic theory of the conductivity of metals it is possible, to a high degree of accuracy, to limit oneself to a semi-classical investigation which does not take account of the quantization of the energy levels of the conduction electrons. This possibility is related to the fact that, for all real cases, the level splitting $\Delta\epsilon$ is considerably smaller than the limiting Fermi energy ϵ_0 of the electrons. In order to have $\Delta\epsilon \sim \epsilon_0$ it would be necessary to have a magnetic field $H \sim \epsilon_0/\mu \sim 10^9$ oersted, or a metallic sample of width $d \sim \hbar/\sqrt{2m^*\epsilon_0} \sim 10^{-8}$ cm (m^* is the effective mass of an electron and $\mu = e\hbar/m^*c$).

However a semi-classical investigation does not permit one to look into an important effect generally absent in the classical case — purely quantum-mechanical oscillations of the conductivity. At the same time the study of these oscillations is of considerable interest, particularly because it gives a convenient method of reconstructing the form of the Fermi surface from experimental data.[1-3]

The papers of I. M. Lifshitz and Kosevich[2,3] appear to be the only ones in which diamagnetic oscillations of the static conductivity of bulk metal in a constant magnetic field were arrived at in a consistent manner. The essential assumption in their papers was that the current density in the metal was isotropic, which permitted them to regard the statistical operator as not depending explicitly on the coordinates.

In the present work a theory is developed for the general case in which there is spatial anisotropy due to a non-stationary electric field. It is assumed that the anisotropy is substantial, that is, that its characteristic dimension — the skin depth δ — is small in comparison with the Larmor radius r and with the electron mean-free-path l (the so-called anomalous skin-effect), so that the relation between the current density $\mathbf{j}$ and the electric field intensity $\mathbf{E}$ is an integral. At helium temperatures, where the quantum oscillations are observed, this is valid already for meter waves.

The study of this case is of special interest because the amplitude of the quantum oscillations of the resistivity tensor turns out, generally speaking, to be considerably greater (by a factor of $\epsilon_0/\mu H$) than in the static case.

At the same time, specific intrinsic difficulties