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THE ANOMALOUS SKIN EFFECT IN THE INFRA-RED REGION

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The surface impedance of metals in the infra-red region at low temperatures has been calculated, taking into account the effect of interelectron collisions. The electrons are considered as a Fermi liquid with an arbitrary dispersion law for the quasi-particles.

THE purpose of this paper is to calculate the effect of interelectron collisions on the optical properties of metals in the infra-red region of the spectrum. To this end we shall assume that the frequency of the incident light satisfies the conditions

$$\omega l / v \gg 1 \tag{1}$$

(v is the speed of the electrons and l is the free path length) and

$$\omega \delta / v \gg 1, \tag{2}$$

where δ is the depth to which the field penetrates in the metal. The physical significance of the first condition is obvious, and the second implies that the field in which the electrons are moving may be considered to be uniform. It is also assumed that the metal has a large negative dielectric constant and a small absorption, which can be treated as a perturbation. This condition limits the validity of the formulas to the high-frequency region. The calculations will be carried out for the case of very low temperatures, where $kT \ll \hbar\omega$.

We have made no assumptions at all about the law of dispersion for the electrons, the type of scattering probability which enters into the theory, nor the magnitude of the interactions between electrons. In particular it must be emphasized that we are not assuming either that the electron dispersion is approximately quadratic, or that the electron interaction is weak.

First of all we must consider the current due to a single moving electron (or, strictly speaking, a single Fermi particle) assuming that the electrons form a Fermi liquid. In the same way that the energy $\epsilon(\mathbf{p})$ of each individual particle is given by the formula

$$\partial E = \int \varepsilon(\mathbf{p})\, \partial n(\mathbf{p})\, d\tau$$

[E is the total energy, n the distribution function of the particles in $\mathbf{p}$-space, and $d\tau \equiv 2\,dp_x dp_y dp_z/(2\pi\hbar)^3$], the current $\mathbf{j}(\mathbf{p})$ due to a single electron is given by the formula

$$\delta\mathbf{J} = \int \mathbf{j}(\mathbf{p})\,\delta n(\mathbf{p})\,d\tau,$$

where

$$\mathbf{J} = e\int \frac{\partial\varepsilon}{\partial\mathbf{p}}\,n\,d\tau = e\int \mathbf{v}n\,d\tau. \tag{3}$$

Assuming that for some definite function f

$$\delta\varepsilon = \int f(\mathbf{p}\mathbf{p}')\,\delta n'\,d\tau'$$

and that $\partial n/\partial\epsilon = -\delta(\epsilon - \epsilon_0)$, we find

$$\mathbf{j}(\mathbf{p}) = e\left(\mathbf{v}(\mathbf{p}) + \int f(\mathbf{p},\ \mathbf{p}')\,\mathbf{v}(\mathbf{p}')\frac{d\tau'}{d\varepsilon'}\right) \tag{4}$$

(Since $kT \ll \hbar\omega$, we can put $T = 0$ in all formulas).

CALCULATION OF THE DIELECTRIC CONSTANT

From the dispersion relation $\epsilon(\omega)$ it follows that in the region of weak absorption in which we are interested the real part of the dielectric constant must be of the form

$$\varepsilon'(\omega) = a - b/\omega^2. \tag{5}$$

(See Landau and Lifshitz,[2] page 338). a and b are to be found in the usual way by calculating the polarization $\mathbf{P}$ of the electrons in the external field. In this case $\dot{\mathbf{P}} = \mathbf{J}$, so that

$$\ddot{\mathbf{P}} = -\omega^2\mathbf{P} = \int \mathbf{j}\dot{n}\,d\tau = e\int \mathbf{j}(\mathbf{n}\cdot\mathbf{E})\frac{d\tau}{d\varepsilon}. \tag{6}$$

Hence for the case of cubic symmetry we find

$$\varepsilon'(\omega) = 1 - \frac{4\pi e^2}{3\omega^2}\int \mathbf{V}\mathbf{v}\,\frac{d\tau}{d\varepsilon} \approx -\frac{4\pi e^2}{3\omega^2}\int \mathbf{V}\mathbf{v}\,\frac{d\tau}{d\varepsilon} = -\frac{4\pi e^2}{\omega^2 m}N, \tag{7}$$

where we have introduced the notation $\mathbf{j}(\mathbf{p}) = e\mathbf{V}(\mathbf{p})$. It can be seen that the so-called "number of free electrons" N depends fundamentally on their interaction.

ABSORPTION

The electrons in the metal can absorb light by colliding with impurity atoms (or with lattice defects), with each other, or with the surface of the metal; and also by simultaneously absorbing or emitting a phonon. In our case, the total absorption is the simple sum of terms corresponding to each of the above processes. The formulas describing absorption by collision can be greatly simplified by making use of the fact that in the infra-red region

$$\omega t_{\text{coll}} \ll 1, \tag{8}$$

where t_{coll} is the duration of a collision (e.g., the time during which the electron is directly interacting with an impurity atom). In this case the Fourier components of $\ddot{\mathbf{d}}$ (where $\mathbf{d}$ is the dipole moment) which determine the absorption do not depend on the frequency, and can be expressed easily in terms of the current change due to the collision (See Landau and Lifshitz,[3] page 207):

$$\ddot{\mathbf{d}}_\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} \ddot{\mathbf{d}}e^{i\omega t}\,dt \approx \frac{1}{2\pi}\int_{-\infty}^{\infty} \ddot{\mathbf{d}}\,dt = \frac{1}{2\pi}(\mathbf{j}' - \mathbf{j}) = \frac{1}{2\pi}\Delta\mathbf{j},$$

$\mathbf{j}$ and $\mathbf{j}'$ being the current before and after the collision. From this we calculate the expression

$$q = 2\pi\frac{\Delta j_i \Delta j_k}{\hbar\omega^3}\frac{\overline{E_i E_k}}{4\pi}. \tag{9}$$

for the energy absorbed in a collision. The derivation of formula (9) is analogous to the derivation of formula (68.10) in Landau and Lifshitz,[3] from which it can readily be obtained by using the principle of detailed balance. In the case of cubic symmetry we obtain

$$dQ = \frac{2\pi}{3}\frac{(\Delta\mathbf{j})^2}{\hbar\omega^3}\frac{\overline{E^2}}{4\pi}\,d\nu, \tag{10}$$

for the energy absorbed per unit time, where $d\nu$ is the number of collisions per unit time in which the quasi-momentum changes by the indicated amount.

COLLISION WITH IMPURITIES

When electrons collide with impurity atoms the number of collisions per second per cubic centimeter is of the form

$$d\nu = N_i\,n(\mathbf{p})(1 - n(\mathbf{p}'))\,\sigma(\mathbf{p}\cdot\mathbf{n}')\,V(\mathbf{p})\,d\tau\,do', \tag{11}$$

where $\sigma\,do$ is the scattering cross-section, $\mathbf{n}'$ is a unit vector in the direction of the scattered electrons, and N_i is the number of scattering centers per unit volume. Since

$$\varepsilon(\mathbf{p}') = \varepsilon(\mathbf{p}) + \hbar\omega, \quad kT \ll \hbar\omega,$$

the expression $n(\mathbf{p})(1 - n(\mathbf{p}'))$ differs from zero only when

$$\varepsilon_0 - \hbar\omega < \varepsilon(\mathbf{p}) < \varepsilon_0$$

(ϵ_0 is the limiting Fermi energy).

By substituting (11) into (10) and integrating with respect to $d\epsilon$, we obtain the energy dissipation per unit volume per unit time

$$Q_{\text{пр}} = \frac{2\pi}{3}\frac{N_i}{\omega^2}\int (\Delta\mathbf{j})^2 \sigma V \frac{d\tau}{d\varepsilon} do' \cdot \frac{\overline{E^2}}{4\pi}. \tag{12}$$

It now becomes easy to find the real part of the impedance. Comparing (12) with the general expression for energy dissipation in a medium with dielectric constant $\epsilon = \epsilon' + i\epsilon''$:

$$Q = \omega\varepsilon''\overline{E^2}/4\pi, \tag{13}$$

we find the imaginary part of the dielectric constant ϵ'', after which we can find an expression for ζ, the surface impedance,* to the corresponding degree of accuracy (since $\epsilon'' \ll \epsilon'$):

$$\zeta = \zeta' + i\zeta'' = -i/\sqrt{\varepsilon'} + \varepsilon''/2|\varepsilon'|^{3/2}. \tag{14}$$

Therefore

$$\zeta' = \frac{2\pi}{3}\frac{\int (\Delta\mathbf{j})^2 \sigma V\, do'\, d\tau/d\varepsilon}{2|\varepsilon'|^{3/2}\omega^3}. \tag{15}$$

As might be expected, equation (15) differs from the well-known light-absorption formulas only in the substitution of $\mathbf{v} = \partial\epsilon/\partial\mathbf{p}$ for $\mathbf{V}$.

COLLISIONS BETWEEN ELECTRONS

This effect is not usually taken into account in calculating the absorption. This is because in the case of an isotropic or quadratic dispersion law, the change of current during the collision process is zero by virtue of the conservation of momentum, since in the first case the current (if we consider the electrons to be non-interacting) is proportional to their momentum, and in the second case the current is a linear function of the momenta which is not altered by the collision.

During the collision of two electrons,

$$d\nu_{\text{el}} = w_{\text{el}}(\mathbf{p}_1, \mathbf{p}_2; \mathbf{p}_1')\, n_1 n_2 (1 - n_1')(1 - n_2')\, d\tau_1' d\tau_2 d\tau_1. \tag{16}$$

Here w_{el} is the probability of the indicated scattering process, $\mathbf{p}_1$ and $\mathbf{p}_2$ are the momenta of the colliding particles, and $\mathbf{p}'$ is the momentum of one of the scattered electrons (the momentum of the other electron is determined by the conservation of momentum). From the law of conservation of energy,

$$\hbar\omega + \varepsilon(\mathbf{p}_1) + \varepsilon(\mathbf{p}_2) = \varepsilon(\mathbf{p}_1') + \varepsilon(\mathbf{p}_2').$$

Substituting (16) into (10) and then integrating with respect to

(1) $d\varepsilon_1'$ between the limits $\varepsilon_1 + \varepsilon_2 + \hbar\omega - \varepsilon_0 > \varepsilon_1' > \varepsilon_0$,
(2) $d\varepsilon_2$ between the limits $\varepsilon_0 > \varepsilon_2 > 2\varepsilon_0 - \varepsilon_1 - \hbar\omega$,
(3) $d\varepsilon_1$ between the limits $\varepsilon_0 > \varepsilon_1 > \varepsilon_0 - \hbar\omega$,

we obtain

$$Q_{\text{el}} = \frac{2\pi}{3}\frac{2\hbar^2}{3}\int (\Delta\mathbf{j})^2 w_{\text{el}} \frac{d\tau_1' d\tau_2 d\tau_1}{d\varepsilon_1' d\varepsilon_2 d\varepsilon_1}\frac{\overline{E^2}}{4\pi}, \tag{17}$$

where

$$\Delta\mathbf{j} = \mathbf{j}(\mathbf{p}_1) + \mathbf{j}(\mathbf{p}_2) - \mathbf{j}(\mathbf{p}_2') - \mathbf{j}(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_1').$$

It must be kept in mind that because of the energy conservation law w_{el} will contain a factor $\delta[\epsilon_0 - \epsilon(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_1')]$, and so the integral in (17) will really be five-dimensional instead of six-dimensional. ζ_{el}' can be found from [17] by the method already described.

COLLISION WITH THE SURFACE

Let $W_{\text{surf}}(\mathbf{p}, o')\,do'$ be the probability that an electron with momentum $\mathbf{p}$ is scattered into the solid angle do'. Then the number of impacts on a unit area of surface per unit time is equal to

$$d\nu_{\text{surf}} = \mathbf{V}\cdot\mathbf{n}\ w_{\text{surf}} n(1 - n')\, do' d\tau \tag{18}$$

$\mathbf{n}$ is the unit vector normal to the surface of the metal.

From this we can obtain the energy dissipated in 1 cm^2 of area in 1 second:

$$Q_{\text{surf}} = \frac{2\pi}{\omega^2}\int \Delta j_i \Delta j_k\, \mathbf{V}\cdot\mathbf{n}\ w_{\text{surf}}\, do' \frac{d\tau}{d\varepsilon}\frac{\overline{E_i E_k}}{4\pi}. \tag{19}$$

(It must be borne in mind that the component of ζ' which we have calculated here is in general a tensor quantity, even in the case of cubic symmetry). Comparing this with the formula for energy dissipation

$$Q = c\zeta_{\alpha\beta}'\varepsilon'\overline{E_\alpha E_\beta}/4\pi,$$

we can find $\zeta_{\alpha\beta\,\text{surf}}'$.

If we put $w = \text{const} = 1/2\pi$ into Eq. (19) we get the usual formula for absorption during diffuse reflection of electrons from a surface.[4] In actual fact, however, there is no particular reason to suppose that w is constant. In this paper we do not

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*The definition of surface impedance which is used here is different from the usual one. Ordinarily the surface impedance is determined from the equation

$$Z(\omega) = R + iX = (4\pi/c)[E_t/H_t]_{\text{surf}}.$$

In this paper, instead of Z we have used the more convenient quantity $\zeta \equiv Zc/4\pi$ (See Landau and Lifshitz[2]).

consider the case of "specular" reflection, i.e., where the integral in (19) becomes identically zero, since this has no physical significance. In this case one would have to consider higher terms in the expansion of ζ'_{surf} in $v/\omega\delta$.

It is interesting to compare the magnitudes of ζ'_{surf} and ζ'_{el}. It stands to reason that they can be estimated only roughly; in any case, no more accurately than the nearest order of magnitude, since at the present time the functions which enter into the formulas are not known. This is particularly true of ζ'_{el}, for which the expression is of an extremely complicated type. The estimates give

$$\zeta'_{\mathrm{surf}} \sim v/c \sim 10^{-2}; \quad \zeta'_{\mathrm{el}} \sim m^{3/2}\sigma\omega^2/\hbar e n^{1/2} \sim 10^{-30}\omega^2.$$

It can be seen that even up to a frequency of $\omega \sim 10^{14}$, ζ'_{el} is, generally speaking, less than ζ'_{surf}.

As for the absorption of light accompanied by the emission of phonons, Holstein[5] has shown, in the case $\hbar\omega \gg k\Theta$ (Θ being the Debye temperature) that the corresponding component of ζ', like ζ'_{surf}, is independent of ω and is of the same order of magnitude as ζ'_{surf}, and is correspondingly smaller at lower frequencies. It would be very difficult to obtain an exact formula for this, since it depends chiefly on the short wavelength phonons with $\hbar\omega \sim k\Theta$, i.e., with wavelengths of the order of the lattice spacing.

In conclusion I would like to thank Academician L. D. Landau for advice and discussions.

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SCATTERING OF PARTICLES OF ARBITRARY SPIN

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We study the geometrical consequences for elastic scattering of the fact that nuclear particles possess a spin. The scattering matrix for particles of arbitrary spin is constructed, and those quantities which are experimentally measurable (cross section, polarization, and polarization correlation) are expressed in terms of its matrix elements. We consider the question of the completeness of a polarization experiment. We show that to reconstruct the scattering matrix it is necessary to measure either the cross section for scattering of a polarized beam by a polarized target, or the polarization correlation after scattering (with an initially unpolarized state), or finally measure the change of polarization of the incident particles after scattering (repeated scattering). The last experiments will be sufficient only if the spin of the particles in the beam is not less than the target spin.

THE analysis of angular distributions and polarization in nuclear reactions is done by two methods. The first method, that of phase analysis, has been investigated in detail and generalized to the case of arbitrary spins.[1] The second method, that of Dalitz, Wolfenstein and Ashkin,[2] which constructs the scattering amplitude as a function of the initial and final wave vectors and spin operators, has been investigated for reactions involving particles of spin 0, ½ and 1.[3-6] The present paper gives the