INTERACTION OF FIELDS IN THE OVERHAUSER EFFECT

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IN this work we show, within the framework of the relaxation theory of Bloch,[1] that relaxation transitions can change substantially the character of the saturation when two resonant fields act simultaneously on a system with a discrete spectrum. According to Ref. 1, when saturation by the field changes the population of the resonant levels, a change takes place in the population of all the levels that are connected with the resonant ones through relaxation transitions (generalized Overhauser effect). Let us consider the interaction of two fields of frequencies ω_1 and ω_2, $|\omega_{pq} - \omega_1| \leq \Gamma$, $|\omega_{mn} - \omega_2| \lesssim \Gamma$, which have no common resonant level.*

We start with an equation for the addition $\hat{D}(t)$ to the equilibrium density matrix $\hat{\rho}^0$, which determines the resonant portion of the average dipole moment of the system, $\mathbf{d}(t) = \mathrm{Sp}(\hat{\mathbf{d}}\hat{D}(t))$. Let ω_{ab} be the frequency of a transition between levels a and b in the unperturbed system, and let the Hamiltonian of the perturbation be $\hbar\hat{V}(t)$, where

$$\hat{V}(t) = \hat{F}^{\pm}e^{\mp i\omega_1 t} + \hat{\Phi}^{\pm}e^{\mp i\omega_2 t}$$

(summation with respect to the ± sign). According to Ref. 1

$$(\partial/\partial t + i\omega_{ab})D_{ab} + \Gamma_{ab}(\hat{D}) = i[\hat{V}(t),$$

$$\hat{D}]_{ab} = i(\rho_a^0 - \rho_b^0)V_{ab}(t). \qquad (1)$$

The relaxation is described by a linear transformation $\Gamma(\hat{D})$ of a definite type. The coefficients of this transformation represent the reciprocals of the relaxation times for the various transitions, and their final combinations need merely be considered by us as phenomenological parameters

$$\Gamma_{ab}(\hat{D}) = -\sum_{\tau\neq 0} 2\exp(\hbar\tau/kT)\,\Gamma_{ab}^{\tau}D_{a+\tau,b+\tau} + \Gamma_{ab}D_{ab}. \qquad (2)$$

If $\omega_{mn} \neq \omega_{pq}$ the first term must be retained only for $a = b$, for, in any event, when $a \neq b$ it does not contain resonant terms:

$$\Gamma_{ab} = \sum_{\tau}(\Gamma_{aa}^{\tau} + \Gamma_{bb}^{\tau}) - 2\Gamma_{ab}^{0}.$$

We seek the solution in the form

$$\hat{D}(t) = \hat{R} + \hat{P}^{\pm}e^{\mp i\omega_1 t} + \hat{Q}^{\pm}e^{\mp i\omega_2 t}, \qquad (3)$$

retaining only the resonance matrix elements

$$R_{aa} \equiv R_a, \quad P_{pq}^{+}, \; P_{qp}^{-}, \quad Q_{mn}^{+}, \; Q_{nm}^{-}.$$

From the equations for the diagonal terms of the system (1) we find, using the normalization condition $\Sigma R_a = 0$,

$$R_a = T_a^{\nu}\mathrm{Im}\,\nu + T_a^{\mu}\mathrm{Im}\,\mu, \quad \nu = P_{qp}^{-}F_{pq}^{+}, \quad \mu = Q_{nm}^{-}Q_{mn}^{+}. \qquad (4)$$

Here T_a are the coefficients of a transformation that is the inverse of $\Gamma'(D)$.†

We introduce

$$T_{ab} = T_a - T_b, \; \alpha_{ab}^{\pm} = \Gamma_{ab}^{+} \pm i\Delta\omega_{ab}, \; \gamma_{ab} = \rho_a^0 - \rho_b^0,$$

$$\Delta\omega_{pq} = \omega_{pq} - \omega_1, \; \Delta\omega_{mn} = \omega_{mn} - \omega_2.$$

For the "line form" we obtain

$$\frac{Q_{mn}^{+}}{\Phi_{mn}^{+}} = \frac{i\alpha_{mn}^{-}\{\gamma_{mn} - \gamma_{pq}s_{pq}(T_{mn}^{\nu}/T_{pq}^{\nu})\}}{|\alpha_{mn}|^2 + \Gamma_{mn}T_{mn}^{\mu}|\Phi_{mn}|^2\{1 - s_{pq}T_{mn}^{\nu}T_{pq}^{\mu}/T_{pq}^{\nu}T_{mn}^{\mu})\}}, \qquad (5)$$

where the saturation parameter is

$$s_{pq} = \Gamma_{pq}T_{pq}^{\nu}|F_{pq}|^2/(|\alpha_{pq}|^2 + \Gamma_{pq}T_{pq}^{\nu}|F_{pq}|^2);$$

a simple transformation of (5) produces a formula for P_{pq}^{+}/F_{pq}^{+}

It was shown in Ref. 1 that T_{mn}^{μ} and T_{pq}^{ν}, $1/\Gamma_{mn}$ and $1/\Gamma_{pq}$, which have the meanings of effective longitudinal and transverse relaxation times for the corresponding transitions, are positive. From the reciprocity theorem for the equivalent circuit for T_a, it follows that when $\hbar a/kT \ll 1$,

$$T_{mn}^{\nu} = T_{pq}^{\mu}, \; |T_{mn}^{\nu}|/T_{pq}^{\nu} < 1, \quad |T_{pq}^{\mu}|/T_{mn}^{\mu} < 1.$$

In this case, which usually holds for a nuclear spin system, the interaction of the fields, as can be seen from the denominator of formula (5), leads to a narrowing of the lines broadened by the saturation effect.

The extent of the effect is determined by the combination of the "crossover" relaxation times

$$\varkappa = (T_{pq}^{\mu}/T_{mn}^{\nu})/(T_{mn}^{\mu}/T_{pq}^{\nu})$$

and has no longer dependent on the distance between the resonant levels, in contradistinction with the effective level population. In connection with this, if $\varkappa$ is on the order of unity, the influence of the low-frequency resonant field on the resonant field of higher frequency should become noticeable.‡

In conclusion, I thank V. L. German and I. M. Lifshitz for discussion of the results.

*In the presence of a common resonant level, the situation becomes considerably more complicated, owing to the resonant contribution of the transitions through the common level, which acts as an intermediate one with the difference frequency.[1,2] In the usual electron-nuclear level scheme in the Overhauser effect, when the resonant fields for the electrons and nuclei have a common level, failure to take these transitions into account causes the constant hyperfine structure to tend to zero in the final result, which, as is known, gives a finite effect.[1] There is no common level if both resonant fields are due to electron transitions at different nuclear orientations.

†The prime indicates that one of the levels should be replaced by the normalization condition. Formally, the system of equations for T_a corresponds to a certain equivalent dc circuit.

‡We disregard level shifts (for example, the influence of polarization on nuclei on the position of the electron resonance[3]). We notice also that in experiments in which the Overhauser effect is measured with two fields,[4] there is no interaction between the fields, in view of the smallness of one of the resonant fields.

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ON THE THEORY OF PLASMA WAVES IN A DEGENERATE ELECTRON LIQUID

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PLASMA waves in a degenerate electron gas were, apparently, first considered by Gol' dman.[1] However, the electrons in metals can hardly be considered as a gas. It is thus of interest to study the plasma oscillations of a degenerate electron liquid. According to Landau's theory of a Fermi liquid[2] the transport equation for the non-equilibrium correction δn to the distribution function of the quasi-particles (electrons) of a degenerate electron liquid has the form,[3]

$$\frac{\partial \delta n}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}}\left\{\delta n - \delta\varepsilon\frac{\partial n_0}{\partial \varepsilon_0}\right\} + e\mathbf{E}\mathbf{v}\frac{\partial n_0}{\partial \varepsilon_0} = 0. \quad (1)$$

Here n_0 is the equilibrium distribution function, ϵ_0 the electron energy in the equilibrium state, and

$$\delta\varepsilon = \int \Phi(\mathbf{p},\mathbf{p}')\,\delta n(\mathbf{p}',\mathbf{r})\,d\mathbf{p}', \quad (2)$$

where Φ is typical for the theory of a Fermi liquid, reflecting the short-range correlation of the particles. Finally $\mathbf{E}$ is the electric field which is determined from the equation

$$\operatorname{div}\mathbf{E} = 4\pi e\int \delta n d\mathbf{p}. \quad (3)$$

In Eq. (1) collisions are neglected since it is assumed that the frequency of the plasma oscillations is much larger than the collision frequencies.

Considering solutions of Eq. (1) of the form $\delta n_k e^{i(\mathbf{k}\cdot\mathbf{r})-i\omega t}$, and restricting ourselves to the case of long wavelengths, which allows us to expand in powers of k, we obtain from Eqs. (1) to (3), assuming that the Fermi surface is spherical, the following dispersion relation for the dependence of the frequency ω of the plasma waves on the wave vector at long waves

$$\omega^2 = \omega_0^2 + v_0 V_0\left(\frac{3}{5} + A_0 + \frac{4}{25}A_2\right)k^2, \quad (4)$$

where v_0 and p_0 are the velocity and momentum of an electron on the Fermi surface, and A_0 and A_2 coefficients in the expansion in Legendre polynomials

$$\frac{8\pi p_0^2\Phi}{3(2\pi\hbar)^2 v_0} = \sum_n A_n P_n(\cos\chi)$$

(χ is the angle between the vectors $\mathbf{p}$ and $\mathbf{p}'$). Finally

$$\omega_0^2 = 4\pi e^2\cdot 8\pi p_0^2 V_0/3(2\pi\hbar)^3,\ V_0 = v_0(1 + A_1).$$

For a perfect Fermi gas of electrons, $A_n = 0$ and (4) goes over into the corresponding formula of Gol' dman's paper. The author gave in Ref. 4 an estimate of the coefficient A_1 for a number of real metals. It was then shown that it was not at all allowed to neglect this quantity compared to unity.

It is useful to make an estimate for the coefficients A_n for the case when the function Φ is determined by the forward-scattering amplitude calculated in Born approximation for a screened Coulomb potential. In that case

$$A_n = \frac{4}{3\pi}\frac{e^2}{\hbar v_0}\frac{2n+1}{2}\int_{-1}^{1} dx\, P_n(x)\left\{\xi - \frac{1}{4}\frac{1}{1 - x + {}^1/_2\xi}\right\}$$

and correspondingly