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VISCOSITY OF LIQUID He^3 *IN THE RANGE 0.35 — 3.2° K AND* He^4 *ABOVE THE LAMBDA-POINT*

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The viscosity of liquid He^3 was measured in the temperature range 0.35 — 3.2° K by means of a capillary tube viscosimeter. A slight increase in the viscosity was observed in lowering the temperature in the range 1.0 — 3.2° K, and a strong increase was noted below 1° K. The temperature dependence of η in the range 0.5 — 1.1° K is satisfactorily described by the formula $T^{-1/2}$ and only below 0.5° K does the dependence become stronger; however, it is less strong than T^{-2}. The measurements of liquid He^4 above the λ-point show that its temperature dependence is not different from that of ordinary liquids, the region near the λ-point being excluded.

1. THE VISCOSITY OF He^3

In 1950, Pomeranchuk,[1] basing his work on the general quantum theory of liquids developed by Landau, predicted, from qualitative considerations, a viscosity coefficient for He^3 with a temperature dependence of the form $1/T^2$. Later, Abrikosov and Khalatnikov[2] considered in detail the kinetic phenomena in liquid He^3 on the basis of the theory of a Fermi liquid constructed by Landau.[3] In accord with the theory of Abrikosov and Khalatnikov, the viscosity of liquid He^3 below 0.1 — 0.2° K has a temperature dependence of the form $1/T^2$.

Up to the present time there have existed only qualitative measurements in the literature on the viscosity of He^3. These were carried out by Abraham, Osborne, and Weinstock.[4] In their research, the fluidity of liquid He^3 across a thin slit was compared with the fluidity of He^4. From their observations, the authors estimated the value of the viscosity coefficient, and established a weak temperature dependence — an increase in the viscosity from 22 to 30.4μ poise for a temperature drop from 2.79 to 1.04° K. These data were in need of verification. Moreover, we were interested in extending the range of measurement to much lower temperatures.

Recently, we learned of the measurements of Taylor and Dash[5] which were carried out by the method of the torsional vibrations of a disc. The data of Taylor and Dash, which were obtained in the range 1.3 — 2.15° K and at $T = 3.34$° K, agree both in absolute magnitude and in temperature dependence with the results of Abraham, Osborne, and Weinstock.

Below, we shall give a description of the method and tabulate the results of our measurements.

The Method

For the measurement of the viscosity of liquid He^3, we chose the method of the capillary viscosimeter as the most useful for research with very small quantities of the liquid. All the measurements were made in two glass thick-walled capillaries (outside diameter 4 — 5 mm), carefully selected and calibrated. With the help of a metal microscope and an ocular micrometer, we determined the shape and dimensions of the openings at the ends of the capillaries. The form of the opening was shown to be slightly elliptical, but the difference in the dimensions of the ellipses at the ends of the capillaries did not exceed 1%. The mean values of the axes of the ellipses were equal. For the first capillary, of length 35.00 mm, $a = 97.8\mu$ and $b = 105.4\mu$; for the second capillary of length 18.13 mm, $a = 74.1\mu$ and $b = 75.0\mu$. For verification of the form of the cross section at all points along the capillary, a definite amount of mercury was introduced and its length was measured. Such a method of checking showed that the first capillary had an almost barrel-shaped form, while the second capillary had a taper in its length of ~ 1%. Calibration of both capillaries by the flow of gaseous hydrogen and helium at room temperature gave a value of 102.1μ as the effective diameter of the first and 74.5μ for the second which, within the limits of accuracy, coincided with the values obtained from direct measurements. The error in the determination of the diameter of the capillaries amounted to ± 1%.

The principal difficulty of measurement lay in the observation of extraordinary precaution against the contamination of the capillary with foreign particles. In spite of the freedom of He^3 from carbon impurities when cooled to the temperature of liquid air, there was always a sufficient number of foreign particles which easily contaminated the capillaries if protective measures were not taken. Two arrangements (I, II) were used in the research. These are shown in their final form in Fig. 1. Measurements were carried out with apparatus I in the temperature range 1.0 — 3.2°K, while apparatus II served for measurements below 1°K. In this and the other apparatus, both capillaries were used in turn.

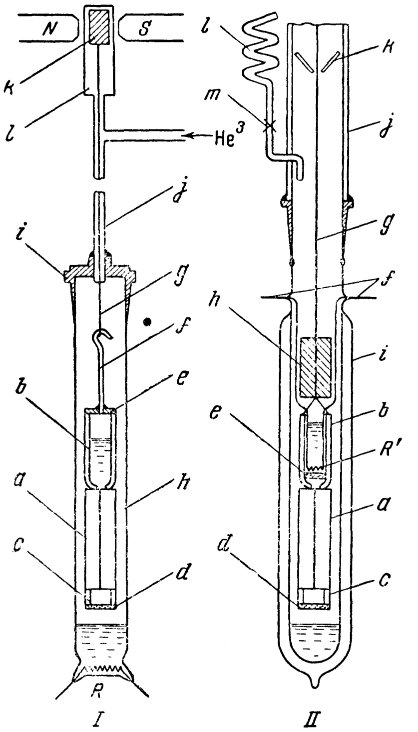

Fig. 1. Schematic drawing of apparatus for the measurement of the viscosity.

Apparatus I for measurements above 1°K contained a capillary a with a glass cylindrical reservoir b attached to it. This cylinder had an internal diameter of 5.35 mm and length 20 mm. For protection against the entrance of impurities from above, there was attached to it a covar cover e with a 1 mm diameter metal pipe f leading into it. A cylinder c of height 2 mm was cemented from below to the capillary with a filter d of fine cotton fiber cemented to its end. This filter trapped particles $\sim 0.1\mu$. Thanks to the large area of the filter (ϕ 5 mm), its resistance was negligibly small in comparison with the resistance of the capillary (the calibration of the capillaries with gas did not reveal differences in the resistance of the capillaries when the filters were present). The reservoir was suspended by the fiber g in a cylindrical test tube h of internal diameter 8 mm, at the bottom of which was placed a resistance thermometer R of 40μ phosphor bronze. The test tube h was connected with a copper-nickel tube j by a copper connector i. The tube was of diameter 2 mm and passed through the cap of the dewar. A steel cylinder k was mounted at the upper end of the fiber and could be adjusted by the magnet NS in the tube, raising and lowering the reservoir b.

He^3 was condensed in the test tube so that the level of the liquid in the emptied capillary was lower than the highest end of the tube f. The liquid flowed into the reservoir below through the capillary, first passing the filter d. A decrease in the temperature of He^3 was achieved by pumping out the He^4 vapors in the surrounding tank. The double helium dewar had a construction similar to that described in Ref. 6. The temperature of the liquid was determined from the vapor pressure of He^3, which was measured by a mercury manometer.

The apparatus for measurements below 1°K (II, Fig. 1) consisted of a dewar i with a volume of about 3 cm^3 and diameter 8 mm, from which was carried out a pumping of the vapors of liquid He^3. The lowest temperature achieved by this method was 0.35°K. In contrast to the apparatus I, the reservoir b was open at the top and had a filter e at the entry to the capillary to serve as protection against contamination. This was in addition to the filter d. The temperature of the liquid was measured by a thirty micron phosphor-bronze resistance thermometer R′, which was located inside the reservoir b. The leads of the thermometer were taken out through the platinum bushing f. The thermometer was first calibrated by means of the vapor pressure of He^3 which was measured by a McLeod gauge. In the temperature interval 0.3 — 1.0°K, the resistance of the thirty micron phosphor-bronze has a linear temperature dependence, which was checked in a separate experiment by means of measurements of the magnetic susceptibility of a paramagnetic salt.

Just as in the apparatus I, the reservoir b was suspended by the filament g and was turned by means of a magnet. Pumping out of the He^3 vapor was carried out by means of a mercury diffusion pump through a copper tube j of 15 mm diameter which was later changed to steel. To decrease thermal conduction inside the tube j, a copper screen k was employed. It was placed in thermal contact with the outside He^4 tank and had a temperature ~ 1°K. The He^3 vapors from the openings of the diffusion pump were condensed in the coil ℓ and the liquid He^3, cooled to 1°K, was transferred to the dewar by means of the valve m. With the exception of the lowest temperatures, the measurements in apparatus II were carried out with the pumping disconnected. In order that the temperature not change significantly during the measuring time of 1 — 2 minutes, a pellet of paramagnetic salt, cooled by the vapors of He^3, was fastened under the reservoir b. Because of its large heat capacity, the salt served as a buffer, lowering the heating rate of the liquid. With the salt in place, it required not less than three hours for the liquid to heat from 0.35 to 1°K.

The process of measurement in both pieces of apparatus was as follows. After filling the reservoir b with the liquid, the capillary was raised until its lower end did not make contact with the level of the liquid outside. Values of the position of the level in the reservoir as a function of time were determined by means of a KM-5 cathetometer and a stop watch. Usually the level in b fell off by 3 — 8 mm, depending on the diameter of the given capillary and the temperature, in the course of about two minutes. At temperatures close to the critical, where the surface tension of He^3 is small, the liquid flowed from the capillary in the form of fine strips of visible flow. At low temperatures, where the surface tension becomes appreciable, drops were formed at the end of the capillary. These were released periodically when they achieved critical dimensions. Observation was carried out with the liquid rendered visible by a luminescent lamp. The cathetometer enabled us to make measurements with the accuracy of ± 0.01 mm. Because of the distortions brought about by the optical inhomogeneities of the glass of the dewar, the errors were somewhat increased; however, they did not exceed ± 0.05 mm. This fact was specially checked by us in Ref. 7.

Results of Measurement

Determination of the coefficient of viscosity was carried out by means of the formula

$$\eta = \frac{(\rho_l - \rho_v)\,g}{2.3\cdot 8\,lR^2}\,\frac{a^3b^3}{8(a^2+b^2)}\,\frac{t}{\log(H_0/H)}\,, \qquad (1)$$

where a and b are the elliptical axes in the cross section of the capillary, ℓ is the length of the capillary, R = radius of reservoir b, ρ_ℓ and ρ_V are respectively the liquid and vapor densities, H = height of the liquid level in the reservoir below the lower end of the capillary, t = time, and g = acceleration due to gravity. In the research, we used He^3 with a purity not lower than 99.98%. Values of the density of the liquid and vapor He^3 in Ref. 1 were taken from the work of Kerr.[8] The temperature was determined according to the scale of Sydoriak and Roberts,[9] with accuracy down to 0.01°K. The correction required by the finite velocity of flow from the capillary, which is about 1%, was not taken into account by us. We also did not consider corrections connected with the formation of small droplets at the end of the capillary. This correction would be of the order of 0.1%.

For the calculation of the viscosity, we constructed a graph of $\log(H_0/H)$ vs. t for each point. A typical graph of this form is shown in Fig. 2. The straight-line dependence of $\log(H_0/H)$ on t, observed at all temperatures, testifies to the laminar flow of the liquid. An estimate of the Reynolds numbers for the capillaries of 102.1 and 74.5 μ gave 600 and 350, respectively.

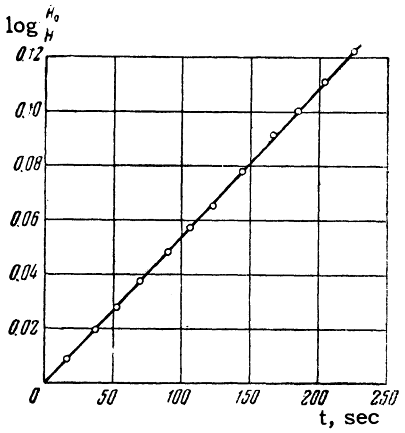

Fig. 2. Dependence of the position of the level in the reservoir on the time (in semilogarithmic plot) for the capillary of diameter 102.1μ at T = 1.105°K.

Figure 3 shows the results of measurements obtained with the two capillaries. The same dependence in logarithmic plot is shown in Fig. 4. The spread of measurements increases at the lower temperatures, which is the result of the instability of the temperature over the measuring interval (~ 0.01 — 0.02° at 0.4 — 0.5°K). The absolute values of the viscosity coefficient were determined with an accuracy to within ± 5% in the region above 1° K, and ± 10% for the very low temperatures. The mean values of η are given in Table I.

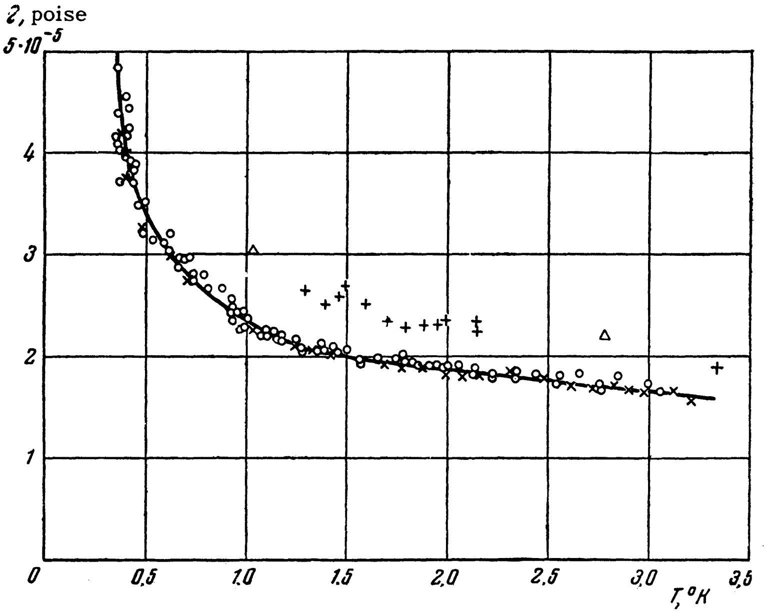

Fig. 3. Viscosity of liquid He³: o = capillary of diameter 102.1μ, × = capillary of diameter 74.5μ, Δ = results of Abraham, Osborne and Weinstock,[4] + = data of Taylor and Dash.[5]

TABLE I. Mean Values of Viscosity of He³

T°K	$\eta\cdot10^5$ poise	T°K	$\eta\cdot10^5$ poise	T°K	$\eta\cdot10^5$ poise
0.35	4.8	0.8	2.63	2.0	1.87
0.40	4.00	0.9	2.46	2.2	1.83
0.45	3.64	1.0	2.33	2.4	1.78
0.50	3.40	1.2	2.14	2.6	1.74
0.55	3.22	1.4	2.02	2.8	1.70
0.6	3.07	1.6	1.96	3.0	1.66
0.7	2.82	1.8	1.92	3.2	1.61

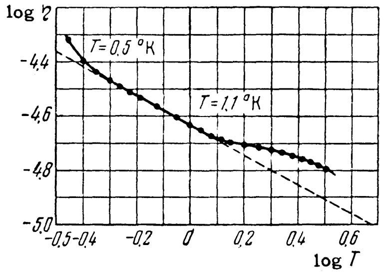

Fig. 4. Viscosity of He³ in semilogarithmic plot.

As is seen from Fig. 3, the viscosity in the temperature range from 1 to 3.2°K increases slightly with decrease in temperature, changing from 1.6×10^{-5} to 2.32×10^{-5} poise, which is in qualitative agreement with one of the variants of the theory of Abrikosov and Khalatnikov.[2] In the temperature range below 1°K, η increases sharply, reaching a value of 4.8×10^{-5} poise at T = 0.35°K. The temperature dependence of η in the interval 1.1 — 0.5°K, as is seen from Fig. 4, is approximately described by the formula $1/\sqrt{T}$ and only below 0.5°K does the dependence become much steeper; however, it is weaker than the dependence $1/T^2$ predicted theoretically, which also agrees with the estimate given in Ref. 2.

We did not succeed in carrying out measurements of the viscosity in the immediate vicinity of the critical point. Measurements in this region are extremely difficult becuase of the small density difference $\rho_l - \rho_v$. It is quite evident, however, that as $T \to T_{cr}$, the viscosities of the liquid and the gas ought to approach the same value. In accord with the measurements of Becker, Misenta, and Schemissner,[10] the viscosity of gaseous He³ at T = 3.35°K has the value of 1.25×10^{-5} poise, which agrees within 20% with our extrapolated values. In Fig. 3 we have also plotted the results of Abraham, Osborne, and Weinstock,[4] and the data of Taylor and Dash.[5] From a comparison of the results, it is evident that while the temperature dependence of the viscosity in Refs. 4 and 5 agrees with our measurements, the absolute values are approximately 20% higher, which is apparently explained by systematic errors in the determination of the constants of the apparatus.

TABLE II. Mean Values of the Viscosity of He⁴

T°,K	$\eta\cdot10^5$ poise	T°,K	$\eta\cdot10^5$ poise
2.2	2.55	3.0	3.30
2.3	2.83	3.2	3.27
2.4	3.00	3.4	3.25
2.5	3.15	3.6	3.20
2.6	3.21	3.8	3.15
2.7	3.26	4.0	3.07
2.8	3.28	4.2	3.00

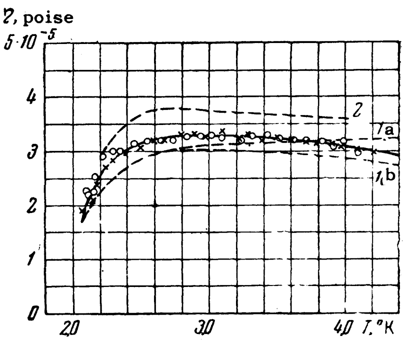

Fig. 5. Viscosity of liquid He^4: o = capillary 102.1μ, × = capillary 74.5μ; 1a – data of Bowers and Mendelssohn,[13] 1b – the same data, recomputed by us with account of the density of the vapor; 2 – temperature curve of Taylor and Dash.[5]

2. VISCOSITY OF He^4 ABOVE THE λ-POINT

To check our method, we simultaneously carried out measurements of the viscosity of He^4 in the temperature range above the λ-point. The measurements were carried out with apparatus I. The dependence of $\log(H_0/H)$ on t had the form of a straight line, similar to that shown in Fig. 2. The viscosity was calculated by Eq. (1), in which the values of ρ_l were taken from the work of Kamerlingh-Onnes and Bok (see Ref. 11), the values of ρ_v from the measurements of Keesom[12] and R. Berman.*

The results of the measurements, taken with the two capillaries, are given in Fig. 5. The average values of the viscosity coefficient are tabulated in Table II. For comparison, the data of Bowers and Mendelssohn[13] (broken line 1a) are also plotted in Fig. 5. These were obtained by the same method. The results of Taylor and Dash (broken line 2) are also given. The results of our measurements agree with the data of Ref. 13 in absolute value (within the limits of error); however, instead of a monotonic increase in the viscosity, we observed a slight falling off, beginning at the temperature of 2.8°K. The cause of the divergence lies in the fact that in Ref. 13, the vapor density of He^4 was not taken into consideration. Close to the critical point, this quantity becomes comparable with the density of the liquid. The data of Bowers and Mendelssohn, recomputed by us with account of the density correction, give the curve 1b, which in its temperature dependence does not differ from our curve. The measurements of Taylor and Dash,[5] as is seen in Fig. 5, have a temperature dependence which agrees with ours, but the absolute value of the viscosity coefficient is approximately 15% higher. We note that even in the region below the λ-point, the values of the viscosity of the normal component of He II (in the measurements of Dash and Taylor[14]) exceed by 10% our data obtained from measurements of surface attenuation of second sound.[15] It should be emphasized that the temperature dependence of the viscosity of He I, contrary to established opinion, does not differ from the same for ordinary liquids, with the exception of the region close to the λ-point. The monotonic decrease of the viscosity of He I with temperature does not contradict the value of the vapor viscosity at the critical point, which is equal to $\sim 15\mu$ poise according to the measurements of Ref. 10.

In conclusion I express my gratitude to Professor V. P. Peshkov for his discussion of the results and his constant interest in the work, and to N. I. Iakovlev, who helped in assembling the apparatus and in the measurements.

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