ON THE QUANTUM-KINETIC EQUATION FOR A SYSTEM OF CHARGED PARTICLES OF MANY KINDS

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Submitted to JETP editor November 26, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 523-524 (February, 1958)

By the method of Bogoliubov[1,2] one can obtain a natural generalization of the quantum-kinetic equation for the description of stochastic processes in a system of charged particles in which the particles belong to an arbitrary number $M \geq 2$ of different kinds. If in the spatially homogeneous case (cf. Ref. 3) one substitutes into the quantum equation for the distribution function $F_a(p;t)$ of any single chosen particle of type a the solution of the system of quantum equations for the quantum correlation-deviation functions g_{ab}, then one can obtain the quantum-kinetic equation for the plasma of many kinds of particles. Since, however, the determination of the exact solution for the functions g_{ab} is a difficult task, it is expedient to introduce the simplifying assumption that the Coulomb and exchange interactions are weak and that the chosen particle exerts only a small reaction influence on the behavior of the large number of charged particles surrounding it. With these assumptions, since an exchange interaction is possible only between particles of the same kind, we find

$$\frac{\partial w_a(p;t)}{\partial t} = \sum_{(1\leqslant b\leqslant M)} \frac{2\pi n_b}{(2\pi\hbar)^6\hbar} \int \left\{ \frac{\nu_{ab}(|p-p'|/\hbar)\,\nu_{ab}(|p'-p|/\hbar)}{1+B_{ab}((p'-p)/\hbar;\ (p+p')/2)} \right.$$
$$\left. \pm \frac{\nu_{ab}(|p-p_1'|/\hbar)\,\nu_{ab}(|p'-p_1|/\hbar)}{1+B_{ab}((p-p_1')/\hbar;\ (p_1'+p)/2)} \right\} \delta(p+p_1-p'-p_1')$$
$$\times\,\delta(E_a+E_{1,b}-E_a'-E_{1,b}') \qquad (1)$$
$$\times[(1\pm n_a w_a(p;t))(1\pm n_b w_b(p_1;t))\,w_a(p';t)\,w_b(p_1';t)$$
$$-(1\pm n_a w_a(p';t))(1\pm n_b w_b(p_1';t))$$
$$\times\, w_a(p;t)\,w_b(p_1;t)]\,dp_1dp'dp_1' + \delta R_a.$$

Here

$$w_a(p;t) = \frac{(2\pi)^3}{v}F_a(p;t);\quad E_a = \frac{p^2}{2\mu_a};$$
$$\nu_{ab}(|k|) = \int e^{ikq}\Phi_{ab}(|q|)\,dq; \qquad (2)$$
$$B_{ab}(k,\ p) = |\,\mathrm{Re}\, i\sum_c n_c \frac{\nu_{cc}(|k|)}{(2\pi)^9 h^4}\iint_0^\infty [e^{i\hbar k\tau/2}-e^{-i\hbar k\tau/2}]$$
$$\times\exp\{i\theta k(p'/\mu_b - p/\mu_a)+i\tau(\eta-p')\}\,w_b(\eta)\,d\theta\, d\tau\, d\eta\, dp'\,|;$$

n_c, μ_c are the concentration and particle mass of the particles of type c in the plasma, $c = 1, 2, \ldots, M$; v is the average volume per particle; and δR_a, a term taking into account the actions on each other of the set of neighboring particles, can be approximately evaluated by a method analogous to that used in Ref. 4.

For heterogeneous systems of large numbers of charged particles obeying the Fermi statistics, under conditions of complete degeneracy the approximate expression for the screening coefficient B_{ab} takes the form

$$B_{ab}(k,\ p) = \frac{1}{r_D^2k^2} + \frac{1}{2r_D^2k^2}\left(\frac{\mu_b kp}{\mu_a kp_0}\right)\ln\left|\frac{1-\mu_b kp/\mu_a kp_0}{1+\mu_b kp/\mu_a kp_0}\right|. \qquad (3)$$

Here $r_D = (p_0^2 v/12\pi\mu_b\sum_c n_c e_c^2)^{1/2}$ is the Debye radius for the distribution of the Fermi particles of type b, p_0 is the average upper limit momentum for the system of many kinds of particles, and e_c is the charge of a particle of type c in the plasma.

I express my deep gratitude to Academician N. N. Bogoliubov for directing this work.

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Translated by W. H. Furry
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THE MAGNETIC SUSCEPTIBILITY OF A UNIAXIAL ANTIFERROMAGNETIC

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Submitted to JETP editor November 26, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 524-525 (February, 1958)

In a large number of papers,[1] the high frequency magnetic susceptibility of an antiferromagnetic is found by using the concept of the precession of the magnetic moments of the sublattices. These authors do not take into account relaxation processes. Our problem is to take relaxation processes into account, in the spirit of the equations of Landau and Lifshitz,[2] and to connect the constants entering into the phenomenological equations of motion of the magnetic moments with the experimentally-observed quantities (static magnetic susceptibility, resonance frequency, and antiferromagnetic resonance line width).

We consider for the sake of simplicity a uniaxial ferromagnetic (we take the chosen axis along the z axis). The equation of motion of the magnetic moments of each of the two sublattices has the form

$$\partial \mathbf{M}_j / \partial t = g \mathbf{M}_j \times \mathbf{H}_j^{\text{eff}} - \gamma M^{-2} \mathbf{M}_j [\mathbf{M}_j \times \mathbf{H}_j^{\text{eff}}], \quad (j = 1,\ 2). \tag{1}$$

Here

$$\mathbf{H}_1^{\text{eff}} = \mathbf{H} - \alpha \mathbf{M}_2 - \lambda (\mathbf{M}_1 - \mathbf{M}),$$
$$\mathbf{H}_2^{\text{eff}} = \mathbf{H} - \alpha \mathbf{M}_1 - \lambda (\mathbf{M}_2 + \mathbf{M}); \tag{2}$$

α is a constant describing the exchange interaction between the sublattices (the order of magnitude of α is determined by the ratio $\Theta_C / \mu M \sim 10^2$, where $\mu = g\hbar$ is the Bohr magneton, Θ_C the Curie temperature, and M the magnetic moment of each of the sublattices), λ is the anisotropy constant ($\lambda \sim 10^{-2}\alpha$, since the anisotropy energy is not determined by the large isotropic exchange interaction but by a relativistic interaction of the spin-orbit coupling type), while the constant γ describes relaxation processes. In the absence of an external field, which we assume here to be applied at right angles to the z axis, the magnetic moments of the sublattices are respectively equal to $\mathbf{M}_1 = \mathbf{M}$, $\mathbf{M}_2 = -\mathbf{M}$.

Assuming the variable magnetic field $\mathbf{H} = \mathbf{H}' e^{-i\omega t}$ to be small we linearize (2)

$$\partial \boldsymbol{\mu}_1 / \partial t = -g [\mathbf{M},\ \alpha(\boldsymbol{\mu}_1 + \boldsymbol{\mu}_2) + \lambda \boldsymbol{\mu}_1] + \alpha\gamma(\boldsymbol{\mu}_1 + \boldsymbol{\mu}_2) + \lambda\gamma\boldsymbol{\mu}_1 + g[\mathbf{M} \times \mathbf{H}] + \gamma \mathbf{H},$$
$$\partial \boldsymbol{\mu}_2 / \partial t = g [\mathbf{M},\ \alpha(\boldsymbol{\mu}_1 + \boldsymbol{\mu}_2) + \lambda \boldsymbol{\mu}_2] + \alpha\gamma(\boldsymbol{\mu}_1 + \boldsymbol{\mu}_2) + \lambda\gamma\boldsymbol{\mu}_2 - g[\mathbf{M} \times \mathbf{H}] + \gamma \mathbf{H}. \tag{3}$$

Taking into account the fact that μ_1 and μ_2 are proportional to $e^{-i\omega t}$ we find (we use the fact that $\lambda \ll \alpha$, $gM \gg \gamma$)

$$\boldsymbol{\mu} = \boldsymbol{\mu}_1 + \boldsymbol{\mu}_2 = 2 \frac{\lambda (gM)^2 + i\omega\gamma}{2\alpha\lambda g^2 M^2 - \omega^2 - 2i\omega\alpha\gamma} \mathbf{H}. \tag{4}$$

Hence

$$\chi_\perp = 2 \frac{\lambda (gM)^2 + i\omega\gamma}{2\alpha\lambda (gM)^2 - \omega^2 - 2i\omega\alpha\gamma}$$

Or, since $1/\alpha = \chi_\perp(\omega = 0) \equiv \chi_\perp(0)$,

$$\chi_\perp(\omega) = \chi_\perp(0) \frac{\omega_0^2(\omega_0^2 - \omega^2) + 2i\omega\Gamma(2\omega_0^2 - \omega^2)}{(\omega_0^2 - \omega^2)^2 + 4\omega^2\Gamma^2}, \tag{5}$$

where

$$\omega_0 = gM\sqrt{2\alpha\lambda}, \quad \Gamma = \alpha\gamma. \tag{6}$$

For $\omega \ll \omega_0$

$$\chi(\omega) = \chi_\perp(0)(1 + 4i\omega\Gamma / \omega_0^2),$$

and for $\omega \sim \omega_0$ (near the resonance frequency)

$$\chi_\perp(\omega) = \chi_\perp(0) \frac{2i\omega_0^3\Gamma}{(\omega_0^2 - \omega^2)^2 + 4\omega_0^2\Gamma^2}.$$

Let us note some facts:

1. $\chi_\perp(\omega)$ does not contain gyrotropy: the rotation of the moments of the sublattices proceeds in such a manner that the total magnetic moment is directed along the magnetic field. This will not be the case in a strong magnetic field, applied along the specimen axis, since $\mathbf{M}_1(H_0) \neq -\mathbf{M}_2(H_0)$.

2. The line width Γ is determined not only by relativistic effects as a ferromagnetic (γ), but also by the exchange interaction energy (α).

3. If the antiferromagnetic is a metal, the exchange interaction will lead to an additional line broadening, produced by the effect of the inhomogeneity of the magnetic moments (see Refs. 3 – 5).

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Translated by D. ter Haar
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