Mathieu equation fall off exponentially indicate that the reduction of the discontinuities with ϵ for the case of an analytic function $\kappa_1(k_2)$ is exponential.

The author wishes to take this opportunity to thank I. M. Lifshitz for a discussion of the results and A. Ia. Povzner for illuminating remarks concerning the mathematical aspects of the problem considered herein.

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Translated by H. Lashinsky
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LATERAL DISTRIBUTION OF PHOTONS NEAR THE AXIS OF EXTENSIVE ATMOSPHERIC SHOWERS

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Submitted to JETP editor November 20, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 516-518 (February, 1958)

It is known that it is very difficult to calculate the lateral-distribution function of the soft component of extensive atmospheric showers with allowance for the cascade processes and for the ionization losses. However, at small values of r, the principal role is played by high-energy particles, for which the ionization losses can be neglected. In the works by Pomeranchuk[1] and Migdal[2] the lateral distribution function of electrons with energies greater than a given value has been calculated for small r, with the ionization losses neglected*

$$N(t,\ E,\ r) \sim 1 / r^{2-s}, \tag{1}$$

and with the parameter s determined from the condition

$$-\lambda_1'(s)\, t = \ln(E_0/\beta) - \ln(R/r)$$

(the same result was obtained by Nishimura and Kamata[4]).

Let us determine the photon density corresponding to such an electron distribution. For this purpose we use one of the Landau equations (see Ref. 5):

$$\partial\Gamma(t,\ E,\ \mathbf{r},\ \theta)/\partial t + \theta\partial\Gamma(t,\ E,\ \mathbf{r},\ \theta)/\partial\mathbf{r}$$

$$= -\sigma_0\Gamma(t,E,\mathbf{r},\theta) + \int_E^\infty P(t,\ E',\ \mathbf{r},\ \theta)\,\varphi_{\mathrm{rad}}(E',\ E)\,dE', \tag{2}$$

where $\varphi_{\mathrm{rad}}(E', E)$ is the probability that an electron with energy E' will radiate a photon with energy E, r is the radius vector in the transverse plane, and θ is the projection of the direction of motion of the particles on this plane.

To solve our problem it is quite enough to put $\varphi_{\mathrm{rad}}(E', E) = 1/E$. Then, integrating over all θ, and also over the azimuth in the plane perpendicular to the axis of the shower (taking account of the symmetry of the problem in the last equation), Eq. (2) can be rewritten†

$$\partial N_\Gamma(t,\ E,\ r)/\partial t = -\sigma_0 N_\Gamma(t,\ E,\ r) + N(t,\ E,\ r)/E. \tag{3}$$

We assume for N(t, E, r) the expression given in Ref. 3

$$N(t,\ E,\ r) \approx e^{\lambda_1(s)t} E_k^{-s}\,[1 - (rE/E_k)^{2-s}]/r^{2-s}(2-s).$$

A solution of Eq. (3), with boundary conditions at t = 0, $N_B = 0$, is

$$N_\Gamma(t,\ E,\ r) \approx \frac{e^{\lambda_1(s)t} - e^{-\sigma_0 t}}{\lambda_1(s) + \sigma_0}\,\frac{E_k^{-s}[1 - (rE/E_h)^{2-s}]}{r^{2-s}(2-s)E}.$$

This expression is correct for $E_k/E_0 < r < E/E_k$.

Let us now determine the ratio $\dfrac{N_1(t, > E, r)}{N(t, E, r)}$, where $N_1(t, > E, r)$ is the number of photons with energies greater than the given value. Taking it into account that at a fixed value of r the particle energy E cannot be greater than E_k/r (see Ref. 3), we obtain the following value for N_1:

$$N_1 \sim \ln(E_k/rE)/r^{2-s}. \tag{4}$$

Let us note that in the derivation of Eq. (3) we used the condition $rE/E_k \ll 1$, which determines the boundary of applicability of formula (1). Thus, we obtain

$$N_1/N \sim \ln(E_k/rE). \tag{5}$$

*The symbols used here are the same as in the book by Belen'kii.(Ref. 3).

†It can be shown that for our problem it is possible to neglect the term containing the derivative with respect to r.

This result is in good agreement with the data given by Moliere[6] for $rE/E_k \ll 1$.

By way of an example it is easy to calculate that for $E = 10^8$ ev, at a distance of 1 m from the axis of the shower and at a primary-electron energy of $E_0 = 10^{14}$ ev, we have $N_1/N \approx 7-8$. This effect can be explained by the fact that the high-energy electrons located near the shower axis are accompanied by a greater number of photons. For distances $r \geq E_k/E$ it is necessary to take ionization losses into account.

In conclusion I express my gratitude to I. L. Rozental' for advice and aid in this work, and also to I. P. Ivanenko for useful discussions.

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Translated by J. G. Adashko
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CONTRIBUTIONS TO THE THEORY OF DISPERSION RELATIONS

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Submitted to JETP editor November 19, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 518-519 (February, 1958)

IT was shown by Bogoliubov, Logunov, and the author[1] that for those processes in which μ, e, and ν particles participate on par with strongly interacting particles, the anti-Hermitian part of the amplitudes expressed through the action of such weakly-interacting particles is equal to zero to first order in the weak-interaction constant C. This simplifies considerably the consequences of the dispersion relations as well as their form. Namely, the dispersion relations lead in this case to the statement that the amplitude for the process depends polynomially upon the sum of the 4-momenta of the weakly interacting particles (for decay, upon the difference), while the polynomial coefficients depend only upon the momenta of the strong-interacting particles. If as usual, the interaction Lagrangian does not contain derivatives of the fields, then the amplitudes are independent of the momenta of the weakly-interacting particles. It is easy to consider similarly processes in which only weakly-interacting particles participate (for example, the decay of the μ meson). The causality principle leads in this case to a Lagrangian which is local in all the fields, and the dispersion relations lead to the statement that the amplitude depends only polynomially upon the momenta.

It is worth noting that the weak interaction cases allow simple analyses on the basis of dispersion theory. For example, the wide spread opinion that in order to obtain the dispersion relations it suffices to apply the principle of causality formulated through the vanishing of the probability current commutator of space-like points, is easily seen to be incorrect.

Indded, to first order in C, the probability current commutator for weakly-interacting particles is zero over all space for any Lagrangian including a non-local one. Generally, non-local Lagrangians do not lead to polynomial dependence. Consider, for example, the Lagrangian:

$$L(x) = \int K(\xi^2)\,\varphi(x+\xi)\,\varphi(x-\xi)\,\psi(x+\xi)\,\psi(x-\xi)\,d\xi.$$

Applying to it perturbation theory, we obtain the following expression for the scattering amplitude:

$$S(p,q;\,p',q') = \delta(p+q-p'-q')\,\overline{K}((q+p)^2),$$

where q, q′ are the momenta of the scattered particles, and p, p′ are the momenta of the scatterers.

In this fashion, the dependence of the amplitude upon the momenta is determined from the kernel of the interaction Lagrangian and, in general, is not polynomial. Note that in deriving the dispersion relations, Goldberger et al.[2] also make use of time-ordered operators, in addition to the causality principle in the form of a commutator. These two conditions are combined in the generalized formulation of the principle of causality, and the formulation now proves sufficient to obtain the dispersion relations. From the example of weak interactions, it is easy to verify that the dispersion relations may similarly arise in certain non-local interactions. Consider for example the