ture gradient can be set up which will persist for some time. A set of two glass test tubes, containing condensed mixtures whose composition lay within the concentration limits given above, were placed in a Dewar flask over liquid helium. The test tubes were so placed that the level corresponding to the temperature of onset of crystallization passed through them. The boundary between the solid and liquid phases established itself at different levels in mixtures of different composition, being at a lower level in mixtures richer in hydrogen (see a in the phase diagram, which is a schematic representation of the test tubes). The boundary could be caused to shift by moving the test tubes to a region of lower temperature. As the amount of the solid phase increased, the liquid phase became increasingly richer in hydrogen, and the boundary in tubes with hydrogen-rich mixtures overtook the boundary in tubes containing more deuterium, corresponding to the equalization of liquid-phase concentrations when the peritectic region is reached. After the solid-liquid boundary had reached the same level in both tubes (see b in the diagram), further crystallization proceeded at the same temperatures in both tubes.

In addition to the thermal analysis, an x-ray study of the hydrogen-deuterium mixtures, and of the pure isotopes, was made. Some improvements in the method of taking the photographs made it possible to eliminate the parasitic lines which were present in the patterns of previous work, as has been explained. As a result, two lines were found to be present due to the hydrogen lattice, corresponding to spacings $d = 3.15$ A and $d = 2.79$ A; but from the deuterium lattice there was only one, corresponding to $d = 2.84$ A. No lines were found at wide angles, because of the rapid falling off of the scattered intensity. The x-ray patterns from mixtures with hydrogen concentrations lying in the interval 20 to 80% contained lines from both the hydrogen and the deuterium lattices, with parameters only slightly altered. This confirms the conclusion we have expressed already, that the structure of hydrogen was incorrectly determined by the workers at Leyden, and that there is a region of concentration in which hydrogen and deuterium exist as a mixture of two solid phases. As to the accurate determination of the structures of hydrogen and deuterium, the question must remain open for the time being, because of the difficulty of assigning definite indices to x-ray patterns which contain so few lines. It is, however, possible to state unequivocally that they do have different lattices.

The results which we have obtained, indicating a separation of the solid mixture of hydrogen isotopes into two phases, agree with the conclusions of Prigogine et al.[3] that there is a crticial temperature below which a solid solution of isotopes must separate. For the hydrogen isotopes, however, these authors estimated a critical temperature below 1° K; I. Lifshitz and Stepanova[4] have shown that the critical temperature for separation is equal to $T = \epsilon^2 \Theta_D$ ($\epsilon = \Delta m/m$ and Θ_D is the Debye temperature); i.e., for hydrogen isotopes it is of the same order as their melting point. This agrees completely with the shape of the phase diagram for the hydrogen-deuterium system, as reported in this letter.

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Translated by D. C. West
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CORRECTION TO THE ARTICLE "ON THE STRUCTURE OF THE ELECTRON SPECTRUM IN LATTICES OF THE TELLURIUM TYPE"

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IN this paper[1] the conclusion was drawn, from the relations (22) and the fact that Λ_2 and Λ_3 are complex conjugate quantities, that the terms E_2 and E_3 are in contact along the entire k_z axis. It is, however, easy to convince oneself that for the "rotating" elements $U_2^{(j)}$ the equation

$$\sum_{j=1}^{3} \chi_{i} [(U_2^{(j)})^2] = 3, \tag{1}$$

holds. According to Herring[2] this indicates the absence of supplementary contact of the zones because of time reversal. This is due to the fact that only $\psi_{k_z,3}$ and $\psi_{-k_z,2}$ belong to a single value of the energy, and not $\psi_{k_z,3}$ and $\psi_{-k_z,3}$. The statements just made follow from the relations

$$\widetilde{C}_3^1\psi_{k_z,3} = \omega \exp\{ibk_z/3\}\,\psi_{k_z,3};$$

$$\widetilde{C}_3^1\psi_{-k_z,3} = \exp\{-ibk_z/3\}\,\omega^2\psi_{-k_z,2} \tag{2}$$

The functional dependence $E(k_z)$ for $k_x = k_y = 0$ can be found by the following considerations. It is well known that displacement by a vector of the reciprocal lattice takes the entire spectrum over into itself, i.e., we have the equations

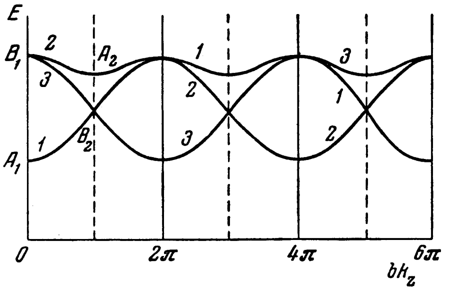
$$E(k_z + 2\pi/b) = E(k_z), \quad \psi_{n,\,k_z+2\pi/b} = \psi_{n,\,k_z}. \tag{3}$$

For $k_z \to 0$ we get the result that $U_{j,2\pi/b} = e^{\frac{2\pi i}{b}z} U_{j,0}$. Thus for the "open" element C_3^1 the laws of transformation of $U_{j,0}$ and $U_{j,2\pi/b}$ will be different:

$$\widetilde{C}_3^1 U_{1,\,2\pi/b} = \omega^2 U_{1,\,2\pi/b}; \quad \widetilde{C}_3^1 U_{3,\,2\pi/b} = U_{3,\,2\pi/b};$$

$$\widetilde{C}_3^1 U_{3,\,2\pi/b} = \omega U_{2,\,2\pi/b}. \tag{4}$$

But if we agree to characterize the n-th term of Eq. (1) of Ref. 1 by the transformation law $U_{n,k}$, then the term $E_1(2\pi/b)$ must be regarded as the continuation of $E_3(k_z)$ for $k_z \to 2\pi/b$, and so on. In the notations adopted, E_j will be periodic functions of $k_z b$ with the period 6π.



The diagram shows the pattern of the term splitting, and indicates the course of the terms through the interval $0 \le k_z b \le 6\pi$ by the numbers 1, 2, 3. As follows from the diagram, the calculations of the dispersion law carried out in Ref. 1 for points of type Λ have meaning only in case of accidental crossing of the terms E_2 and E_3 (which is permissible in principle, because of their different symmetries). From considerations of symmetry one can show that the points A_1 and A_2 in the diagram are points of zero slope. But it makes sense to suppose that there can also be present additional points of zero slope, connected with the concrete form of the potential $U(\mathbf{r})$, as occurs in electronic germanium, in which the minimums are not located at the point $\mathbf{k} = 0$, although this is a point of zero slope. These assumptions impose the following limitations on the functions U_1 and U_2 that transform according to the representation Γ_3: $p_{1,2}^z = 0$. All the remaining quantities $p_{i,j}^x = p_{i,j}^y = p_{i,j}^z = 0$ by the selection rules (here $i, j = 1, 2$). Then the points B_1, B_2 (see diagram) can be extremal points. The functional dependence of $E(\mathbf{k})$ for the points Γ, A, K, H was obtained in Ref. 1 on just this assumption. It must be kept in mind that the contact of the bands along the z axis is maintained only in the approximation quadratic in k. All the results of Ref. 1 are correct with the reservations that have been indicated. As regards the valence band, at the point B_2 in the diagram the degeneracy is in all probability removed on account of the spin-orbit interaction; otherwise for the four p electrons of tellurium the band would not be completely filled, and tellurium would possess metallic properties. For the conduction electrons the assumption of a strong spin-orbit splitting does not appear indispensable.

It is interesting to note that the results obtained in Ref. 1 are entirely applicable for lattices of the type $C_{3v}^1 - C_{3v}^6$, for which the group of the wave vector $\mathbf{k} = 0$ is C_{3v}. In this case the contact of the bands along the k_z axis is due to the fact that the group of the wave vector is not changed by displacement along the k_z axis and the point $\mathbf{k} = 0$ is a point of zero slope. The only difference is that in Eq. (33) one must replace all terms of the form Ak_zk_x by Ak_zk_y. A similar treatment can be carried through for the point $\mathbf{k} = 0$ in lattices for which the space groups are related to the point groups C_{4v}, D_{2d}, D_4. In this case $\mathbf{k} = 0$ is a point of zero slope and the zones make contact* along the z axis (for D_{2d} this is a consequence of the symmetry with respect to time reversal[2]). The dispersion law of $E(\mathbf{k})$ is obtained from a secular equation of the form:

$$\begin{vmatrix} (C_1 + C_2)\,k_x^2 + C_4k_y^2 + C_3k_z^2 - E, & (C_1 - C_2 + C_4)\,k_xk_y \\ (C_1 - C_2 + C_4)\,k_xk_y, & (C_1 + C_2)\,k_y^2 + C_4k_x^2 + C_3k_z^2 - E \end{vmatrix} = 0. \tag{5}$$

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*For D_4 the zones make contact only in the approximation quadratic in k_z.

For $C_1 = 0$ Eq. (5) is the same as Eq. (33) of Ref. 1 with $A = 0$. Thus the two ellipsoids[3] tangent at the k_z axis are the limiting case of the equipotential surfaces for the whole sequence of space groups $D_{2d}^1 - D_{2d}^{12}$, $C_{3v}^1 - C_{3v}^{12}$, $C_{4v}^1 - C_{4v}^{12}$, $D_4^1 - D_4^{10}$, and this can hold for tellurium type lattices if the relation $p_{1,2}^z = 0$ is fulfilled.

In conclusion I take occasion to thank E. I. Rashba, who called my attention to the illegitimacy of the conclusions drawn in Ref. 1 from the relations (22).

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Translated by W. H. Furry
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POTENTIAL WELLS FOR CHARGED PARTICLES IN A HIGH-FREQUENCY ELECTROMAGNETIC FIELD

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As is well known, in source-free regions of an electrostatic field there can be no absolute maxima or minima of the potential; this fact excludes the possibility of maintaining a charged particle in a state of stable equilibrium (Earnshaw's theorem). This same situation also excludes the possibility of localizing particles, if by localization we mean a state in which a particle with energy smaller than some given magnitude cannot go beyond the limits of a bounded region, no matter what the initial conditions.

The above statement does not apply in the case of a high-frequency electromagnetic field where, as we shall show below, localization of particles can be accomplished.

Consider a particle of charge e and mass m moving in an external electromagnetic field $\mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r})e^{i\omega t}$, $\mathbf{H}(\mathbf{r}, t) = \mathbf{H}(\mathbf{r})e^{i\omega t}$. In the nonrelativistic approximation the equation of motion is

$$\ddot{\mathbf{r}} = \eta\mathbf{E}(\mathbf{r}, t) + (\eta/c)\dot{\mathbf{r}}\times\mathbf{H}(\mathbf{r}, t), \qquad (1)$$

where $\eta = e/m$. If the frequency of the external field ω is sufficiently high, the solution of Eq. (1) can be written as a sum of a slowly varying (in terms of the oscillation period of the external field) function $\mathbf{r}_0(t)$ and an oscillating function $\mathbf{r}_1(t)$ (frequency ω). Assuming that $\mathbf{r}_1(t)$ is much smaller than the distance L over which the amplitude of the external field changes markedly,

$$|\mathbf{r}_1(t)| \ll L, \qquad (2)$$

and neglecting terms of order $|\mathbf{r}_1/L|$ and $|\dot{\mathbf{r}}_0/L|$, averaging Eq. (1) over the period of the high-frequency field we obtain an equation for $\mathbf{r}_0(t)$:

$$\ddot{\mathbf{r}}_0(t) = -\nabla\Phi, \quad \Phi = (\eta/2\omega)^2|\mathbf{E}|^2 \qquad (3)$$

Thus, the time average of the force acting on the particle is derivable from a potential; the potential is proportional to the square of the modulus of the electric intensity and is independent of the sign of the charge.

There are an unlimited number of possibilities for creating potential wells $\Phi(\mathbf{r})$. The simplest of these are realized in quasi-electrostatic multipole fields

$$\mathbf{E}(\mathbf{r}, t) = \nabla\{r^n P_n^m(\cos\theta)\cos m\varphi\}\, e^{i\omega t},$$

where r, θ, and φ are the spherical ordinates and the P_n^m are the associated Legendre polynomials. For example, the potential Φ in the field of a quasi-static axial quadrupole ($m = 0$, $n = 2$) is of the form $\Phi = \text{const}\, r^2(1 + 3\cos^2\theta)$, i.e., there is an absolute minimum at the origin.*

To determine the motion of the particle inside the potential well we consider the first integral of Eq. (3):

$$\frac{m}{2}\left(|\dot{\mathbf{r}}_0|^2 + \frac{\eta^2}{2\omega^2}|\mathbf{E}|^2\right) = \frac{m}{2}\left(|\dot{\mathbf{r}}_0|^2 + \frac{1}{2}|\dot{\mathbf{r}}_1|^2\right)$$
$$= \text{const} = |e|V_0. \qquad (4)$$

The left-hand part of Eq. (4) is equal to the time average of the kinetic energy of the particle, where the kinetic energy of the oscillatory (with frequency ω) motion plays the role of the potential energy.

If the $E = 0$ at the center of the potential well, particles with energy less than or equal to V_0, are localized within a region at the boundaries of which the following conditions are satisfied

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*It is interesting to note that in an axially-symmetric quadrupole field the original equation (1) in Cartesian coordinates leads to three Mathieu equations; this allows us to analyze the properties of the solution without the limitation imposed by (2).