experiment.[7] We therefore believe that computations of the total cross section for a diffuse surface nucleus is of definite interest.

In conclusion we take this opportunity to thank I. A. Kropin and B. F. Chumakov for carrying out a large part of the numerical calculations.


[1] Frank, Grammel, and Watson, Phys. Rev. **101**, 891 (1956).

[2] R. M. Sternheimer, Phys. Rev. **101**, 384 (1956).

[3] Anderson, Davidson, and Kruse, Phys. Rev. **100**, 339 (1955).

[4] Fernbach, Serber, and Taylor, Phys. Rev. **75**, 1352 (1949).

[5] A. I. Akhiezer and I. Ia. Pomeranchuk, Некоторые вопросы теории ядра (Certain Problems in Nuclear Theory), GITTL, M., 1950.

[6] J. O. Kessler and L. M. Lederman, Phys. Rev. **94**, 689 (1954).

[7] Byfield, Kessler, and Lederman, Phys. Rev. **86**, 17 (1952).

[8] D. H. Stork, Phys. Rev. **93**, 868 (1954).

[9] A. M. Shapiro, Phys. Rev. **84**, 1063 (1951).

[10] Chedester, Isaacs, Sachs, and Steinberger, Phys. Rev. **82**, 958 (1951).

[11] R. L. Martin, Phys. Rev. **87**, 1052 (1952).

[12] Ivanov, Osipenkov, Petrov, and Rusakov, J. Exptl. Theoret. Phys. (U.S.S.R.) **31**, 1097 (1956), Soviet Phys. JETP **4**, 922 (1957).

[13] Dzhelepov, Ivanov, Kozodaev, Osipenkov, Petrov, and Rusakov, J. Exptl. Theoret. Phys. (U.S.S.R.) **31**, 923 (1956), Soviet Phys. JETP **4**, 864 (1957).

[14] Ignatenko, Mukhin, Ozerov, and Pontecorvo, J. Exptl. Theoret. Phys. (U.S.S.R.) **31**, 546 (1956), Soviet Phys. JETP **4**, 351 (1957).

[15] Isaacs, Sachs, and Steinberger, Phys. Rev. **85**, 718 (1952).

[16] Pevsner, Rainwater, Williams, and Lindenbaum, Phys. Rev. **100**, 1419 (1955).

[17] Fowler, Fowler, Shutt, Thorndike, and Whittemore, Phys. Rev. **91**, 135 (1953).

[18] Kozodaev, Suliaev, Fillipov, and Shcherbakov, J. Exptl. Theoret. Phys. (U.S.S.R.) **31**, 701 (1956), Soviet Phys. JETP **4**, 580 (1957).

[19] Nikol'skii, Kudrin, and Ali-Zade, J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 48 (1957), Soviet Phys. JETP **5**, 93 (1957); L. P. Kudrin and B. A. Nikol'skii, Dokl. Akad. Nauk SSSR **111**, 795 (1956), Soviet Phys. "Doklady" **1**, 708 (1956).

[20] Fregeau, Helm, and Hofstadter, Physica **22**, 1195 (1956).

[21] L. S. Kisslinger, Phys. Rev. **98**, 761 (1955).


Translated by M. A. Melkanoff
33

## *RADIATIVE CORRECTIONS TO BREMSSTRAHLUNG*


P. I. FOMIN

Khar'kov State University



Submitted to JETP editor October 11, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 227-228 (January, 1958)


FEYNMAN'S method,[1] which is usually used to obtain radiative corrections, cannot be applied in practice to bremsstrahlung because of the much greater calculational difficulties than those encountered, for instance, in the application to the corrections to Compton scattering.[2] In the present work, we use the so-called mass-operator method,[3,4] which has many advantages over the Feynman method.

The expression for the renormalized mass operator to the necessary order in $e^2$ has been obtained by Newton.[4] Not only the mass operator, but also vacuum polarization contributes to the radiative corrections to bremsstrahlung. In order to avoid the infrared catastrophe, as usual, we make use of the fictitious photon mass $\lambda$. We eliminate $\lambda$ from the final expression by adding to the usual bremsstrahlung cross section the cross section for double bremsstrahlung, when two photons are emitted simultaneously and the energy of one of these photons is less than some quantity $\Delta E$ determined by the accuracy of the measurement.

The total cross section is conveniently written in the form

$$d\sigma \doteq d\sigma_0 [1 - (e^2/\pi)(\delta_R + \delta_D)], \quad e^2 = 1/137,$$

where $d\sigma_0$ is the cross section for the basic process and is given by the Bethe-Heitler formula,[5,6] $\delta_R$ gives the radiative corrections, and $\delta_D$ gives the double bremsstrahlung.

Exact expressions for $\delta_R$ and $\delta_D$, together with a description of the method by which they have been calculated, will be presented in a detailed article. Here we shall give only some limiting values of $\delta_R$ and $\delta_D$. We choose units in which $\hbar = c = m = 1$, and make use of the following notation: $\mathbf{p}_1$ and $\epsilon_1$ are the initial, and $\mathbf{p}_2$ and $\epsilon_2$ are the final electron momentum and energy; $\mathbf{k}$ and $\omega = \epsilon_1 - \epsilon_2$ are the momentum and energy of the emitted photon;

$$\varkappa = -2\omega(\varepsilon_2 - p_2\cos\theta_2), \quad \theta_2 = \widehat{\mathbf{kp}_2};$$

$$\tau = 2\omega(\varepsilon_1 - p_1\cos\theta_1), \quad \theta_1 = \widehat{\mathbf{kp}_1};$$

$$\alpha = \varkappa + \tau;$$

$$\rho = -2 + \alpha + 2(\varepsilon_1\varepsilon_2 - p_1p_2\cos\theta), \quad \theta = \widehat{\mathbf{p}_1\mathbf{p}_2};$$

$$4\sinh^2 x = \rho, \quad 4\sinh^2 y = \rho - \alpha; \quad xh(x) = \int_0^x u\coth u\, du.$$

1. The limiting case of low frequencies ($\omega\epsilon_1 \ll 1$). In this case

$$|\varkappa|, \tau \ll 1, \quad \rho \approx 4p^2\sin^2(\theta/2);$$

$$\delta_R = 2[1 - 2x\coth(2x)](\ln\lambda + 1) + x\tan x + 4x\coth(2x) \times [h(2x) - h(x)] + \frac{2x}{\sinh(2x)}\frac{\rho}{4\varepsilon^2 - \rho} + w(x) + O(\omega\varepsilon_1); \quad (1)$$

$$\delta_D = 2[1 - 2x\coth(2x)]\ln\frac{2\Delta E}{\lambda} + \frac{1}{v}\ln\frac{1-v}{1+v} + \frac{(1-v^2)\cosh(2x)}{v\sin\theta/2}G(v,\theta) + O(\omega\varepsilon_1); \quad (2)$$

where

$$G(v, \theta) = \int_{\cos(\theta/2)}^{1} \frac{du}{(1 - v^2u^2)(u^2 - \cos^2(\theta/2))^{1/2}}\ln\frac{1+vu}{1-vu};$$

$$p_1 \approx p_2 = p, \quad \varepsilon_1 \approx \varepsilon_2 = \varepsilon, \quad v = p/\varepsilon.$$

Here and below we use

$$w(x) = 2(1 - x\coth x)(1 - {}^1/_3\coth^2 x) - {}^2/_9$$

to denote the contribution from vacuum polarization. Expressions (1) and (2) coincide, as may have been expected, with the radiative corrections to elastic scattering as calculated by Schwinger[7] (see also Akhiezer and Berestetskii,[6] Sec. 45).

2. The relativistic case ($\epsilon_1$ and $\epsilon_2 \gg 1$) with low energy losses ($\omega \ll \epsilon_1$ and $\epsilon_2$) at small angles ($\theta$, $\theta_1$, and $\theta_2 \sim 1/\epsilon \ll 1$; we note that at high energies it is just such small angles that give the main contribution). In this case

$$|\varkappa| = -\varkappa = \omega/\varepsilon_2 + \omega\varepsilon_2\theta_2^2 \ll 1,$$

$$\tau = \omega/\varepsilon_1 + \omega\varepsilon_1\theta_1^2 \ll 1, \quad \rho = \varepsilon_1\varepsilon_2\theta^2 \sim 1;$$

$$\delta_R = 2[1 - 2x\coth(2x)](\ln\lambda + 1) + x\tanh x + 4x\coth(2x) \times [h(2x) - h(x)] + w(x) + O(\varkappa, \tau); \quad (3)$$

$$\delta_D = 2[1 - 2x\coth(2x)]\ln\frac{2\Delta E}{\lambda} - 4x\coth(2x)[h(2x) - h(x)] + 2\ln(2\varepsilon)[2x\coth(2x) - 1] + O\left(\frac{\omega}{\varepsilon}\ln\varepsilon, \frac{\omega}{\varepsilon}\right). \quad (4)$$

It should be noted that cases 1 and 2 overlap in the region $\epsilon_1$, $\epsilon_2 \gg 1$; $\omega\epsilon_1 \ll 1$; $\theta$, $\theta_1$, $\theta_2 \sim 1/\epsilon \ll 1$), and that in this region Eqs. (1) and (2) become (3) and (4).

3. In the extreme relativistic limit, when

$$\varepsilon_1, \varepsilon_2, \omega \gg 1; \quad \ln(\rho - \alpha), \quad \ln\rho, \quad \ln|\varkappa|, \quad \ln\tau \gg 1,$$

but

$$\ln\frac{\rho - \alpha}{|\varkappa|}, \quad \ln\frac{\rho - \alpha}{\tau}, \quad \ln\frac{\rho}{|\varkappa|}, \quad \ln\frac{\rho}{\tau} \sim 1,$$

the quantities $\delta_R$ and $\delta_D$ are

$$\delta_R = 2(1 - 2y)\ln\lambda + 2y^2 - 3y + {}^4/_3 x + O(1); \quad (5)$$

$$\delta_D = 2(1 - 2y)\ln\frac{2\Delta E}{\lambda} + 2y^2 - 2y\ln\frac{\rho - \alpha}{4\varepsilon_1\varepsilon_2} - \ln\varepsilon_1\varepsilon_2 + O(1), \quad (6)$$

where

$$2x = \ln\rho, \quad 2y = \ln(\rho - \alpha).$$

In the nonrelativistic limit

$$\delta_R = {}^2/_3\{-(\mathbf{p}_1 - \mathbf{p}_2)^2(\ln\lambda + {}^1/_5) + 2(\mathbf{p}_1 - \mathbf{p}_2)\mathbf{k}\ln\omega\} + O(p^3); \quad (7)$$

$$\delta_D = {}^2/_3(\mathbf{p}_1 - \mathbf{p}_2)^2\left(\frac{5}{6} - \ln\frac{2\Delta E}{\lambda}\right) + O(p^3); \quad (8)$$

$$2\omega = p_1^2 - p_2^2.$$

In the limit as $\omega \to 0$ when $\mathbf{p}_1 = \mathbf{p}_2 = \mathbf{p}$ these expressions lead to the nonrelativistic limit of the corrections to elastic scattering,[7] namely

$$\delta_R + \delta_D = \frac{8}{3}p^2\sin^2\frac{\theta}{2}\left[\frac{19}{30} - \ln(2\Delta E)\right].$$

From (7) and (8) it follows that in the limit as $p_1 \to 0$, the corrections vanish.

The author is grateful for valuable advice and discussions to Professors A. I. Akhiezer, V. F. Aleksin, D. V. Volkov, and S. V. Peletminskii.

[1] R. P. Feynman, Phys. Rev. **76**, 749, 769 (1949).

[2] L. M. Brown and R. P. Feynman, Phys. Rev. **85**, 231 (1952).

[3] J. Schwinger, Proc. Natl. Acad. Sci. **37**, 455 (1951).

[4] R. G. Newton, Phys. Rev. **94**, 1773 (1954).

[5] H. Bethe and W. Heitler, Proc. Roy. Soc. (London) **146**, 83 (1934).

[6] A. I. Akhiezer and V. B. Berestetskii, Квантовая электродинамика (Quantum Electrodynamics), M., 1953.

[7] J. Schwinger, Phys. Rev. **76**, 790 (1949).

Translated by E. J. Saletan
34