N_3 and M_3 are the concentration and mass of particles of the third component and Q_{13} is the diffusion interaction cross section for ions of the first isotope with particles of the third component. The second equation for v_1 and v_2 is obtained from the first, as usual, by the interchange of the indices 1 and 2. In the presence of several additional components, each of them gives an additional term of the type of Eq. (8) in the right side of Eq. (4).

For the case of strong fields where it is possible to disregard the thermal motion of the atoms we obtain, by a method completely analagous to that applied in Ref. 3, an exact result for the drift velocity of an ion of an isotope in an isotopic mixture

$$v_1 = [2eE / \pi M_1 (N_1 + N_2) q]^{1/2}. \qquad (9)$$

This result is obtained under the assumption that the charge exchange cross section is independent of velocity (in the method applied this need not necessarily be so, and is assumed for simplicity). The drift velocities in this case are inversely proportional to the square roots of the masses of the ions. This conclusion is physically obvious inasmuch as in the assumption of quiescent atoms the difference in their masses plays no role.

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POLARIZATION OF SLOW NEUTRONS SCATTERED IN CRYSTALS

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Formulas are derived for the polarization and cross section of slow neutrons scattered in crystals with polarized nuclei.

ROSE[1] has examined the polarization effects arising in the scattering of slow neutrons in crystals, and has obtained formulas for the scattering cross section of polarized neutrons in crystals with polarized nuclei. He considered only crystals of atoms with a single isotope. In the present note, his results are generalized to include crystals of several isotopes, and expressions obtained for the change in neutron polarization due to scattering.

It is well known[2] that the scattering amplitude for slow neutrons in a crystal is proportional to the quantity

$$\left(f\mu' \left| \sum_j e^{i\mathbf{q}\mathbf{R}_j} \left(A_j + \frac{1}{2} B_j \sigma \mathbf{I}_j\right) \right| i\mu\right) = F, \qquad (1)$$

where i and f are the initial and final states of the scatterer; μ and μ' are the initial and final neutron spin projections; $\sigma/2$ is the neutron spin; $\mathbf{q}$ is the momentum transferred to the scatterer; $\mathbf{R}_j$ and $\mathbf{I}_j$ are the coordinates and spins of the j-th nucleus; and A_j, B_j are complex constants. The summation is over all the nuclei in the crystal.

From (1) it is easy to obtain the following expression for the coherent and incoherent parts of the scattering cross section, averaged over all orientations of the neutron and nuclear spins, and over all possible distributions of isotopes in the lattice:

$$\sigma_{\mathrm{coh}} = \sigma_{\mathrm{coh}}^{(0)} \{1 + |\langle A \rangle|^{-2} [(\mathbf{pN}) \operatorname{Re} \langle A^* \rangle \langle BIP \rangle + {}^1/_4 |\langle BIP \rangle|^2]\}; \qquad (2)$$

$$\sigma_{\text{incoh}} = \sigma^{(0)}_{\text{incoh}}\{1 + [\langle |A|^2\rangle - |\langle A\rangle|^2$$

$$+ {}^1/_4 \langle |B|^2 I(I+1)\rangle]^{-1} [(\mathbf{pN})\,\mathrm{Re}\,(\langle A^*BIP\rangle - \langle A^*\rangle\langle BIP\rangle)$$

$$- {}^1/_4 (\mathbf{pN}) \langle |B|^2 IP\rangle - {}^1/_4 |\langle BIP\rangle|^2]\}, \tag{3}$$

Here $\mathbf{p}$ is the polarization vector of the neutron, $\mathbf{N}$ is a unit vector defining the direction in which the nuclei are polarized, P is their polarization, $\langle\ldots\rangle$ means an average over all distributions of isotopes in the lattice, and $\sigma^{(0)}$ is the coherent or incoherent cross section for unpolarized nuclei. It is clear that formulas (2) and (3) hold for both the differential and total scattering cross sections. If the scatterer consists of one isotope only, (2) and (3) reduce to the expressions obtained by Rose.

It is not difficult to compute the polarization of the neutrons after they have been scattered in a definite direction. We use the formula

$$\mathbf{p}_1 = \mathrm{Sp}\,(F^+\sigma F\rho)\,/\,\mathrm{Sp}\,(F^+F\rho),$$

where ρ is the density matrix describing the polarization of the neutrons before scattering. We obtain the formula

$$\mathbf{p}_1 = (\sigma_{\text{coh}} + \sigma_{\text{incoh}})^{-1}\{\sigma^{(0)}_{\text{coh}}[\mathbf{p} + |\langle A\rangle|^{-2}(\mathbf{N}\,\mathrm{Re}\,\langle A^*\rangle\langle BIP\rangle$$
$$+ \mathrm{Im}\,\langle A^*\rangle\langle BIP\rangle[\mathbf{Np}] + {}^1/_4|\langle BIP\rangle|^2[2(\mathbf{Np})\mathbf{N} - \mathbf{p}])]$$
$$+ \sigma^{(0)}_{\text{incoh}}[\mathbf{p} + (\langle |A|^2\rangle - |\langle A\rangle|^2 + {}^1/_4\langle |B|^2 I(I+1)\rangle)^{-1}$$
$$\times \{\mathbf{N}\,\mathrm{Re}\,(\langle A^*BIP\rangle - \langle A^*\rangle\langle BIP\rangle) + [\mathbf{Np}]\,\mathrm{Im}\,(1^*BIP\rangle$$
$$- \langle A^*\rangle\langle BIP\rangle) + {}^1/_2\langle |B|^2\hat{D}\rangle\mathbf{p} + {}^1/_4\langle |B|^2 IP\rangle\mathbf{N}$$
$$- {}^1/_4|\langle BIP\rangle|^2[2(\mathbf{pN})\mathbf{N} - \mathbf{p}]\}]\}. \tag{4}$$

In this formula, $\hat{\mathrm{D}}$ is a tensor related to the polarization tensor $\hat{\mathrm{T}}$ of the nuclei by the equation

$$D_{\alpha\beta} = \frac{1}{3} I(2I-1)\left(T_{\alpha\beta} - 2\frac{I+1}{2I-1}\,\delta_{\alpha\beta}\right), \tag{5}$$

where

$$T_{\alpha\beta} = \frac{3}{2I(2I-1)}\left(\overline{I_{j\alpha}I_{j\beta} + I_{j\beta}I_{j\alpha}} - \frac{2}{3}I(I+1)\,\delta_{\alpha\beta}\right), \tag{6}$$

and $\hat{\mathrm{D}}\mathbf{p}$ is a vector with components $(\hat{\mathrm{D}}\mathbf{p})_{\alpha\beta} = \mathrm{D}_{\alpha\beta}\mathrm{p}_{\beta}$. The bar means average over spin projections.

If the incident neutron had not been polarized, then its polarization after scattering would be

$$\mathbf{p}_1 = \mathbf{N}(\sigma_{\text{coh}} + \sigma_{\text{incoh}})^{-1}\{\sigma^{(0)}_{\text{coh}}|\langle A\rangle|^{-2}\,\mathrm{Re}\,\langle A^*\rangle\langle BIP\rangle$$
$$+ \sigma^{(0)}_{\text{incoh}}\frac{\mathrm{Re}\,(\langle A^*BIP\rangle - \langle A^*\rangle\langle BIP\rangle) + {}^1/_4\langle |B|^2 IP\rangle}{\langle |A|^2\rangle - |\langle A\rangle|^2 + {}^1/_4\langle |B|^2 I(I+1)\rangle}\}. \tag{7}$$

If $\sigma_{\text{coh.}} \gg \sigma_{\text{incoh.}}$, which will hold, for example, in the direction of a Bragg peak,

$$\mathbf{p}_1 = \mathbf{N}\,\frac{\mathrm{Re}\,\langle A^*\rangle\langle BIP\rangle}{|\langle A\rangle|^2 + {}^1/_4|\langle BIP\rangle|^2}. \tag{8}$$

If, on the other hand, $\sigma_{\text{coh.}} \ll \sigma_{\text{incoh.}}$, which will hold, for example for very slow neutrons,[2] then

$$\mathbf{p}_1 = \mathbf{N}\,\frac{\mathrm{Re}\,(\langle A^*BIP\rangle - \langle A^*\rangle\langle BIP\rangle) + {}^1/_4\langle |B|^2 IP\rangle}{\langle |A|^2\rangle - |\langle A\rangle|^2 + {}^1/_4\langle |B|^2 I(I+1)\rangle - {}^1/_4\langle BIP\rangle|^2}. \tag{9}$$

Expression (9) reduces to

$$\mathbf{p}_1 = \mathbf{N}\,P/[I(1-P^2)+1]. \tag{10}$$

if all the scatterer atoms are of one isotope.

If the nuclei are unpolarized, but are oriented $(\mathrm{P} = 0,\ \hat{\mathrm{T}} \neq 0)$, then

$$\mathbf{p}_1 = \frac{\sigma^{(0)}_{\text{incoh}}\langle |B|^2\hat{D}\rangle\mathbf{p}}{2(\sigma^{(0)}_{\text{coh}} + \sigma^{(0)}_{\text{incoh}})[\langle |A|^2\rangle - |\langle A\rangle|^2 + {}^1/_4\langle |B|^2 I(I+1)\rangle]}. \tag{11}$$

If, in addition, $\sigma_{\text{incoh.}} \gg \sigma_{\text{coh.}}$ and the scatterer consists of one isotope only, then

$$\mathbf{p}_1 = 2\hat{D}\mathbf{p}/I(I+1). \tag{12}$$

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