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POSSIBLE EXPERIMENTS ON INELASTIC SCATTERING OF NUCLEONS. I

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Possible experiments are considered for the investigation of inelastic collisions of nucleons with the reaction scheme $N + N \to \pi + N + N'$, which represents the most intense inelastic processes in the ~ 650-Mev region. Some nontrivial experiments are suggested on the angular correlation of π mesons and nucleons based on the existence of azimuthal asymmetry in the emission of these particles. Analogous effects in a polarized nucleon beam are also considered. The calculations are performed in terms of ten inelastic transition amplitudes which are assumed to provide the principal contribution to the total cross section for $N + N \to \pi + N + N'$ at 650 Mev.

NUCLEON scattering experiments ~ 650 Mev which are intended to serve as the basis of a quantitative phase analysis include both elastic and inelastic scattering. We are already well acquainted with the program of experiments on elastic scattering including the observation of various polarization effects.[1,2] However, for inelastic processes whose total cross section at ~ 650 Mev is about equal to the total elastic cross section[3] the experimental program has not yet been completely considered. Theoretical articles have been concerned with only the reaction $p + p \to \pi^+ + d$,[4-8] and have not considered the more intense inelastic processes in the ~ 650-Mev region: $p + p \to \pi^+ + n + p$ and $p + p \to \pi^0 + p + p$. It is the purpose of the present article to fill this gap and to determine the experimental possibilities for the observation of the reactions represented by $N + N \to \pi + N + N'$ In the first part the angular π-meson distribution and the π-meson-nucleon angular correlation are analyzed for both polarized and unpolarized nucleon beams. Calculations are carried through for the two reactions $p + p \to \pi^+ + n + p$ and $p + p \to \pi^0 + p + p$.

1. GENERAL COMMENTS

We know that strong absorption by a scattering center complicates substantially the character of elastic scattering. When it is necessary to determine the phase shifts of different partial waves through observation of an elastic process, the absorption intensities of these partial waves are also required. The latter information can be obtained by analyzing the angular and spin characteristics of secondary particle emission for all possible elastic process channels. The most complicated case is that of π-meson production: $N + N \to \pi + N + N'$, which is characterized by the final emission of three particles.

Any specific instance of this process for which the energy of the incident nucleon is known can be characterized completely by giving the energy of one of the particles, the polar angles (θ, φ) of the direction of flight of this particle in the center-of-mass system of the colliding nucleons, and the polar angles of emission for the remaining two particles in their center-of-mass system. The differential cross section will thus depend on five arguments.

For the purpose of determining the angular part of the wave function for the three ultimate particles, the three-body problem is usually reduced to a two-body problem by introducing two subsystems. This can be accomplished in two ways. In the first, two nucleons constitute one subsystem. This division enables us to add separately the nucleon spins in one subsystem and the orbital angular momenta in each subsystem. Such a procedure involving two subsystems is therefore usually called an ℓs-scheme. The angular part of the combination of two subsystems is written as

$$\Psi^{(ls)} = \sum_{m+\mu=M} (l_\pi j m \mu \mid l_\pi j J M)\, Y_{l_\pi}^{m}(\pi)\, Y_j^{\mu}(1,2)\,.$$

Considering the orbital angular momentum of relative motion of the two nucleons and their spins, we finally obtain

$$\Psi^{(ls)} = \sum_{\substack{m+\mu=M \\ m_l+m_s=\mu \\ m_{s_1}+m_{s_2}=m_s}}\sum\sum (l_\pi j m \mu \mid l_\pi j J M)(l s m_l m_s \mid l s j \mu)$$
$$\times (s_1 s_2 m_{s_1} m_{s_2} \mid s_1 s_2 s m_s) \times Y_{l_\pi}^{m}(\pi)\, Y_l^{m_l}(1,2)\, \chi_{s_1}^{m_{s_1}}(1)\, \chi_{s_2}^{m_{s_2}}(2), \quad (1)$$

where

$$Y_{l_\pi}^{m}(\pi) \equiv Y_{l_\pi}^{m}(\theta_\pi, \varphi_\pi), \quad Y_l^{m_l}(1,2) \equiv Y_l^{m_l}(\theta_{12}, \varphi_{12})$$

are the spherical harmonics of the respective arguments. The angles θ_π, φ_π characterize the outgoing direction of the π meson in the c.m.s. of the two colliding nucleons and the angles θ_{12}, φ_{12} denote the outgoing direction of nucleons "1" in the coordinate system which is associated with the center of mass of inelastically scattered nucleons. Nucleon "2" is ejected in the opposite direction so that $\theta_{21} = \pi - \theta_{12}$ and $\varphi_{21} = \varphi_{12} + \pi$.

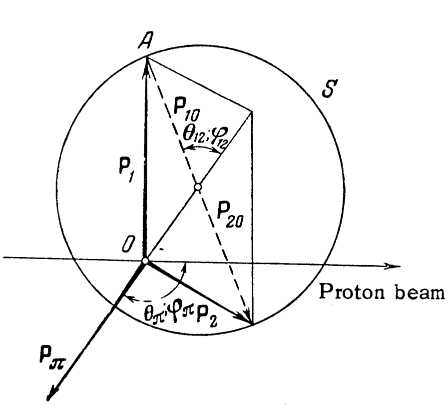

FIG. 1

The representation of the angular part of the wave function by Eq. (1) is most suitable for experiments which observe the π mesons. On the other hand, when we are interested in experiments where only the nucleons are observed, it is most convenient to employ the jj-scheme of two subsystems where one of the subsystems contains the π meson and one of the nucleons. It must be emphasized that the ℓs- and jj-schemes for writing the angular part correspond to the two possible expansions of the angular part of the three-particle wave function in the eigenfunctions of the two subsystems. Which of these two expansions is more suitable, i.e., which contains the smaller number of amplitudes that provide the principal contribution to the cross section, depends on the mechanism of π-meson production and on the character of interaction of the ejected particles, and can only be decided experimentally.

The angular part of the three-particle wave function in the jj-scheme is written as follows:

$$\Psi^{(jj)} = \sum_{\substack{m+m_j=M \\ m_l+m_{s_1}=m_j \\ m_2+m_{s_2}=m}}\sum\sum (j L m_j m \mid j L J M)(l_2 s_2 m_2 m_{2s} \mid l_2 s_2 L m)$$
$$\times (l_\pi s_1 m_l m_{1s} \mid l_\pi s_1 j m_j) \times Y_{l_2}^{m_2}(2)\, Y_{l_\pi}^{m_l}(\pi 1)\, \chi_{s_1}^{m_{s_1}}(1)\, \chi_{s_2}^{m_{s_2}}(2). \quad (2)$$

Figure 1 is a vector diagram of the secondary particle momenta for the reaction N + N → π + N + N′ in the c.m.s. of the colliding nucleons. When the secondary nucleons can be considered approximately, neglecting their relativistic mass increase, the geometrical locus A of the end of a nucleon vector is a sphere of radius $R = [M(W - \sqrt{p^2+m^2} - p^2/4]^{1/2}$, where W is the total kinetic energy in the c.m.s., p is the momentum of the π meson, and M and m are the nucleon and π-meson masses. For this case it is easily shown that the differential of the phase volume density for two particles in the ℓs-scheme, $d^2N/d\Omega\,(\theta_{12}, \varphi_{12})\,dW$, is independent of the angles θ_{12}, φ_{12} and is determined by the magnitude of the radius R. For the jj-scheme this no longer holds true: $d^2N/d\Omega\,(\theta_{\pi 1}, \varphi_{\pi 1})\,dW$ depends on $\theta_{\pi 1}$ but is independent of $\varphi_{\pi 1}$.

2. STATEMENT OF THE PROBLEM

In our calculations we have used Berestetskii's formalism[6] for the study of polarization effects in inelastic collisions of nucleons.

The initial function for two protons, one of which (in the target) is unpolarized, has the following form:

$$\Psi_{\text{init}} = \varphi_1 + \varphi_2,$$

$$\varphi_1 = \frac{\varepsilon_1}{\sqrt{2}}\sqrt{4\pi}\Big\{\sum_{l=0}^{\infty}(-i)^l\sqrt{2l+1}\,[1 - (-1)^l]\Big[q_1\sum_J^{\infty}(l101 \mid l1J1)\,Y_{J(l1)}^{1} + \frac{q_2}{\sqrt{2}}\sum_J^{\infty}(l100 \mid l1J0)\,Y_{J(l1)}^{0}\Big]$$
$$+ \sum_{l=0}^{\infty}(-i)^l\sqrt{2l+1}\,[1 + (-1)^l]\,\frac{q_2}{\sqrt{2}}(l000 \mid l0l0)\,Y_{l(l0)}^{0}\Big\}; \quad (3)$$

$$\varphi_2 = \frac{\varepsilon_2}{\sqrt{2}}\sqrt{4\pi}\Big\{\sum_{l=0}^{\infty}(-i)^l\sqrt{2l+1}\,[1 - (-1)^l] \times \Big[q_2\sum_J^{\infty}(l10-1 \mid l1J-1)\,Y_{J(l1)}^{-1} + \frac{q_1}{\sqrt{2}}\sum_J^{\infty}(l100 \mid l1J0)\,Y_{J(l1)}^{0}\Big] \quad (4)$$
$$- \sum_{l=0}^{\infty}(-i)^l\sqrt{2l+1}\,[1 + (-1)^l]\,\frac{q_1}{\sqrt{2}}(l000 \mid l0l0)\,Y_{l(l0)}^{0}\Big\}.$$

The coefficients ϵ_1 and ϵ_2 are subject to the condition $|\epsilon_1|^2 = |\epsilon_2|^2 = \frac{1}{2}$; $\epsilon_1^* \epsilon_2 = 0$ and describe the unpolarized target, while $q_1 = \cos\frac{\vartheta}{2} e^{-i\delta/2}$; $q_2 = \sin\frac{\vartheta}{2} e^{i\delta/2}$ are coefficients that describe the spin function of a proton beam which is completely polarized in the direction (ϑ, δ).

In this article, the possibilities of different independent experiments are considered only for one special case, which takes into account a limited number of inelastic transition amplitudes. Certain qualitative generalizations are then based on this special case. Specifically, we shall assume that for the ℓs-scheme with incident proton energies $\sim$ 650 Mev it is sufficient to consider the transitions listed in the table.

The amplitude of each transition is a function of the π-meson momentum and incident proton energy. The expression for the cross sections will not contain the momenta of the nucleons, because the phase volume of a subsystem of two nucleons does not depend on their emission angles $(\theta_{12}, \varphi_{12})$. In other words, the final angular and momentum parts are completely separated.

3. DISCUSSION OF RESULTS

We have calculated the mean values of the spin tensors which are directly related to the experimentally observed physical quantities. For example, the mean value of the zero-rank spin tensor $\langle T_{00} \rangle$ determines the differential cross section for the emission of a π meson in the direction $(\theta_\pi, \varphi_\pi)$, when the nucleons are emitted in the direction $(\theta_{12}, \varphi_{12})$ measured from the π-meson direction in the c.m.s. of the two nucleons. The spin tensor $\langle T_{00} \rangle$ and the differential cross section are then related by

$$\langle T_{00} \rangle (\theta_\pi, \varphi_\pi; \theta_{12}, \varphi_{12}; p_\pi) = \frac{k^2}{\rho_E} \frac{d^3\sigma(\theta_\pi, \varphi_\pi; \theta_{12}, \varphi_{12}; p_\pi)}{d\Omega(\theta_\pi, \varphi_\pi)\, d\Omega(\theta_{12}, \varphi_{12})\, dp_\pi}. \quad (5)$$

Here $k = 1/\lambda\!\!\!^-$, p_π is the momentum of the π meson in the c.m.s. of the colliding nucleons, ρ_E

Type of transition	Transition scheme	Transition amplitude
σ_{10}	${}^1S_0 \to ({}^3S_1p)_0$	c_1
	${}^1D_2 \to ({}^3S_1p)_2$	c_2
	${}^3P_0 \to ({}^1P_1p)_0$	b_1
	${}^3P_1 \to ({}^1P_1p)_1$	b_2
	${}^3P_2 \to ({}^1P_1p)_2$	b_3
	${}^3F_2 \to ({}^1P_1p)_2$	b_4
σ_{11}	${}^3P_0 \to ({}^3P_1p)_0$	d_1
	$\nearrow ({}^3P_0p)_1$	d_2
	${}^3P_1 \to ({}^3P_1p)_1$	d_3
	$\searrow ({}^3P_2p)_2$	d_4

is the phase volume of the three particles, $\rho_E = p_\pi^2 R(p_\pi; W)$.

The final wave function is

$$F_1 = [A_1\chi_0^0 + \alpha_1\chi_1^1 + \beta_1\chi_1^0 + \gamma_1\chi_1^{-1}] / \sqrt{4\pi},$$
$$F_2 = [A_2\chi_0^0 + \alpha_2\chi_1^1 + \beta_2\chi_1^0 + \gamma_2\chi_1^{-1}] / \sqrt{4\pi}, \quad (6)$$

where the coefficients A_1, α_1 etc. are related to the transition amplitudes as follows:

$$A_1 = -iq_1\frac{3}{\sqrt{2}}\Big[\Big(b_- + \frac{\sqrt{3}}{2}b_2\Big)\sin\theta_\pi\cos\theta_{12}e^{i\varphi_\pi} + \Big(b_- - \frac{\sqrt{3}}{2}b_2\Big)\cos\theta_\pi\sin\theta_{12}e^{i\varphi_{12}}\Big] - 3iq_2\Big[\Big(\frac{b_1}{\sqrt{6}} - b_+\Big)\sin\theta_\pi\sin\theta_{12}\cos(\varphi_\pi - \varphi_{12})$$
$$+ \Big(\frac{b_1}{\sqrt{6}} + 2b_+\Big)\cos\theta_\pi\cos\theta_{12}\Big];$$

$$\alpha_1 = iq_2\Big[\frac{c_-}{2i}\sin\theta_\pi e^{-i\varphi_\pi} + \frac{1}{2}\sqrt{\frac{3}{2}}d_1(\cos\theta_\pi\sin\theta_{12}e^{-i\varphi_{12}} - \sin\theta_\pi\cos\theta_{12}e^{-i\varphi_\pi})\Big]$$
$$+ \frac{3}{2}iq_1\Big[\Big(\sqrt{\frac{3}{2}}d_3 - \frac{3}{\sqrt{10}}d_4\Big)\cos\theta_\pi\cos\theta_{12} - D_-\sin\theta_\pi\sin\theta_{12}e^{i(\varphi_\pi - \varphi_{12})} - \frac{3}{\sqrt{10}}d_4\sin\theta_\pi\sin\theta_{12}e^{-i(\varphi_\pi - \varphi_{12})}\Big];$$

$$\beta_1 = iq_2\Big[-\frac{c_+}{2i}\cos\theta_\pi + i\frac{\sqrt{3}}{2}d_1\sin\theta_\pi\sin\theta_{12}\sin(\varphi_\pi - \varphi_{12})\Big]$$
$$+ \frac{3}{2}iq_1\Big[\Big(d_2 - \frac{d_4}{\sqrt{5}}\Big)\sin\theta_\pi\cos\theta_{12}e^{i\varphi_\pi} + \Big(\frac{\sqrt{3}}{2}d_3 + \frac{3}{\sqrt{20}}d_4\Big)\cos\theta_\pi\sin\theta_{12}e^{i\varphi_{12}}\Big];$$

$$\gamma_1 = iq_2\Big[-\frac{c_-}{2i}\sin\theta_\pi e^{i\varphi_\pi} + \frac{1}{2}\sqrt{\frac{3}{2}}d_1(\cos\theta_\pi\sin\theta_{12}e^{i\varphi_{12}} - \sin\theta_\pi\cos\theta_{12}e^{i\varphi_\pi})\Big] + iq_1\frac{3}{2}D_+\sin\theta_\pi\sin\theta_{12}e^{i(\varphi_\pi + \varphi_{12})};$$

$$A_2 = \frac{3}{\sqrt{2}}iq_2\Big[\Big(b_- + \frac{\sqrt{3}}{2}b_2\Big)\sin\theta_\pi\cos\theta_{12}e^{-i\varphi_\pi} + \Big(b_- - \frac{\sqrt{3}}{2}b_2\Big)\cos\theta_\pi\sin\theta_{12}e^{-i\varphi_{12}}\Big]$$
$$- 3iq_1\Big[\Big(\frac{b_1}{\sqrt{6}} - b_+\Big)\sin\theta_\pi\sin\theta_{12}\cos(\varphi_\pi - \varphi_{12}) + \Big(\frac{b_1}{\sqrt{6}} + 2b_+\Big)\cos\theta_\pi\cos\theta_{12}\Big];$$

$$\alpha_2 = iq_1\left[-\frac{c_-}{2i}\sin\theta_\pi e^{-i\varphi_\pi} + \frac{1}{2}\sqrt{\frac{3}{2}}d_1(\cos\theta_\pi\sin\theta_{12}e^{-i\varphi_{12}} - \sin\theta_\pi\cos\theta_{12}e^{-i\varphi_\pi})\right] - \frac{3}{2}iq_2D_+\sin\theta_\pi\sin\theta_{12}e^{-i(\varphi_\pi+\varphi_{12})};$$

$$\beta_2 = iq_1\left[\frac{c_+}{2i}\cos\theta_\pi + i\frac{\sqrt{3}}{2}d_1\sin\theta_\pi\sin\theta_{12}\sin(\varphi_\pi - \varphi_{12})\right] + \frac{3}{2}iq_2\left[\left(d_2 - \frac{d_4}{\sqrt{5}}\right)\sin\theta_\pi\cos\theta_{12}e^{-i\varphi_\pi} + \left(\frac{\sqrt{3}}{2}d_3 + \frac{3}{\sqrt{20}}d_4\right)\cos\theta_\pi\sin\theta_{12}e^{-i\varphi_{12}}\right];$$

$$\gamma_2 = iq_1\left[\frac{c_-}{2i}\sin\theta_\pi e^{i\varphi_\pi} + \frac{1}{2}\sqrt{\frac{3}{2}}d_1(\cos\theta_\pi\sin\theta_{12}e^{i\varphi_{12}} - \sin\theta_\pi\cos\theta_{12}e^{i\varphi_\pi})\right] - \frac{3}{2}iq_2\left[\left(\sqrt{\frac{3}{2}}d_3 - \frac{3}{\sqrt{10}}d_4\right)\cos\theta_\pi\cos\theta_{12} - D_-\sin\theta_\pi\sin\theta_{12}e^{-i(\varphi_\pi-\varphi_{12})} - \frac{3}{\sqrt{10}}d_4\sin\theta_\pi\sin\theta_{12}e^{i(\varphi_\pi-\varphi_{12})}\right]. \quad (7)$$

Here we have the relations

$$c_+ = c_1 + \sqrt{10}c_2; \quad b_+ = \frac{b_3}{\sqrt{6}} + \frac{1}{2}b_4; \quad D_\pm = \frac{d_2}{\sqrt{2}} \pm \frac{1}{2}\sqrt{\frac{3}{2}}d_3 + \frac{d_4}{2\sqrt{10}};$$
$$c_- = c_1 - \sqrt{\frac{5}{2}}c_2; \quad b_- = -\frac{\sqrt{3}}{2}b_3 + \frac{1}{2}b_4. \quad (8)$$

Furthermore,

$$\langle T_{00}\rangle = |F_1|^2 + |F_2|^2 = \frac{1}{4\pi}\{|A_1|^2 + |\alpha_1|^2 + |\beta_1|^2 + |\gamma_1|^2 + |A_2|^2 + |\alpha_2|^2 + |\beta_2|^2 + |\gamma_2|^2\}. \quad (9)$$

The results can be represented by

$$\langle T_{00}\rangle = \langle T_{00}\rangle_{\text{unpol}} + P\langle T_{00}\rangle_{\text{pol}}. \quad (10)$$

Here P is the degree of polarization of the incident proton beam along the y axis, and

$$\langle T_{00}\rangle_{\text{unpol}} = \frac{1}{4\pi}[f_0 + f_1\cos^2\theta_\pi + f_2\cos^2\theta_{12} + f_3\cos^2\theta_\pi\cos^2\theta_{12} + f_4\sin\theta_\pi\cos\theta_\pi\sin\theta_{12}\cos\theta_{12}\cos(\varphi_\pi - \varphi_{12}) + f_5\sin^2\theta_\pi\sin^2\theta_{12}\cos 2(\varphi_\pi - \varphi_{12})]; \quad (11)$$

$$\langle T_{00}\rangle_{\text{pol}} = \frac{1}{4\pi}\{\sin\theta_\pi\cos\theta_\pi[(g_0 + g_1\cos\theta_{12} + g_2\cos^2\theta_{12})\cos\varphi_\pi + g_3\sin^2\theta_{12}\cos(\varphi_\pi - 2\varphi_{12})] + \sin\theta_{12}\cos\theta_{12}[(g_4 + g_5\cos^2\theta_\pi)\cos\varphi_{12} + g_6\sin^2\theta_\pi\cos(\varphi_{12} - 2\varphi_\pi)] + \sin\theta_{12}[(g_7 + g_8\cos^2\theta_\pi)\cos\varphi_{12} + g_9\sin^2\theta_\pi\cos(\varphi_{12} - 2\varphi_\pi)]\}. \quad (12)$$

The coefficients f_i and g_i are given in the Appendix. Equation (11), unlike the differential cross section for inelastic processes in which two particles are ejected, contains asymmetries with respect to the relative azimuth $(\varphi_\pi - \varphi_{12})$ in the forms of $\cos(\varphi_\pi - \varphi_{12})$ and $\cos 2(\varphi_\pi - \varphi_{12})$. Both asymmetries must, of course, be measured to determine the two independent coefficients f_4 and f_5. The kinematic scheme of this experiment in the c.m.s. of the colliding nucleons is represented in Fig. 2. The relation between the radius R and the π-meson momentum in this diagram corresponds to a π-meson energy which is close to the maximum. For fixed angles θ_π and θ_{12}, as well as for $\varphi_\pi = 0$, the cross section (11) becomes

$$\langle T_{00}\rangle_{\text{unpol}} = h_0 + h_1\cos\varphi_{12} + h_2\cos 2\varphi_{12}.$$

To obtain the coefficients h_1 and h_2 (and thus f_4 and f_5), measurements must be made at the angles $\varphi_{12} = 0$, 90, and 180°. In measuring the asymmetry associated with $\cos 2(\varphi_\pi - \varphi_{12})$ it is expedient to have the angles θ_π and θ_{12} equal 90°, in which case the term containing $\cos(\varphi_\pi - \varphi_{12})$ disappears.

Measurement of f_5 is very important because this coefficient depends only on the transitions which are associated with the initial states $^3P_0$, $^3P_2$, and $^3F_2$ of two protons and involves no other transition. The following general conclusion is evidently justified: the term containing f_5 receives contributions from those triplet states of the nucleons, in which the orbital and spin angular momenta are either parallel or antiparallel.

As an example we can consider the reaction $p + p \to \pi^0 + p + p$, to which only σ_{11} transitions can contribute. In our particular selection of transitions the measurement of f_5' directly gives

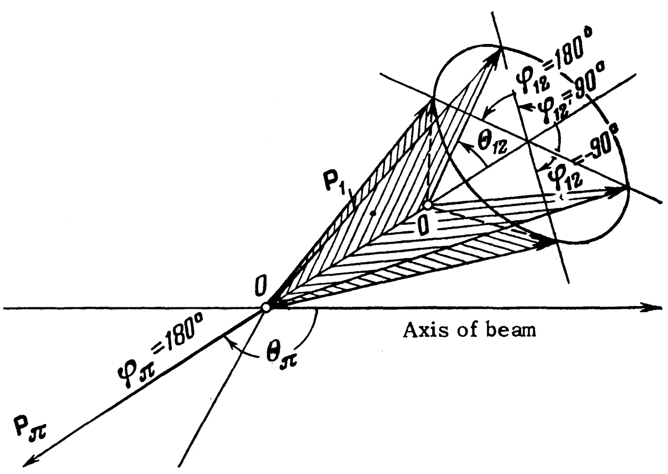

FIG. 2

the amplitude of the transition $^3P_0 \rightarrow (^3P_1p)_0$. In this case we have

$$f_5' = -{}^3/_8 |d_1|^2. \tag{13}$$

The minus sign in (13) indicates that emission is more probable when $(\varphi_\pi - \varphi_{12})$ equals 90° than when it equals 0°.

Experiments in which only the π meson is recorded and the nucleons can be ejected in any directions, give less information than when one of the nucleons is also recorded. Indeed, when only the π meson is observed we have

$$\langle T_{00}\rangle_{\rm unpol}(\theta_\pi, \varphi_\pi; p_\pi) = (f_0 + {}^1/_3 f_2) + (f_1 + {}^1/_3 f_3)\cos^2\theta_\pi, \tag{14}$$

where the coefficients f_4 and f_5 have disappeared completely and the remaining coefficients occur in pairs.

From consideration of the coefficients which determine $\langle T_{00}\rangle_{\rm pol}$ it is seen that g_1, g_7, g_8, and g_9 contain terms that result from the interference of σ_{10}- and σ_{11}-transitions, respectively: namely between the transitions

$$^1S_0 \rightarrow (^3S_1p)_0 \text{ and } ^1D_2 \rightarrow (^3S_1p)_2$$

and the transitions

$$^3P_1 \rightarrow (^3P_0p)_1, \quad ^3P_1 \rightarrow (^3P_1p)_1, \quad ^3P_1 \rightarrow (^3P_2p)_1.$$

Experiments with polarized beams when only π mesons are recorded enable us to determine

$$\langle T_{00}\rangle_{\rm pol}(\theta_\pi, \varphi_\pi; p_\pi) = \sin\theta_\pi\cos\theta_\pi\left(g_0 + {}^1/_3 g_2\right)\cos\varphi_\pi, \tag{15}$$

where seven of the nine coefficients have disappeared and the remaining two appear in linear combination. This is a convincing illustration of how much more information is obtained from experiments in which one of the nucleons is observed in addition to the π meson.

In conclusion the author wishes to thank S. M. Bilen'kii, L. I. Lapidus, and R. M. Ryndin for valuable discussions and suggestions.

APPENDIX*

The Coefficients f_i and g_i for the Reaction $p + p \rightarrow \pi^+ + n + p$

$$f_0 = \frac{1}{2}c_-^2 + \frac{9}{4}\left\{\frac{1}{6}d_1^2 + d_2^2 + \frac{3}{4}d_3^2 + \frac{5}{4}d_4^2 + \frac{4}{\sqrt{5}}d_2d_4\cos(d_2, d_4) - \frac{3}{2}\sqrt{\frac{3}{5}}d_3d_4\cos(d_3, d_4) + \frac{1}{3}(b_1^2 + b_3^2) + \frac{1}{2}b_4^2 \right.$$
$$\left. + \sqrt{\frac{2}{3}}b_3b_4\cos(b_3, b_4) - \sqrt{\frac{2}{3}}b_1b_4\cos(b_1, b_4) - \frac{2}{3}b_1b_3\cos(b_1, b_3)\right\};$$

$$f_1 = \frac{c_+^2 - c_-^2}{2} + \frac{9}{4}\left\{\frac{1}{6}d_1^2 - d_2^2 - \frac{3}{4}d_4^2 - \frac{4}{\sqrt{5}}d_2d_4\cos(d_2, d_4) - \frac{1}{3}b_1^2 + \frac{7}{6}b_3^2 + \frac{3}{2}b_2^2 + \frac{1}{2}b_4^2 - 4\sqrt{\frac{3}{2}}b_3b_4\cos(b_3, b_4)\right.$$
$$\left. + \sqrt{\frac{2}{3}}b_1b_4\cos(b_1, b_4) + \frac{2}{3}b_1b_3\cos(b_1, b_3) - 2\sqrt{\frac{3}{2}}b_2b_4\cos(b_2, b_4) + 3b_2b_3\cos(b_2, b_3)\right\};$$

$$f_2 = \frac{9}{4}\left[\frac{1}{6}d_1^2 - \frac{3}{4}d_3^2 - \frac{21}{20}d_4^2 - \frac{6}{\sqrt{5}}d_2d_4\cos(d_2, d_4) + \frac{3}{2}\sqrt{\frac{3}{5}}d_3d_4\cos(d_3, d_4) - \frac{1}{3}b_1^2 + \frac{3}{2}b_2^2 + \frac{7}{6}b_3^2 + \frac{1}{2}b_4^2\right.$$
$$\left. + \sqrt{\frac{2}{3}}b_1b_4\cos(b_1, b_4) + \frac{2}{3}b_1b_3\cos(b_1, b_3) - 4\sqrt{\frac{2}{3}}b_3b_4\cos(b_3, b_4) + \sqrt{6}\,b_2b_4\cos(b_2, b_4) - 3b_2b_3\cos(b_2, b_3)\right];$$

$$f_3 = \frac{9}{4}\left[-\frac{1}{2}d_1^2 + \frac{3}{2}d_3^2 + \frac{3}{2}d_4^2 + \frac{6}{\sqrt{5}}d_2d_4\cos(d_2, d_4) - 6\sqrt{\frac{3}{5}}d_3d_4\cos(d_3, d_4) + b_1^2 - 3b_2^2 - \frac{1}{2}b_4^2\right.$$
$$\left. + 5\sqrt{6}\,b_3b_4\cos(b_3, b_4) + \sqrt{6}\,b_1b_4\cos(b_1, b_4) + 2b_1b_3\cos(b_1, b_3)\right];$$

$$f_4 = 3\left[-\frac{1}{2}d_1^2 - \frac{63}{80}d_4^2 - \frac{9}{8}d_3^2 + \frac{3}{4}\sqrt{3}\,d_2d_3\cos(d_2, d_3) + \frac{3}{4}\sqrt{15}\,d_3d_4\cos(d_3, d_4) - \frac{3}{4}\sqrt{\frac{1}{5}}d_2d_4\cos(d_2, d_4)\right.$$
$$\left. + b_1^2 - \frac{9}{4}b_2^2 + \frac{1}{3}b_3^2 - \frac{3}{2}b_4^2 + \sqrt{\frac{3}{2}}b_1b_4\cos(b_1, b_4) + b_1b_3\cos(b_1, b_3) - \frac{21}{\sqrt{6}}b_3b_4\cos(b_3, b_4)\right];$$

*In all formulas of the Appendix the letters b, c and d denote absolute values.

$$f_5=\frac{3}{2}\Big[-\frac{1}{4}d_1^2+\frac{1}{2}(b_1^2+b_3^2)+\frac{3}{4}b_4^2+\sqrt{\frac{3}{2}}b_3b_4\cos(b_3,b_4)-\sqrt{\frac{3}{2}}b_1b_4\cos(b_1,b_4)-b_1b_3\cos(b_1,b_3)\Big];$$

and also

$$g_0=\frac{9}{4}\Big[\frac{1}{2}d_1d_3\sin(d_1,d_3)-\frac{1}{\sqrt{20}}d_1d_4\sin(d_1,d_4)-b_1b_2\sin(b_1,b_2)-b_1b_3\sin(b_1,b_3)+\frac{1}{\sqrt{3}}b_1b_4\sin(b_1,b_4)$$
$$-b_2b_3\sin(b_2,b_3)-\sqrt{\frac{3}{2}}b_2b_4\sin(b_2,b_4)-\frac{5}{\sqrt{6}}b_3b_4\sin(b_3,b_4)\Big];$$

$$g_1=\frac{3}{2}\Big[\sqrt{\frac{3}{2}}c_-d_3\cos(c_-,d_3)-\frac{3}{\sqrt{10}}c_-d_4\cos(c_-,d_4)-c_+d_2\cos(c_+,d_2)+\frac{1}{\sqrt{5}}c_+d_4\cos(c_+,d_4)\Big];$$

$$g_2=\frac{9}{4}\Big[\frac{1}{2}d_1d_3\sin(d_1,d_3)-\frac{1}{2}\sqrt{\frac{3}{5}}d_1d_4\sin(d_1,d_4)+3b_1b_2\sin(b_1,b_2)-b_1b_3\sin(b_1,b_3)+b_1b_4\sin(b_1,b_4)$$
$$+5b_2b_3\sin(b_2,b_3)-3\sqrt{\frac{3}{2}}b_2b_4\sin(b_2,b_4)+\frac{25}{\sqrt{6}}b_3b_4\sin(b_3,b_4)\Big];$$

$$g_3=\frac{9}{4}\Big[\frac{1}{2}d_1d_3\sin(d_1,d_3)-\frac{1}{2}\sqrt{\frac{3}{5}}d_1d_4\sin(d_1,d_4)-b_1b_2\sin(b_1,b_2)-b_1b_3\sin(b_1,b_3)+\sqrt{\frac{2}{3}}b_1b_4\sin(b_1,b_4),$$
$$-b_2b_3\sin(b_2,b_3)-\sqrt{\frac{3}{2}}b_2b_4\sin(b_2,b_4)-\frac{5}{\sqrt{6}}b_3b_4\sin(b_3,b_4)\Big];$$

$$g_4=\frac{9}{4}\Big[-\frac{1}{2}d_1d_3\sin(d_1,d_3)+\frac{1}{2}\sqrt{\frac{3}{5}}d_1d_4\sin(d_1,d_4)+b_1b_2\sin(b_1,b_2)-b_1b_3\sin(b_1,b_3)+\sqrt{\frac{2}{3}}b_1b_4\sin(b_1,b_4)$$
$$+b_2b_3\sin(b_2,b_3)+\sqrt{\frac{3}{2}}b_2b_4\sin(b_2,b_4)-\frac{5}{\sqrt{6}}b_3b_4\sin(b_3,b_4)\Big];$$

$$g_5=\frac{9}{4}\Big[-\frac{1}{2}d_1d_3\sin(d_1,d_3)+\frac{1}{2}\sqrt{\frac{3}{5}}d_1d_4\sin(d_1,d_4)-3b_1b_2\sin(b_1,b_2)-b_1b_3\sin(b_1,b_3)+\sqrt{\frac{2}{3}}b_1b_4\sin(b_1,b_4)$$
$$+3b_2b_3\sin(b_2,b_3)+3\sqrt{\frac{3}{2}}b_2b_4\sin(b_2,b_4)+\frac{25}{\sqrt{6}}b_3b_4\sin(b_3,b_4)\Big];$$

$$g_6=\frac{9}{4}\Big[-\frac{1}{\sqrt{3}}d_1d_3\sin(d_1,d_3)+\frac{1}{2}\sqrt{\frac{3}{5}}d_1d_4\sin(d_1,d_4)+b_1b_2\sin(b_1,b_2)-b_1b_3\sin(b_1,b_3)+\sqrt{\frac{2}{3}}b_1b_4\sin(b_1,b_4)$$
$$+b_2b_3\sin(b_2,b_3)-\frac{5}{\sqrt{6}}b_3b_4\sin(b_3,b_4)+\sqrt{\frac{3}{2}}b_2b_4\sin(b_2,b_4)\Big];$$

$$g_7=\frac{9}{4}\Big[-\frac{1}{3\sqrt{2}}c_-d_2\cos(c_-,d_2)-\frac{1}{\sqrt{6}}c_-d_3\cos(c_-,d_3)-\frac{7}{3}\sqrt{\frac{1}{10}}c_-d_4\cos(c_-,d_4)\Big];$$

$$g_8=\frac{9}{4}\Big[\frac{7}{3}\sqrt{\frac{1}{10}}c_-d_4\cos(c_-,d_4)+\frac{\sqrt{2}}{3}c_-d_2\cos(c_-,d_2)+\frac{1}{\sqrt{6}}c_-d_3\cos(c_-,d_3)$$
$$-\frac{1}{\sqrt{3}}c_+d_3\cos(c_+,d_3)-\frac{1}{\sqrt{5}}c_+d_4\cos(c_+,d_4)\Big];$$

$$g_9=\frac{3}{2}\Big[\frac{1}{2}\sqrt{\frac{3}{2}}c_-d_3\cos(c_-,d_3)-\frac{1}{\sqrt{2}}c_-d_2\cos(c_-,d_2)-\frac{1}{2\sqrt{10}}c_-d_4\cos(c_-,d_4)\Big].$$

The Coefficients f_i' and g_i' for the Process $p+p\to\pi^0+p+p$

$$f_0'=\frac{9}{4}\Big[\frac{1}{6}d_1^2+d_2^2+\frac{3}{4}d_4^2+\frac{5}{4}d_4^2+\frac{4}{\sqrt{5}}d_2d_4\cos(d_2,d_4)-\frac{3}{5}\sqrt{\frac{3}{5}}d_3d_4\cos(d_3,d_4)\Big];$$

$$f_1'=\frac{9}{4}\Big[\frac{1}{6}d_1^2-d_2^2-\frac{3}{4}d_3^2-\frac{4}{\sqrt{5}}d_2d_4\cos(d_2,d_4)\Big];$$

$$f_2'=\frac{9}{4}\Big[\frac{1}{6}d_1^2-\frac{3}{4}d_3^2-\frac{21}{20}d_4^2-\frac{6}{\sqrt{5}}d_2d_4\cos(d_2,d_4)+\frac{3}{2}\sqrt{\frac{3}{5}}d_3d_4\cos(d_3,d_4)\Big];$$

$$f_3'=\frac{9}{4}\Big[-\frac{1}{2}d_1^2+\frac{3}{2}d_3^2+\frac{3}{2}d_4^2+\frac{6}{\sqrt{5}}d_2d_4\cos(d_2,d_4)-6\sqrt{\frac{3}{5}}d_3d_4\cos(d_3,d_4)\Big];$$

$$f_4'=3\Big[-\frac{1}{2}d_1^2-\frac{63}{80}d_4^2-\frac{9}{8}d_3^2+\frac{3}{4}\sqrt{3}d_2d_3\cos(d_2,d_3)+\frac{3}{4}\sqrt{15}d_3d_4\cos(d_3,d_4)\Big];$$

$$f_5'=-\frac{3}{8}d_1^2;$$

and also

$$g'_0 = \frac{9}{4}\left[\frac{1}{2} d_1 d_3 \sin(d_1, d_3) - \frac{1}{\sqrt{20}} d_1 d_4 \sin(d_1, d_4)\right];$$

$$g'_2 = \frac{9}{4}\left[\frac{1}{2} d_1 d_3 \sin(d_1, d_3) - \frac{1}{2}\sqrt{\frac{3}{5}} d_1 d_4 \sin(d_1, d_4)\right];$$

$$g'_6 = \frac{9}{4}\left[-\frac{1}{\sqrt{3}} d_1 d_3 \sin(d_1, d_3) + \frac{1}{2}\sqrt{\frac{3}{5}} d_1 d_4 \sin(d_1, d_4)\right];$$

$$g'_3 = g'_2 = -g'_4 = -g'_5;$$

$$g'_1 = g'_7 = g'_8 = g'_9 = 0.$$

[1]Chamberlain, Segre, Tripp, Wiegand, and Ypsilantis, Phys. Rev. 105, 288 (1957); Stapp, Ypsilantis, and Metropolis, Phys. Rev. 105, 302 (1957).

[2]V. V. Vladimirskii and Ia. A. Smorodinskii, Dokl. Akad. Nauk SSSR 104, 713 (1955).

[3]Dzhelepov, Moskalev, and Medved', Dokl. Akad. Nauk SSSR 104, 380 (1955).

[4]L. Wolfenstein, Phys. Rev. 98, 766 (1955).

[5]K. M. Watson and K. A. Brueckner, Phys. Rev. 83, 1 (1951).

[6]V. B. Berestetskii, Dokl. Akad. Nauk SSSR 94, 421 (1954).

[7]S. M. Bilen'kii and L. I. Lapidus, Report, Inst. of Nuclear Problems (1955).

[8]F. Mandl and T. Regge, Phys. Rev. 99, 1478 (1955).

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EXTENSION OF THE BOGOLIUBOV-TIABLIKOV PERTURBATION METHOD TO A NON-STATIONARY CASE

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The perturbation theory in the second-quantization representation, developed by Bogoliubov and Tiablikov in the stationary case for use with the polar model of crystals, is generalized to include the case of perturbation by the electromagnetic field of a light wave. A general expression is derived for the deformed operator of the electric current density excited by the radiation perturbation. A possible application of the method in the theory of optical properties of cyrstals is indicated.

1. INTRODUCTION

THE perturbation-theory method of Bogoliubov and Tiablikov[1-3] is based on the introduction of an operator that projects an arbitrary wave function of the system of valence electrons of the crystal in the homopolar* functions of the problem. In this variant of the theory the mean value of the electric current produced in the crystal by a weak constant external field is non-vanishing only in third approximation. This makes the method unsuitable for the consideration of the electrical and optical properties of metals, and also for the consideration of the strongly excited "current" states of semiconductors within the framework of the polar model, although it does not diminish its usefulness for the description of the properties of crystals that are determined by exchange interactions, for example.

Nevertheless it seems to us that the Bogoliubov-Tiablikov method can be used for the treatment of the electrical and optical properties of electronic semiconductors at low temperatures. Here we have

*By the homopolar states in the polar model of a crystal we mean states in which there is always one valence electron close to each lattice point of the crystal.