As is seen from (19), the minus sign does not agree with the normalization condition (20).

Thus all the E are positive (this can be seen directly from the fact that the quadratic form under consideration is positive definite) and are separated from zero by the gap

$$E = 2E_e(k) \geqslant 2E_e(k_F) = 2C = 4\omega e^{-1/\rho}. \qquad (22)$$

Here again we obtain the results of Bardeen as in the previous papers.[1,2]

Since we have confined ourselves only to diagrams consisting of pairs, we cannot decide directly from (22) that the excitation with energy $2E_e(k)$ [Eq. (22)] consists indeed of two excitations of the Fermi type, which was shown in Ref. 1.

As we see, the method of summation of diagrams is shown to be quite lucid and permits us to establish the connection with the ideas of the work of Bardeen, Cooper, and Schrieffer.

However, in our opinion, the method of canonical transformation is more flexible, allowing us easily to obtain the higher approximations. Moreover, it achieves various generalizations, for example, in the calculation of thermodynamic quantities.

In conclusion, I should thank D. N. Zubarev, V. V. Tolmachev, S. V. Tiablikov, and Iu. A. Tserkovnikov for their valued discussion.

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BREMSSTRAHLUNG OF PI MESONS AND PRODUCTION OF PI-MESON PAIRS BY GAMMA QUANTA IN COLLISION WITH NONSPHERICAL NUCLEI

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The effective cross-sections are calculated for a number of radiative processes occurring in the interaction of high-energy π mesons with nonspherical nuclei. The nonspherical shape of the nuclei leads to a change of the angular distributions and the appearance in the cross-sections of factors that depend only on the geometrical shape of the nuclei.

In papers by Landau and Pomeranchuk,[1] Pomeranchuk,[2] and Vdovin[3] treatments have been given of the processes of bremsstrahlung in the interaction of π mesons with nuclei, production of π-meson pairs from nuclei by γ quanta, and production of nuclear stars by γ quanta, for very large π-meson energies E and γ-quantum energies ω ($E \gg \mu$; $\omega \gg \mu$, where μ is the mass of the π meson; we set $\hbar = c = 1$ throughout). A peculiarity of these processes at such energies is that very large distances from the nucleus ($r_{eff} \sim E/\mu^2 \gg R$) contribute to the matrix elements that give the probabilities of the processes, and one can use in the calculation the asymptotic form of the π-meson wave functions outside the region of their interaction with the nucleus. At large energies one can take these functions to be diffraction functions. The functions used in the papers mentioned are those of the diffraction by a black or a gray sphere.

As is well known, the equilibrium shape of a heavy nucleus is not that of a sphere, but that of an ellipsoid of revolution. The change of the shape and the related appearance of a spectrum of rotational levels causes considerable change in the diffraction solution, as has been shown by Drozdov[4] for the scattering of fast neutrons by nuclei.

In the present paper we find the changes made by the nonspherical shape of the nucleus in radiative processes occurring in the collisions of π mesons with nuclei and in the production of π-meson pairs by γ quanta. We use in this work as a model of the nucleus an ellipsoid of revolution which is black relative to the π mesons.

1. EMISSION OF A GAMMA QUANTUM BY A PI MESON

The radiation in question consists of radiation emitted when the π meson is scattered and radiation emitted during the absorption of the meson (stopping radiation). As has been shown by Landau and Pomeranchuk,[1] both parts of the radiation can be obtained from the wave equation for the π meson outside the nucleus

$$(\partial^2/\partial t^2 - \Delta + \mu^2)\psi = -2ie(\mathbf{A}\nabla)\psi, \tag{1}$$

$$\mathbf{A} = \sum_{\omega \mathbf{j}} \sqrt{\frac{2\pi}{\omega}}\, \mathbf{j}\,(a_{\omega \mathbf{j}} e^{i(\omega \mathbf{r} - \omega t)} + a^{+}_{\omega \mathbf{j}} e^{-i(\omega \mathbf{r} - \omega t)}).$$

If we regard the interaction with the radiation field as a perturbation, we can replace ψ in the right member of Eq. (1) by the π-meson wave function $\Phi_{\mathbf{p}_1}$ in zeroth approximation with respect to e. Then the solution of Eq. (1) that corresponds to the emission of a photon with momentum $\boldsymbol{\omega}$ and polarization $\mathbf{j}$ has the form

$$\Phi_{\mathbf{p}_2} = \int G(\mathbf{r}, \mathbf{r}') \frac{2e}{i} \sqrt{\frac{2\pi}{\omega}} (\mathbf{j}\nabla\Phi_{\mathbf{p}_1}) e^{-i\omega \mathbf{r}'} d\mathbf{r}', \tag{2}$$

$$p_2^2 = (E_1 - \omega)^2 - \mu^2,$$

where $G(\mathbf{r}, \mathbf{r}')$ is the Green's function of the wave equation for the π meson in the absence of interaction with the radiation field. When the radiation is emitted in a collision with a nonspherical nucleus, we can use for $G(\mathbf{r}, \mathbf{r}')$ and $\Phi_{\mathbf{p}_1}$ the functions for the problem of diffraction by a stationary ellipsoid:

$$G(\mathbf{r}, \mathbf{r}') = \frac{\exp\{ip_2|\mathbf{r} - \mathbf{r}'|\}}{4\pi|\mathbf{r} - \mathbf{r}'|} - \frac{p_2}{2\pi i}\int \frac{\exp\{ip_2|\mathbf{r} - \mathbf{s}|\}}{|\mathbf{r} - \mathbf{s}|}\frac{\exp\{ip_2|\mathbf{s} - \mathbf{r}'|\}}{4\pi|\mathbf{s} - \mathbf{r}'|} ds; \tag{3}$$

$$\Phi_{\mathbf{p}_1} = e^{i\mathbf{p}_1\mathbf{r}} - \frac{p_1}{2\pi i}\int \frac{e^{ip_1|\mathbf{r} - \rho_1|}}{|\mathbf{r} - \rho_1|} d\rho_1. \tag{4}$$

The integration is taken over those parts of planes, taken to pass through the center of the nucleon and to be perpendicular to the vector $\mathbf{r} - \mathbf{r}'$ for Eq. (3) and to $\mathbf{p}_1$ for Eq. (4), which lie inside the projections of the nuclear ellipsoid onto these planes. These regions of integration are functions of ϑ and ϕ — of the direction of the axis of symmetry of the nuclear ellipsoid relative to the chosen coordinate system. The projection is an ellipse. If in the plane in question we take the Y axis along the projection of the axis of symmetry, the equation of the ellipse is

$$X^2/a^2 + Y^2/b^2 = 1,$$
$$b^2 = a^2\xi^2(\vartheta); \quad \xi^2(\vartheta) = z^2\sin^2\vartheta + \cos^2\vartheta; \quad z = c/a, \tag{5}$$

where c is half the length of the axis of symmetry and a is the radius of the largest circular section.

The use of the functions (3) and (4) is permissible in the adiabatic approximation, in which the energies of the particles are much larger than the rotational energy of the nucleus; this condition is fulfilled here, since the entire theory is valid only for E_1, $E_2 \gg \mu$.

When the meson is outside the range of the nuclear forces, the wave function of the whole system of π meson + nucleus, for the emission of a γ quantum, can be written in the adiabatic approximation in the form

$$\Phi_{\mathbf{p}_2}\Omega_0(\vartheta, \phi) = \int G(\mathbf{r}, \mathbf{r}') \frac{2e}{i} \sqrt{\frac{2\pi}{\omega}} (\mathbf{j}\nabla\Phi_{\mathbf{p}_1}) e^{-i\omega\mathbf{r}'} d\mathbf{r}'\, \Omega_0(\vartheta, \phi), \tag{6}$$

where $\Omega_0(\vartheta, \phi)$ corresponds to the ground state of the nucleus. Starting with this formula, we can obtain the probability of emission of radiation in scattering with excitation of a definite rotational state, the total probability of radiation with scattering, and the probability of the "stopping" radiation.

Finding the asymptotic form of Eq. (6) for $r \to \infty$, we determine the amplitude for scattering with emission of radiation and with excitation of the n-th rotational state:

$$\langle \Omega_n(\vartheta, \phi) | M(\vartheta, \phi) | \Omega_0(\vartheta, \phi) \rangle = \int \Omega_n(\vartheta, \phi) \frac{2e}{i}\sqrt{\frac{2\pi}{\omega}} \Phi^*_{\mathbf{p}_2}(\mathbf{j}, \nabla\Phi_{\mathbf{p}_1}) e^{-i\omega\mathbf{r}'} d\mathbf{r}'\, \Omega_0(\vartheta, \phi) \sin\vartheta\, d\vartheta\, d\phi, \tag{7}$$

$$\Phi_{\mathbf{p}_2} = e^{i\mathbf{p}_2\mathbf{r}} + \frac{p_2}{2\pi i}\int \frac{\exp\{-ip_2|\mathbf{r} - \rho_2|\}}{|\mathbf{r} - \rho_2|} d\rho_2, \quad \mathbf{p}_2 = p_2\frac{\mathbf{r}}{r}.$$

The integration with respect to ρ_2 is taken over the projection of the nucleus onto a plane perpendicular to $\mathbf{p}_2$. The calculation of the probability of emission of a γ quantum in diffraction scattering with the excitation of rotational states is made by means of Eq. (7), in analogy with the calculation of the probability of radiation in scattering made by Landau and Pomeranchuk in Ref. 1. On carrying out these calculations and using the relations

$$E_1 \sim p_1; \quad E_2 \sim p_2; \quad E_1,\ E_2,\ \omega \gg \mu;$$

$$\mathbf{p}_2 = (\mathbf{p}_2\mathbf{p}_1)\,\mathbf{p}_1 / p_1^2 + p_2\theta'; \quad \omega = (\omega\mathbf{p}_1)\,\mathbf{p}_1 / p_1^2 + \omega\theta,$$

$$\theta \text{ and } \theta' \perp \mathbf{p}_1; \quad |\theta|,\ |\theta'| \leqslant \mu / E,$$

we get, neglecting terms of the order μ/E in comparison with unity:

$$d\sigma_{n0} = \frac{e^2}{\pi^2}\frac{d\omega\, do_\omega}{\omega}\frac{E_2\, do_{p_2}}{E_1} a^2 |F| \frac{|J_{n0}(|\omega\theta + E^2\theta'|\, a;\, z)|^2}{|\omega\theta + E_2\theta'|^2} \times \left|\frac{(\theta-\theta')E_1}{(\mu/E_2)^2+(\theta-\theta')^2} - \frac{\theta E_2}{(\mu/E_1)^2+\theta^2}\right|^2, \tag{8}$$

F is the form factor of the π meson,[1] and

$$J_{n0}(ka;\, z) = \int \Omega_n(\vartheta,\, \Phi)\,\xi^2(\vartheta)\,\frac{J_1\left(ka\sqrt{\xi^2(\vartheta)\sin^2\Phi + \cos^2\Phi}\right)}{\sqrt{\xi^2(\vartheta)\sin^2\Phi + \cos^2\Phi}}\,\Omega_0(\vartheta,\, \Phi)\sin\vartheta\, d\vartheta\, d\Phi. \tag{9}$$

As has been shown by Drozdov,[4b] the functions (9) determine the excitation of the rotational states of nuclei by fast particles. He has computed them by numerical integration for a number of states of a nucleus with spin zero [$\Omega(\vartheta,\, \phi)$ are then ordinary spherical harmonics] and for various values of the parameters ka and z. Their main property is a rapid decrease with increase of the number n.

Drozdov's results[4] are entirely applicable in the present case also. The only difference is that now k is not the transverse momentum $p_2\theta'$ of the scattered particle, but is given by $|\omega\theta + E_2\theta'|$, which is equal to the transverse momentum only for small ω.

For such high energies it is of more interest to consider the total cross-section for emission of radiation in collisions with the nonspherical nucleus

$$d\sigma = \sum_n d\sigma_{n0}. \tag{10}$$

These cross-sections have the same form for all axially symmetrical nuclei with a given ratio of the semiaxes, independent of the spin,[5] i.e., independent of the lowest rotational state:

$$d\sigma = \frac{e^2}{\pi^2}\frac{d\omega do_\omega}{\omega}\frac{E_2 do_{p_2}}{E_1} a^2 |F|^2 \frac{I_{00}\,(|\omega\theta + E_2\theta'|\, a;\, z)}{|\omega\theta + E_2\theta'|^2} \times \left|\frac{(\theta-\theta')E_1}{(\mu^2/E_2^2)+(\theta-\theta')^2} - \frac{\theta E_2}{(\mu^2/E_1^2)+\theta^2}\right|^2,$$

$$I_{00}(ka;\, z) = \frac{1}{4\pi}\int_0^{2\pi} d\Phi \int_{-1}^{+1} dx\,\xi^2(x)\left|\frac{J_1\left(ka\sqrt{\xi^2(x)\sin^2\Phi + \cos^2\Phi}\right)}{\sqrt{\xi^2(x)\sin^2\Phi + \cos^2\Phi}}\right|^2. \tag{11}$$

The function I_{00}(ka; z) determines the total cross-section for scattering of the particle by the nonspherical nucleus with excitation of all rotational states.[4] It has also been calculated by Drozdov[4b] by numerical integration for a number of values of the parameters ka and z.

For z = 1 the formulas of (11) give the result obtained in Ref. 1 for the spherical nucleus.

Introducing the dimensionless quantities s and η,

$$\theta = (\mu / E_1)\,s, \quad \theta' - \theta = (\mu / E_2)\,\eta; \quad s,\, \eta \leqslant 1,$$

we get instead of (11)

$$d\sigma = \frac{e^2}{\pi^2}\frac{d\omega}{\omega}\frac{E_2}{E_1} a^2 |F|^2 \frac{I_{00}(\mu a\,|s + \eta|;\, z)}{|s+\eta|^2} \times \left|\frac{s}{1+s^2} + \frac{\eta}{1+\eta^2}\right|^2 ds\,d\eta. \tag{12}$$

For heavy nuclei ($\mu a \gg 1$) and large angles s and $\eta \sim 1$, Eq. (12) is considerably simplified if we replace the Bessel function J_1 by the asymptotic expression for large arugments:

$$d\sigma = \frac{e^2}{\pi^3}\frac{d\omega}{\omega}\frac{E_2}{E_1}\frac{a}{\mu}\frac{|F|^2}{|s+\eta|^2}\left|\frac{s}{1+s^2} + \frac{\eta}{1+\eta^2}\right|^2 ds\,d\eta\, I; \tag{13}$$

$$I = \begin{cases} \frac{1}{4}\left\{1 + \frac{z^2+2}{\sqrt{z^2-1}}\sin^{-1}\frac{\sqrt{z^2-1}}{z}\right\} & z \geqslant 1 \\ \frac{1}{4}\left\{1 + \frac{z^2+2}{\sqrt{1-z^2}}\sinh^{-1}\frac{\sqrt{1-z^2}}{z}\right\} & z \leqslant 1. \end{cases} \tag{14}$$

Equation (13) can be written in the form

$$d\sigma = d\sigma_{\text{sph}} I, \tag{15}$$

where $d\sigma_{\text{sph}}$ is the cross-section for emission of radiation in scattering by a spherical nucleus of radius a [Ref. 1, Eq. (30)]. As is shown in Ref. 1, the main contribution to the total cross-section comes from values of s and η of the order of unity. Thus the total cross-section for heavy nuclei can be obtained with F = 1 in Eq. (13) [Ref. 1, Eq. (32)]:

$$\sigma = \sigma_{\text{sph}} I \approx 2{,}3\, e^2 \frac{E_2}{E_1}\frac{a}{\omega\mu} I. \tag{16}$$

Consequently, for heavy nuclei and large angles the cross-section for bremsstrahlung in scattering by nonspherical nuclei differs from the corresponding cross-section for the spherical nucleus only by the factor (14). In the general case the angular distribution for the nonspherical nucleus differs from that for the spherical nucleus by the fact that for $z \neq 1$ the function $I_{00}(ka; z)$, and thus also the cross-section, does not become zero for any value of $k \neq 0$. For spherical nuclei, on the other hand, with $z = 1$, $I_{00}(ka; 1) = J_1^2(ka)$, and thus the corresponding cross-section becomes zero for certain values of k. Curves showing the behavior of the function I_{00} for various values of z and the angles are given in Ref. 4b.

The "stopping" radiation from a nonspherical nucleus can be calculated if one uses Eq. (6) to find the flux of π mesons incident on the nucleus, integrates this flux over the surface of the nucleus as is done in Ref. 1, and then averages over the ground state of rotation of the nucleus. The cross-section obtained does not depend on the spin of the ground state of the nucleus, and differs only by a factor (a function of z) from the corresponding cross-section for the spherical nucleus of radius R = a:

$$d\sigma = d\sigma_{c\Phi} I_1 = \frac{e^2}{\pi} a^2 \frac{\theta^2 do_\omega d\omega}{[(\mu/E_1)^2 + \theta^2]^2} \frac{E_1 - \omega}{E_1\omega} |F|^2 I_1; \qquad (17)$$

$$I_1 = \begin{cases} \frac{1}{2}\left\{1 + \frac{z^2}{\sqrt{z^2-1}} \sin^{-1}\frac{\sqrt{z^2-1}}{z}\right\} & z \geqslant 1 \\ \frac{1}{2}\left\{1 + \frac{z^2}{\sqrt{1-z^2}} \sinh^{-1}\frac{\sqrt{1-z^2}}{z}\right\} & z \leqslant 1, \end{cases} \qquad (18)$$

$\pi a^2 I_1$ is the average area of the projection of the nuclear ellipsoid onto the plane perpendicular to the direction of motion of the incident π mesons.

2. PRODUCTION OF PAIRS OF PI MESONS BY AN INCIDENT GAMMA QUANTUM

This process can also be dealt with by means of Eq. (1), as a process of "absorption" of the γ quantum by a π meson with energy and momentum $-E_1$, $-\mathbf{p}_1$, where E_1 and $\mathbf{p}_1$ are the energy and momentum of one member of the pair.*

When the meson is outside the nucleus, the wave function of the system π meson + nucleus, after the "absorption" of the γ quantum, is

$$\frac{2e}{i}\sqrt{\frac{2\pi}{\omega}} \int G(\mathbf{r}, \mathbf{r}')(\mathbf{j}, \nabla\Phi_{-\mathbf{p}_1}(\mathbf{r}') e^{i\omega r'} d\mathbf{r}' \Omega_0(\vartheta, \phi), \qquad (19)$$

*It is well known that the matrix element for pair production is obtained from that for bremsstrahlung by the replacement $\omega \to -\omega$, $\boldsymbol{\omega} \to -\boldsymbol{\omega}$ and change of sign of the energy and momentum of the incident particle.[6]

$G(\mathbf{r}, \mathbf{r}')$ is given by Eq. (3), and

$$\Phi_{-\mathbf{p}_1}(\mathbf{r}) = e^{-i\mathbf{p}_1\mathbf{r}} - \frac{p_1}{2\pi i} \int \frac{\exp\{ip_1|\mathbf{r} - \mathbf{s}|\}}{|\mathbf{r} - \mathbf{s}|} ds.$$

The integration is taken over the projection of the nucleus onto the plane perpendicular to $\mathbf{p}_1$ [Eq. (5)].

The calculations are carried out in complete analogy with the case of the spherical nucleus,[2a, b] with the nonspherical shape taken into account by a formula analogous to Eq. (7). The calculation can be carried to completion only for the case of heavy nuclei ($\mu a \gg 1$, $\mu c \gg 1$). We proceed at once to give the results. Using the notation of Pomeranchuk[2] we have

$$\mathbf{p}_i = p_i \frac{\boldsymbol{\omega}}{\omega}\left(1 - \frac{k_i^2}{2p_i}\right) + \mathbf{k}_i, \quad \boldsymbol{\omega}\mathbf{k}_i = 0, \quad k_i \ll p_i,$$

$$\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}, \quad i = 1, 2.$$

The cross-section for pair production with excitation of the n-th rotational state is

$$d\sigma_{n0}(\mathbf{p}_1, \mathbf{p}_2) = \frac{e^2}{8\pi^2} a^2 \frac{E_1 E_2}{\omega^3} \frac{|F_{+-}|^2}{k^2} \left| \frac{\mathbf{k}_2 - \mathbf{k}_1}{\mu^2 + (\mathbf{k}_2 - \mathbf{k}_1)^2/4} + \frac{2\mathbf{k}_1}{\mu^2 + k_1^2} - \frac{2\mathbf{k}_2}{\mu^2 + k_2^2} \right|^2 |J_{n0}(ka; z)|^2 \, dE_1 \, d\mathbf{k}_1 \, d\mathbf{k}_2. \qquad (20)$$

The function $J_{n0}(ka; z)$ is defined by Eq. (9). With regard to the form factor F_{+-}, see Ref. 2. The cross-section for pair production from a nonspherical nucleus with excitation of any rotational state is

$$d\sigma(\mathbf{p}_1, \mathbf{p}_2) = \frac{e^2}{8\pi^2} a^2 \frac{E_1 E_2}{\omega^3} \frac{|F_{+-}|^2}{k^2} \left| \frac{\mathbf{k}_2 - \mathbf{k}_1}{\mu^2 + (\mathbf{k}_2 - \mathbf{k}_1)^2/4} + \frac{2\mathbf{k}_1}{\mu^2 + k_1^2} - \frac{2\mathbf{k}_2}{\mu^2 + k_2^2} \right|^2 \times I_{00}(ka; z) \, dE_1 \, d\mathbf{k}_1 \, d\mathbf{k}_2. \qquad (21)$$

The function I_{00} is defined in Eq. (11).

Thus the changes in the angular distributions for pair production from a nonspherical nucleus, as compared with the case of the spherical nucleus, are the same as for the bremsstrahlung.

The total cross-sections can be calculated, as in Ref. 2b, beginning with Eq. (21), if we set $|F_{+-}|^2$ equal to 1. These cross-sections differ from those calculated by Pomeranchuk by the geometrical factor (18) and the replacement of R^2 by a^2. For example, integrating the pair-production cross-section (21) over all $\mathbf{k}_1$ and $\mathbf{k}_2$, up to certain maximum values, and over all possible energies of the pair, we get:

$$\sigma = \frac{e^2 a^2}{12}\left[\ln\frac{\mu^2 + b_{max}^2}{\mu^2} + \frac{\mu^2}{\mu^2 + b_{max}^2}\right] I_1,$$
$$b_{max} = \frac{1}{2}|\mathbf{k}_1 - \mathbf{k}_2|_{max}. \tag{22}$$

3. PRODUCTION OF A PAIR OF PI MESONS BY A GAMMA QUANTUM WITH SUBSEQUENT ABSORPTION OF ONE MEMBER OF THE PAIR BY A NONSPHERICAL NUCLEUS

This process can also be treated by means of Eq. (19), finding the flux falling on unit area of the nucleus, integrating over the surface of the nucleus and averaging over the rotational state and over the polarization of the γ quantum. Thus we get, in analogy with Ref. 3b:

$$d\sigma = \frac{e^2}{2\pi^2}\frac{E_1(\omega - E_1)}{\omega^3} dk_1 dE_1 \int |\Omega_0(\vartheta, \Phi)|^2 \sin\vartheta\, d\vartheta d\Phi$$
$$\times \int d\rho \Big| \frac{\mathbf{k}_1}{\mu^2 + k_1^2}$$
$$- \frac{1}{4\pi^2}\int ds\, dg \frac{\mathbf{g}}{\mu^2 + g^2} \exp\{i(\mathbf{g} - \mathbf{k}_1, \mathbf{s} - \boldsymbol{\rho})\}\Big|^2 |F_{+-}|^2. \tag{23}$$

We integrate with respect to ds and $d\rho$ over the projection of the nucleus onto the plane perpendicular to $\mathbf{p}_1$, and with respect to $\mathbf{g}$ over the whole of the plane perpendicular to $\boldsymbol{\omega}$:

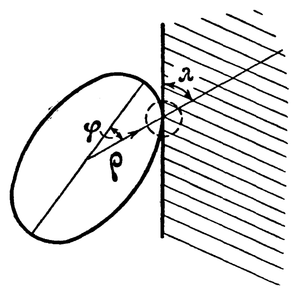
$$\frac{\mathbf{k}_1}{\mu^2 + k_1^2} - \frac{1}{4\pi^2}\int ds\, dg \frac{\mathbf{g}}{\mu^2 + g^2}\exp\{i(\mathbf{g} - \mathbf{k}_1, \mathbf{s} - \boldsymbol{\rho})\} =$$
$$= \frac{\mu}{2\pi}\nabla_{\mathbf{k}_1}\int_{s \geqslant R(\vartheta,\varphi)} ds \exp\{-i\mathbf{k}_1(\mathbf{s} - \boldsymbol{\rho})\}\frac{K_1(\mu|\mathbf{s} - \boldsymbol{\rho}|)}{|\mathbf{s} - \boldsymbol{\rho}|}. \tag{24}$$

$K_1(x)$ is a solution of Bessel's equation for $n = 1$ which falls off exponentially at infinity. The integral (24) is taken over the part of the plane perpendicular to $\mathbf{p}_1$ that is outside the projection of the nuclear ellipsoid; ϑ is the angle between the axis in this plane, measured from the axis of symmetry (the diagram shows the projection of the nuclear ellipsoid onto the plane perpendicular to $\mathbf{p}_1$).

From Eq. (5) we have:

$$R(\vartheta, \varphi) = a\xi(\vartheta) / \sqrt{\xi^2(\vartheta)\cos^2\varphi + \sin^2\varphi}. \tag{25}$$

The integral in Eq. (24) can be calculated for heavy nuclei ($\mu a \gg 1$, $\mu c \gg 1$). Since the important region for this integral is $|s - \rho| \sim 1/\mu$, neglecting terms of order $1/\mu a$ we can use an integration (for each vector $\boldsymbol{\rho}$ making an angle φ with the projection of the axis of symmetry) taken not over the part of the plane outside the ellipse at the point of intersection with the extension of the vector $\boldsymbol{\rho}$ (see diagram); this makes it possible to



do the integration. We then integrate the square of the absolute value of the resulting expression with respect to $\boldsymbol{\rho}$ over the interior of the ellipse (25),

$$\int d\rho \Big|\frac{\mu}{2\pi}\nabla_{\mathbf{k}_1}\int ds \exp\{-i\mathbf{k}_1(\mathbf{s} - \boldsymbol{\rho})\}\frac{K_1(\mu|\mathbf{s} - \boldsymbol{\rho}|)}{|\mathbf{s} - \boldsymbol{\rho}|}\Big|^2$$
$$= \frac{1}{8}\frac{1}{\mu^2 + k_1^2}\int_{-\pi}^{+\pi} d\varphi \Big\{1$$
$$+ \frac{k_1^2\cos^2(\lambda + \varphi + \Phi)}{\mu^2 + k_1^2\cos^2(\lambda + \varphi + \Phi)}\Big\}\frac{R(\vartheta, \varphi)}{\sin\lambda\,[\mu^2 + k_1^2\cos^2(\lambda + \varphi + \Phi)]^{1/2}}, \tag{26}$$

where λ is the angle between the direction of $\boldsymbol{\rho}$ and the tangent to the ellipse (see drawing):

$$\sin\lambda = [\xi^2(\vartheta)\cos^2\varphi + \sin^2\varphi] / [\sin^2\varphi + \cos^2\varphi\,\xi^4(\vartheta)]^{1/2}.$$

Just as was found in Eq. (11), averaging of the expression (26) over the lowest rotational state gives a result that is independent of the spin of the ground state,[5] and the averaging reduces simply to averaging over ϑ and ϕ.

The result differs from that for the case of the spherical nucleus[3a] only by a factor that is a function of z:

$$d\sigma = \frac{e^2}{4\pi^2}\frac{E_1(\omega - E_1)}{\omega^3} d\eta\, dE_1 \frac{a}{\mu}\frac{|F_{+-}|^2}{(1 + \eta^2)^{3/2}}\{2K(\varepsilon) - E(\varepsilon)\} I_2,$$
$$\eta = k_1/\mu;\quad \varepsilon = \eta / \sqrt{1 + \eta^2} \tag{27}$$

(the notation is that of Vdovin[3a]); K and E are the complete elliptic integrals of the first and second kinds; and

$$I_2 = \frac{1}{4\pi}\int_{-1}^{+1} dx \int_{-\pi}^{+\pi} d\varphi \frac{\xi(x)[\sin^2\varphi + \cos^2\varphi\,\xi^4(x)]^{1/2}}{[\sin^2\varphi + \cos^2\varphi\,\xi^2(x)]^{3/2}}; \tag{28}$$
$$I_2 = \begin{cases} \frac{1}{2}\{z + (z^2 - 1)^{-1/2}\sinh^{-1}(z^2 - 1)^{1/2}\} & z \geqslant 1 \\ \frac{1}{2}\{z + (1 - z^2)^{-1/2}\sin^{-1}(1 - z^2)^{1/2}\} & z \leqslant 1. \end{cases}$$

Thus the angular distribution and the total cross-section for the nonspherical nucleus differ from those for the spherical nucleus[3b] only by the factor (28).

Let us now consider the conditions for the applicability of the formulas that have been obtained. The integration of (24) over a half-plane instead of over the part of the plane outside the ellipse amounts to the neglect of terms of order $1/\mu a \ll 1$ only if $\rho_\varphi \gg 1/\mu$, where ρ_φ is the radius of curvature of the ellipse at the point in question. For the spherical nucleus this condition is just $\mu R \gg 1$. For the ellipse the curvature is variable, and the minimum radius of curvature of the projection of the nuclear ellipsoid is $z^2 a$ for $z < 1$ and a/z for $z > 1$. It would appear that the treatment that has been carried out is valid only for

$$z^2 \gg 1/\mu a, \quad z < 1$$
$$1/z \gg 1/\mu a, \quad z > 1. \qquad (29)$$

But in actual fact Eqs. (27) and (28) are valid over a range of values of z considerably larger than that given by the conditions (29), since the radius of curvature ρ_φ fails to be much larger than $1/\mu$ only in small ranges of angles given by $\Delta\varphi \sim z^{-2} \times (z/\mu a)^{1/3}$ for $z > 1$ and $\Delta\varphi \sim z(z/\mu a)^{1/3}$ for $z < 1$, and the contribution from these intervals to the integral (26), for $z \gg 1$ and $z \ll 1$, is small in comparison with the whole integral (26).

Thus (27) and (28) can be used for heavy nuclei with $\mu a \gg 1$, $\mu c \gg 1$, independently of the value of z.

If both mesons of the pair are absorbed by the heavy nucleus, producing a "star," the cross-section for this process with a nonspherical nucleus differs from the corresponding cross-section with a spherical nucleus[3C] by the factor (18) and the replacement of R by a.

In conclusion the writer expresses his deep gratitude to I. M. Shmushkevich for suggesting the problem and for valuable discussions.

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[2] I. Ia. Pomeranchuk, Dokl. Akad. Nauk SSSR **96**, 265 (1954) (a), **96**, 481 (1954) (b).

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[4] S. I. Drozdov, J. Exptl. Theoret. Phys. (U.S.S.R.) **28**, 734 (1955) (a), **28**, 736 (1955) (b); Soviet Phys. JETP **1**, 588 (1955) (a), **1**, 591 (1955) (b).

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Translated by W. H. Furry
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up the crystals as infinitely long unidimensional or two-dimensional atom complexes, bound together by "small" forces of one nature, whereas in the complex itself the atoms are bound by "big" forces of another nature.

6. The difference between the typical molecular crystals (e.g., the CH_4 or C_6H_6 crystals) and the heteropolar molecular crystals (such as the NaCl, $HgCl_2$ or PbS crystals) lies: (1) in the degree of molecularity β; (2) in the nature of the forces in the molecules; (3) in the nature of intermolecular forces. The quantity β is defined as the ratio of the intramolecular energy $U^a \cong D$ (D is the energy of dissociation of the diatomic molecule into ions) to the intermolecular energy U^e per bond. For the substances for which β is given below, it is possible to take $U^e \approx 2S/l$. Example:

$\beta = 300(CH_4)$, $200(HCl)$, $22(HgCl_2)$, $10(NaCl)$ taking $l = 12$ in all four cases.

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ERRATA

Volume 5

Page	Line	Reads	Should Read
1043	Eq. (4)		$W = y^2 a_{14}^2 \sin 2\phi/2\rho\,(a_{11} a_{44} - \alpha_{14}^2 \sin^2 3\phi)$ The coefficient k_2 equals $0.185 \times 10^{-3}\,cm^{-1}$.
1044	3 from bottom (l.h.)	$\Delta y = 2.87 \times 10^{-3}$ cm	$\Delta y = 3.18 \times 10^{-3}$ cm
	4 from top (r.h.)	$\Delta\vartheta_\Sigma = 7.2 \times 10^{-5}$ radians	$\Delta\vartheta_\Sigma = 5.9 \times 10^{-5}$ radians

Volume 6

Page	Line	Reads	Should Read
1090	4 and 5 from top	2—(d, 3n); and of the I_{53}^{127} cross section, 3—(d, 2n); 4—(d, 3n)	2—(d, 3n) on I_{53}^{127} and 3—(d, 3n); 4—(d, 3n) on Bi_{83}^{209}
1091	6 from bottom expression for determinant C(y)	ρ, γp, h, $1/\rho$	ρy_2, $\gamma p y_2$, $h y_2$, y_2/ρ
1094	7 from bottom	For $\gamma = 5/3$, μ has . . .	Here μ has . . .

Volume 7

Page	Line	Reads	Should Read
55	16 from bottom	Correct submittal date is April 5, 1957	
169	17 from bottom	Delete "Joint Institute for Nuclear Research"	
215	Table	Add: Note. Columns 2—9 give the number of counts per 10^6 monitor counts	
215	Table, column headings	1, 2, 3, 4-7, 8	1, 2, 3, 4, 8-7
312	Eq. (8)	. . . $(1 \pm \mu/2M)^2$	. . . $(1 \mp \mu/2M)^2$
313	2, r.h. col.	$\alpha_{33} = 0.235$	$a_{33} = 0.235$
692	Eq. (5)	$m_B/M_B = \ldots \mp [1 + \ldots]$	$m_B/M_B = \mp [1 + \ldots]$
461	Title	. . . Elastically Conducting	. . . Electrically Conducting