THE RELATION BETWEEN DECAY ASYMMETRY AND DIPOLE MOMENT OF ELEMENTARY PARTICLES

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Using a simple model for the emission of nonrelativistic particles, the wave function of a system is constructed in the coordinate representation for real and virtual decay. It is shown that asymmetry in real decay of a polarized particle depends on the imaginary part of the vector coupling constant, and that asymmetry in the virtual decay and the dipole moment both depend on the real part of this constant.

It is shown that elementary particles, which according to Landau have no dipole moment, resemble the enanthiomorphic (optically active) molecules of organic chemistry, rather than molecules in Λ-doubled states.

A consistent description of particle production is developed in the coordinate representation. A convenient expression is presented for the virtual decay probability.

THE assumption that parity is not conserved in weak interactions has led Lee and Yang[1] to new conclusions with respect to the behavior of elementary particles possessing spin. These conclusions are the following.

(1) Asymmetry decay is possible, in which the emitted particles are directed primarily along or against the angular momentum of the decaying particle.

(2) An elementary particle may have a dipole moment, and this is also parallel (or antiparallel) to its angular momentum.

As is well known, the first conclusion of Lee and Yang was brilliantly verified in experiments on the β-decay of oriented nuclei[2] and on μ mesons.[3] The order of magnitude of the dipole moment they predict, however, is too small for experimental observation. Landau[4] has given a complete theory relating parity nonconservation in charged-particle decay with space reflections.*

One of Landau's conclusions is that the dipole moment of elementary particles vanishes identically. It would seem at first that a decay asymmetry would lead necessarily to a dipole moment. Let us consider, for instance, a polarized neutron whose angular momentum is directed vertically upward. We may consider as established the fact that such a neutron decays by emitting electrons primarily in the upward direction. Let us now consider such a polarized neutron in the spherically symmetric field of a nucleus in which the energy relations are such that the neutron is stable and cannot decay. It then becomes possible and necessary for the neutron to undergo virtual decay, emitting an electron and capturing it again instantaneously. We may speak of a cloud of virtual electrons about the nucleus.

It would seem that asymmetry in real decay should correspond to a similar asymmetry in virtual decay. This would lead to asymmetry in the virtual electron cloud, and therefore to a dipole moment. Landau's work shows that such simple concepts are in general mistaken.† Ioffe[6] (who has kindly communicated his work to the present author before its publication) has recently shown that the decay asymmetry and dipole moment depend on whether or not the theory is invariant with respect to time reversal. Essentially the matter reduces to the following. A linear relation between the momentum $\mathbf{p}$ of the emitted particle and the direction of polarization (spin direction) $\mathbf{s}$ of the decaying particle is possible in a theory invariant under time reversal, since both quantities change sign. A static dipole $\mathbf{d}$, however, or the analogous sta-

*Lee and Yang[5] have also independently indicated the possibility of combining space reflections with the transitions to the antiparticle.

†Note added in proof (November 25, 1957). In the absence of a dipole moment, parity nonconservation leads to certain specific magnetic properties, as is shown by the author on p. 1531 of the present journal (p. 1184 of translation).

tic center-of-mass **r** of a virtual-particle cloud, does not change sign under time reversal. Therefore the spin **s** can be related to the static quantities **d** and **r** only in a theory which is not invariant under time reversal.

Section 1 of the present article contains a coordinate-representation treatment of the emission of a nonrelativistic particle in the decay of a spin-$\frac{1}{2}$ particle, assuming parity nonconservation. This example shows clearly the dependence of the decay asymmetry of a polarized spin-$\frac{1}{2}$ particle on the phase of the coupling constants in the expression for the interaction leading to the decay.

It is shown that in the first approximation the decay asymmetry depends on the imaginary part of the vector coupling constant, and that the dipole moment depends on its real part, so that there is no direct connection between the decay asymmetry and dipole moment. It is shown how the interaction potential can lead to asymmetric decay in the case of a real coupling constant, i.e., in the Ioffe-Rudik-Okun' theory[7] (see assumption II of Ioffe[6]).

In Landau's theory charged particles are "odd." In molecular physics two types of odd phenomena are known. These are enanthiomorphic molecules (such as right and left tartaric acid) and diatomic molecules with Λ-doubling (such as a nitric oxide molecule with the projection of the election angular momentum on the axis directed from the N to the O, or the same molecule with the angular momentum directed in the opposite way). Section 2 explains the properties of such molecules with respect to their decay asymmetry and dipole moment. It is shown that according to Landau's theory elementary particles are similar to enanthiomorphic molecules, rather than to molecules with Λ-doubling.

In Sec. 1 it is found convenient to consider decay by treating the wave function of the produced particle in configuration space, rather than in momentum space. In Sec. 3 we show why it seems to us that this approach leads more simply and directly to the known formulas for the decay probability, without calculating the level density in phase space.

1. THE ASYMMETRY OF REAL AND VIRTUAL DECAY IN THE NONRELATIVISTIC MODEL

Let us consider the decay $A = B + C$, where A and C have spin $\frac{1}{2}$, and B has spin 0. We shall denote the states of A and C with $s_z = \frac{1}{2}$ by α, and those with $s_z = -\frac{1}{2}$ by β.† Let us assume that A and C are coupled by some spherically-symmetric potential field. We denote the wave function of A by χ, and that of C by η. We assume further that A and C are in a state with orbital angular momentum $l = 0$. We denote the wave function of B by φ. Let m be the mass of B, and assume that the decay energy is small compared with mc^2.

Assume that in the initial state A is polarized with $s_z = \frac{1}{2}$, so that the state is $\alpha\chi$. The wave function of particles B and C in the final state consists of two terms, and can be written

$$\Phi = \varphi_1\alpha\eta + \varphi_2\beta\eta \tag{1}$$

each term of which corresponds to one of the two possible polarizations of C after decay.

In nonrelativistic quantum mechanics there are two possible types of elementary interactions that do not involve the momenta of A and C. These are the scalar and vector (derivative) interactions. In the theory of second quantization, the corresponding terms and the Hamiltonian density H are

$$H' = g\psi_A^*\psi_C\psi_B + \text{hermitian conjugate}, \tag{2a}$$

$$H'' = f\psi_A^*\sigma\psi_C \operatorname{grad}\psi_B + \text{hermitian conjugate}, \tag{2b}$$

where g and f are coupling constants, ψ_A^*, ψ_B^*, and ψ_C^* are creation operators for particles A, B, and C, and σ is the spin operator.

Previously, the requirement that H be invariant under space conversion has led to the conclusion that it cannot contain a sum of these expressions. We now know, however, that in weak interactions, in which parity is not conserved, it is exactly such a sum $H' + H''$ which must be considered.

The corresponding equations for the wave functions of B are the inhomogeneous Schrödinger equations (we set $\hbar = 1$)

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†In the case of real decay we can think of A as being a Λ particle, C a proton, and B a π meson. For virtual decay, on the other hand, we may consider the original particle A to be a proton, C a Λ particle, and B a meson, so that real decay is forbidden by energy considerations.

$$i\partial\varphi_1/\partial t = -(1/2m)\Delta\varphi_1 + V\varphi_1 + gq + f\partial q/\partial z, \tag{3}$$

$$i\partial\varphi_2/\partial t = -(1/2m)\Delta\varphi_2 + V\varphi_2 + f(\partial/\partial x + i\partial/\partial y)q, \tag{4}$$

where V is the potential of the forces acting on B, and

$$q = q(x, y, z, t) = \chi\eta^* = Q(r)\exp\{-i(E_A - E_C + M_Ac^2 - M_Cc^2 - mc^2)t\}. \tag{5}$$

Equations (3) — (5) are written specifically for the decay of a polarized particle A in which the wave functions χ and η are spherically symmetric, owing to the fact that $l = 0$. In what follows it is also important that Q(r) is real, which also follows from the fact that $l = 0$. The energies E_A and E_C do not include the rest masses.

We seek a stationary solution in which the time dependnence of φ_1 and φ_2 is the same as that of the inhomogeneous terms involving q in (3) and (4), namely

$$\varphi = \varphi(x, y, z)e^{-iEt}, \quad E = E_A - E_C + M_Ac^2 - M_Cc^2 - mc^2. \tag{6}$$

Inserting (6) into (3) and (4), we obtain

$$i\partial\varphi/\partial t = E\varphi = -(1/2m)\Delta\varphi + \ldots$$

Decay is possible when $E > 0$, and then B particles are emitted with momentum p, where $p^2/2m = E$. Far from the source, the solution should be a diverging wave $\varphi \sim e^{ipr}/r$.

If $E < 0$, real decay cannot occur. Virtual decay is described by a solution which decays exponentially with distance, such as $e^{-\kappa r}/r$, where $\kappa^2/2m = -E$.

Let us further simplify the problem, so that we may obtain an exact solution. We assume that the particle produced has no forces acting on it ($V = 0$) and that the region in which q is nonzero is small compared to the wavelength of B. Then for $E > 0$, we have

$$\varphi_1 = ae^{ipr}/r + b\cos\theta\frac{d}{dr}(e^{ipr}/r), \quad \varphi_2 = be^{i\varphi}\sin\theta\frac{d}{dr}(e^{ipr}/r). \tag{7}$$

When $pr \gg 1$,

$$\varphi_1 = (e^{ipr}/r)(a + ipb\cos\theta), \quad \varphi_2 = (e^{ipr}/r)ipbe^{i\varphi}\sin\theta. \tag{7a}$$

For virtual decay, when $E < 0$, we have

$$\varphi_1 = a(e^{-\varkappa r}/r) + b\cos\theta\frac{d}{dr}(e^{-\varkappa r}/r) = (e^{-\varkappa r}/r)\left[a - \varkappa b\cos\theta\left(1 + \frac{1}{\varkappa r}\right)\right],$$
$$\varphi_2 = be^{i\varphi}\sin\theta\frac{d}{dr}(e^{-\varkappa r}/r) = -(e^{-\varkappa r})e^{i\varphi}\varkappa b\sin\theta\left(1 + \frac{1}{\varkappa r}\right). \tag{8}$$

In Eqs. (7) and (8)

$$a = g\int Q(r)4\pi r^2dr, \quad b = f\int\frac{dQ}{dr}4\pi r^2dr. \tag{9}$$

These expressions give information on the decay asymmetry and on the asymmetry of the virtual-particle cloud when decay does not take place. It is characteristic that there appear two terms in the expressions for φ_1: If particle A is in a state with $l = 0$ and $s_z = +\frac{1}{2}$, the final state in which C has $l = 0$ and $s_z = +\frac{1}{2}$ can have B both in the form of an S wave (with $l = 0$) and of a P wave (with $l = 1$ and $l_z = 0$). It is the interference of the two terms of φ_1 which leads to the terms linear in $\cos\theta$, which are those of interest in that they are related to the direction of the angular momentum of the decaying A particle. Here θ is the angle between the direction of polarization of A (the z axis) and the radius vector r. The wave function φ_2 corresponds to the decay of A with $s_z = \frac{1}{2}$ to C with $s_z = -\frac{1}{2}$ and describes particle B in the P state with $l = 1$ and $l_z = +1$.

Since φ_1 and φ_2 belong to different orthogonal states of C [see Eq. (1)], they do not interfere.

Let us find the flux of the B particles at large values of r. This is

$$j = (1/2mi)(\varphi_1^*\nabla\varphi_1 - \nabla\varphi_1^*\cdot\varphi_1 + \varphi_2^*\nabla\varphi_2 - \nabla\varphi_2^*\cdot\varphi_2) = (p/r^2m)[|a^2| + p^2|b|^2 + \cos\theta\cdot ip(a^*b - ab^*)]. \tag{10}$$

For $E < 0$, we are interested only in the density ρ of virtual B particles, since at infinity the flux vanishes. This is

$$\rho = \varphi_1^*\varphi_1 + \varphi_2^*\varphi_2 = r^{-2}e^{-2\varkappa r}\,[|a|^2 + \varkappa^2|b|^2(1 + 1/\varkappa r)^2 - \varkappa\cos\theta\,(1 + 1/\varkappa r)\,(a^*b + ab^*)]. \quad (11)$$

The fundamental result is contained in Eqs. (10) and (11), which show that there is no necessary relation between the decay asymmetry and the asymmetry of the virtual-particle cloud; one of these depends on $(a^*b - ab^*)$, and the other on $(a^*b + ab^*)$.

Let us assume the scalar coupling constant g to be real, so that a is real.

The decay asymmetry depends on the imaginary part of b, and vanishes if f is a real constant and b is real.

If, on the other hand, f is imaginary, and therefore so is b, the decay is asymmetric although the term containing $\cos\theta$ vanishes in the expression for the virtual particle density; the virtual-particle density is therefore spherically symmetric, and there is no dipole moment. Imaginary f corresponds to the Landau case, and real f to the Ioffe-Rudik-Okun' case (Ioffe's[6] assumptions I and II).

If for real f we consider the projection of the C spin along an axis perpendicular to the z axis (which is the polarization direction of the initial A particle), it is easily seen that from the interpretation of φ_1 and φ_2 one obtains the relation Ioffe predicts between the vector product $[\mathbf{S}_A \times \mathbf{S}_C]$ and the direction in which particle B is emitted.

We note that if we do not neglect the action of the potential on particle B, but assume it to be spherical, the conclusion that there is no dipole moment if f is imaginary remains valid. This follows from the fact that in the equation obtained from (3) the real term gQ gives a spherically symmetric real solution, the imaginary term $f\cos\theta\cdot Q'$ gives an imaginary solution proportional to $f\cos\theta$, and, as before, there are no interference terms proportional to $\cos\theta$ in the expression for the density.

Although for real f there is no decay asymmetry in the absence of a potential and when the source is small, a decay asymmetry does appear when V(r) is included. Then in the expression for the diverging wave, the S-wave and P-wave phases shift by different amounts α_S and α_P, and asymptotically when $pr \gg 1$ we obtain the expression†

$$\varphi_1 = a'\exp(ipr + i\alpha_S)/r + ip\cos\theta\cdot b'\exp(ipr + i\alpha_P)/r \quad (12)$$

and real b gives

$$j = (p/mr^2)\,[a'^2 + p^2b'^2 + 2a'b'\cos\theta\sin(\alpha_S - \alpha_P)]. \quad (13)$$

2. ODD MOLECULES

Let us consider the decay A = B + C from the point of view of many-body quantum mechanics. We shall consider A and C to be systems (molecules) in eigenstates with the same angular momentum L. The systems consist, for instance, of nuclei and electrons whose spins we do not take into account, considering only the Coulomb interaction. The theory is invariant with respect to reflection in space, as well as with respect to the transformation of time reversal accompanied by the transition to the complex conjugate wave function according to Wigner. If the states A and C are only (2L + 1)-fold degenerate in accordance with the possible L_z values for a given L, it is easily shown that neither for real nor for virtual decay will there appear terms linear in $\cos\theta$.

In the usual case (I) in which the internal states of A and C are uniquely determined, the eigenstates A_0 and C_0 with angular momentum L are just (2L + 1)-fold degenerate. In addition to this, we may consider enanthiomorphic molecules (II) which may be either right-handed (D) or left-handed (L). We shall designate these states by A_D and A_L respectively. Because spontaneous transitions between these states are possible, they are not, strictly speaking, eigenstates. During a time short with respect to the $A_D \rightleftarrows A_L$ transition time, however, A_D and A_L may be treated as separate eigenstates. It is important only that the A = B + C decay itself not lead to a transition between A_D and A_L. We assume that

† In Eq. (12) a′ and b′ are proportional to g and f, respectively. The proportionality factors are real, but their expressions in terms of Q and dQ/dr are not as simple as (9). In the absence of a potential, but with an extended source Q(r) and real g and f, there is again no asymmetry.

The quantities α_S and α_P depend on the potential V(r). They represent the phase shifts between the S and P waves of the regular solution of the homogeneous equation, and the free-particle S and P waves

$$\psi_S = \sin pr/r,\ \psi_P = \cos\theta\,(\cos pr/r - \sin pr/pr^2).$$

C occurs in two possible states, C_D and C_L, and that the only transitions possible are $A_D = C_D + B$ and $A_L = C_L + B$.

Finally, certain diatomic molecules (III) with fixed nuclei may, in the ground state, have a nonzero projection Λ of the electron angular momentum on the molecular axis. States which differ only in the sign of Λ have the same energy. We shall denote these states by n and m, writing A_n and A_m. We may consider the states A_n, A_m, C_n, and C_m approximately as eigenstates in the same sense as we did with A_D, A_L, C_D, and C_L. Case (II) may be called a static asymmetric molecule, and case (III) may be called a dynamic one. The symbol A now stands for the set consisting of A_0, A_D, A_L, A_n, and A_m.

Under a continuous rotation of coordinates in any of the three cases (I, II, or III), a given state (A_0, A_D, or A_n) with given values of L and L_z remains the same state (A_0, A_D, or A_n) with the same value of L, but with a different L_z. Under time reversal the wave functions of A_0 and A_D are transformed to their complex conjugates, but A_{+L_z} is transformed into A_{-L_z}, so that L remains invariant, but L_z changes sign.

Under a space reflection, A_0 remains invariant, being nondegenerate, A_D is transformed to A_L, A_n is transformed to A_m, etc., and L and L_z remain invariant.

As is seen from Sec. 1, the decay asymmetry and the dipole moment, which are of interest in the present case, depend on the interference of the S wave and P wave (with $l = 1$ and $l_z = 0$) of particle B. Two such waves can be obtained simultaneously in the decay $A(L, L_z) = C(L, L_z) + B$ with equal values of L and L_z for particles A and C, with $L \neq 0$.

Let us consider a Hamiltonian similar to (2a) and (2b), describing the decay $A = B + C$, and let us require that this Hamiltonian be invariant under coordinate rotations, space reflections, and time reversal. By treating continuous rotations under which states with different L_z transform into each other, one can show that the constant coupling A and C with the S wave of particle B cannot depend on L_z, and that the coupling constant for the P wave of particle B is proportional to L_z. The possible asymmetry between positive and negative directions along the axis is proportional to the polarization of A, or to the mean value of L_z.

By treating space reflection, we see that in case I either the S-wave amplitude or the P-wave amplitude can be nonzero, and that these waves cannot interfere. This proves the impossibility of decay asymmetry and the absence of a dipole moment in an ordinary nondegenerate moleucle in an eigenstate of the angular momentum.*

Time-reversal considerations can be used to show that in cases I and II the constant coupling A and C with the S wave of particle B must be real, and that for the P wave must be imaginary (we note that an imaginary f corresponds to a real coupling constant if one goes over from grad ψ_B to the momentum operator, writing $H'' = \sigma_{AC} p_B$).

It follows from this that enanthiomorphic (right-handed and left-handed) molecules also have no dipole moment, as is the case for ordinary molecules and atoms; polarized and enanthiomorphic molecules, however, may have a decay asymmetry.

In case III (Λ-doubling) time-reversal considerations lead to the conclusion that the P-wave coupling constant is real. As is well known, Λ-doubled molecules actually have dipole moments. It follows from Sec. 1 that in the first approximation there will be no decay asymmetry, although such an asymmetry will appear in the second approximation when one takes into account the interaction potential of B and C.

Parity nonconservation in elementary particle interactions can be thought of simply as an internal asymmetry of the elementary particles. Landau's theory, which forbids dipole moments, shows that this internal asymmetry is of the static kind, rather than analogous to Λ-doubling.

3. DESCRIPTION OF PARTICLE PRODUCTION IN CONFIGURATION SPACE

In Sec. 1 the inhomogeneous Schrödinger equation was used to treat particle production, which is usually treated by an expansion in eigenfunctions of the particle produced. In the absence of a potential, Eq.

*By nondegenerate we mean that there is to be no degeneracy (as in case II and III) greater than the (2L + 1)-fold degeneracy in the angular momentum. According to the usual terminology, the dipole moment of a molecule such as HCl means the dipole moment with fixed nuclei, rather than in the free-rotation state treated by Yang and Lee, by Landau, and in the present work.

(7) is analogous to the Born approximation. In scattering theory we write $\psi = \psi_0 + \varphi$, where ψ_0 is the incident wave and φ is the scattered wave, and neglect $V\varphi$ (where V is the scattering potential), assuming it to be a second-order term. We then obtain for φ the inhomogeneous equation

$$i\partial\varphi/\partial t = -(1/2m)\Delta\varphi + V\psi_0.$$

We may attempt to find a solution to this equation by a plane-wave expansion of φ, or by using a Green's-function method, writing

$$\varphi(r_1) = \frac{m}{2\pi}\int V(r_2)\psi_0(r_2)(e^{ipr_{12}}/r_{12})\,d^3r_2.$$

The problem of particle production without taking $V(r)$ into account is entirely analogous to the first approximation of scattering theory.

Let us now use the inhomogeneous Schrödinger equation to treat the problem of particle production in a potential field. This problem has no analogy in scattering theory. The equation is of the form

$$i\partial\varphi/\partial t = -(1/2m)\Delta\varphi + V\varphi + q. \tag{14}$$

If q is the "source" in the equation for φ, then the equation

$$\frac{\partial\rho}{\partial t} = \frac{\partial}{\partial t}\varphi^*\varphi = \operatorname{div}\mathbf{j} + \frac{1}{i}(q\varphi^* - q^*\varphi) \tag{15}$$

shows that its yield is proportional to $q\varphi^* - q^*\varphi$, which means that it depends on the phase difference between φ and q. If Q and φ are both real,† no particles are produced.

For simplicity, let us consider particle production in the S state. Let $V(r)$ be spherically symmetric. The homogeneous equation has for $\ell = 0$, a regular solution ψ which is real, finite, at $r = 0$, and proportional to $\sin(pr + \alpha)/r$ for $pr \gg 1$. The phase shift α is determined by the regularity condition and by the potential. To be specific, let us normalize by writing

$$\psi_1 = \sin(pr + \alpha)/r \text{ for } pr \gg 1. \tag{16a}$$

The second linearly-independent solution ψ_2 of the second-order equation has a singularity at $r = 0$ (singular solution). We shall choose it to make the Wronskian equal to unity:

$$\psi_2 = \cos(pr + \alpha)/r \text{ for } pr \gg 1, \tag{16b}$$

$$r\psi_2\frac{d}{dr}(r\psi_1) - r\psi_1\frac{d}{dr}(r\psi_2) = 1 \text{ for all } pr. \tag{16c}$$

Let us now return to the inhomogeneous equation. Using $q = Q(r)\exp(-iEt)$ and $\varphi = \varphi(r)\exp(-iEt)$ we obtain from Eq. (3)

$$E\varphi + (1/2m)\Delta\varphi(r) - V(r)\varphi(r) = q(r). \tag{17}$$

As is known, the solution of (17) can be expressed by means of quadratures in terms of the solutions ψ_1 and ψ_2 of the homogeneous equation. This solution is found quite simply using the Lagrange method of Eq. (6), since the equations for the coefficients c_1 and c_2 are in this case independent.[8] We obtain

$$\varphi = c_1\psi_1 + c_2\psi_2 = \psi_1(r)\int^r Q(s)\psi_2(s)s^2ds + \psi_2(r)\int^r Q(s)\psi_1(s)s^2ds. \tag{17'}$$

The integrals of (17′) determine c_1 and c_2 up to a constant factor. We apply the condition $c_2(0) = 0$ to the coefficient of ψ_2 (which means that we require the solution to be regular). The coefficient ψ_1 is determined from the condition that when $pr \gg 1$, the solution must represent a diverging wave, so that $c_1(\infty) = ic_2(\infty)$. Thus the final solution in the form of (17′) with the required properties is

$$2m\varphi = -\psi_1(r)\int_r^\infty Q(s)\psi_2(s)s^2ds + \psi_2(r)\int_0^r Q(s)\psi_1(s)s^2ds + i\psi_1(r)\int_0^\infty Q(s)\psi_1(s)s^2ds. \tag{18}$$

For $pr \gg 1$, Eq. (18) gives

† After eliminating the factor e^{-iEt}.

$$2m\varphi = \frac{1}{r} e^{i(pr+\alpha)} \int_0^\infty Q(s)\,\psi_1(s)\,s^2 ds. \tag{19}$$

From this we easily obtain an expression for the total particle flux (decay probability), namely

$$J = \frac{m}{\pi p} M^2, \quad M = 4\pi \int Q\psi_1 s^2 ds = \int Q\psi_1 dV. \tag{20}$$

This is the same as the usual expression

$$J = (2\pi/\hbar)\, M^2 dN/dE, \tag{21}$$

where M is the same matrix element as that given in (20), and dN/dE is the number of levels per unit energy interval.

Let us form the expression

$$J = -i \int (Q\varphi^* - Q^*\varphi)\, dV. \tag{22}$$

Inserting the expression for φ from (18) into this equation, we see that J involves only the imaginary part of φ, which is proportional to M.

The above derivation also shows why the decay probability involves the matrix element $\int Q\psi dV$ taken over the regular function ψ_1 which is finite for small r; this matrix element remains finite even if we let Q approach a δ-function.

On the other hand, for small r and when $Q = \delta$, the wave function φ of the produced particle will itself increase as r^{-1} if it is an S wave (and as r^{-l-1} if $l > 0$), owing to the second term in (18). The question arises as to why the expression for the wave function does not enter into the matrix element. The answer is that particle production depends only on the imaginary part of φ (for a real source), and that only the real part increases as 1/r; as is seen from (18), the imaginary part is regular.

The mathematical expectation value for the number of virtual particles is particularly simply obtained in the coordinate representation. This value may be thought of also as the fraction β of the time that particle A is in state B + C if there is not sufficient energy for real decay. For weak coupling this fraction is

$$\beta = \int \varphi^2 dV, \tag{23}$$

where φ is a solution such as (18) (decaying exponentially for large r) of an inhomogeneous equation such as (17) with $E < 0$. A particularly simple expression is obtained if $V(r) = 0$. In this case we have

$$\varphi(r_1) = -\frac{m}{2\pi} \int \frac{1}{r_{12}} e^{-\varkappa r_{12}} Q(r_2)\, d^3 r_2, \tag{24}$$

so that (23) can be written

$$\beta = \frac{m^2}{2\pi\varkappa} \iint Q(r_1)\, Q(r_2)\, e^{-\varkappa r_{12}} d^3 r_1 d^3 r_2. \tag{25}$$

We note that (25) does not contain r_{12} in the denominator, so that the expression does not diverge as Q approaches a δ-function.

In the usual method of approach, nonrelativistic theory gives

$$\beta = (2\pi)^{-3} \int |C_p|^2 d^3 p, \quad C_p = \frac{\int Q e^{ipr} d^3 r}{(p^2 + \varkappa^2)/2m}. \tag{26}$$

It can be shown that Eqs. (23) – (25) are identical with the more complicated expression (26).

In the decay of a free elementary particle A interacting with no other fields, the source Q becomes a δ-function, and the potential V(r) describes the interaction of the decay products B and C with each other.

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[1] T. D. Lee and C. N. Yang, Phys. Rev. 104, 254 (1956).
[2] C. S. Wu et al., Phys. Rev. 105, 1413 (1957).
[3] Garwin, Ledermann, and Weinrich, Phys. Rev. 105, 1415 (1957).
[4] L. D. Landau, J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 407 (1957), Soviet Phys. JETP 5, 337 (1957).
[5] C. N. Yang, Revs. Mod. Phys. 29, 231 (1957).
[6] B. L. Ioffe, J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 1246 (1957).
[7] Ioffe, Okun', and Rudik, J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 396 (1957), Soviet Phys. JETP 5, 328 (1957).
[8] Ia. B. Zel'dovich, J. Exptl. Theoret. Phys. (U.S.S.R.) 31, 1101 (1956), Soviet Phys. JETP 4, 942 (1957).

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SKIN EFFECT AND FERROMAGNETIC RESONANCE

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We consider the normal and the anomalous skin effect in ferromagnetic metals under conditions of ferromagnetic resonance. The constant magnetic field is taken to be perpendicular to the surface of the sample. The influence of exchange effects on the resonance line width and on the shift of the resonance frequency is taken into account.

I. Ferromagnetic resonance absorption is observed when the frequency of the electromagnetic field incident on the surface of the ferromagnetic is nearly equal to the eigenfrequency of the precession of the magnetization vector **M** around the direction of the magnetic field **H**. This effect is described by the equation of motion of the magnetization vector **M** (Ref. 1) in a certain effective magnetic field,

$$\frac{1}{\gamma}\frac{\partial \mathbf{M}}{\partial t} = \mathbf{M} \times \left(\mathbf{H} + \frac{2A}{M_s^2} \nabla^2 \mathbf{M} - \frac{\beta}{M_s} \mathbf{M} \times \mathbf{H} \right). \tag{1}$$

The first term on the right-hand side of (1) is the true magnetic field inside the specimen. The second term is the effective field due to exchange forces caused by the inhomogeneity of the magnetic moment **M**, and the third term is a relaxation term that describes the approach of the magnetization **M** to an equilibrium position along the field **H**; γ is the gyromagnetic ratio, and M_S the saturation value of the magnetization.

The relaxation term was introduced by Landau and Lifshitz[1] for a phenomenological description of damping processes. Up to the present the physical meaning of this term is not completely clear, since all existing theories lead to values of the dimensionless constant β that are rather low compared with experiment. In most cases, however, such a term gives a fairly good description of the experimentally observed effects that are connected with relaxation.