SOVIET PHYSICS JETP VOLUME 6 (33), NUMBER 6 JUNE, 1958

THEORY OF CYCLOTRON RESONANCE IN METALS

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Submitted to JETP editor June 21, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) 33, 1461-1471 (December, 1957)

The surface impedance of a metal is calculated, with arbitrary dispersion law and collision probability, for the case of radio-frequency and constant magnetic fields both parallel to the metal surface. The field-dependence and temperature-dependence of the surface impedance are studied over the whole range of possible field strengths. It is explained how one can determine the form of the Fermi surface, the velocity of electrons in the surface, and the probability of electron transitions caused by collisions. The predictions of the theory are compared with experiments on cyclotron resonance in metals.

1. INTRODUCTION

THE authors[1,2] predicted and investigated theoretically the phenomenon of cyclotron resonance in metals. This resonance differs basically from the well-known diamagnetic resonance[3] in semiconductors in two respects: (1) Cyclotron resonance occurs not only when the imposed frequency ω is equal to the cyclotron frequency Ω, but also at the multiple frequencies $\omega \approx \Omega,\ 2\Omega,\ \ldots,$ whether the metal is isotropic or not. (2) Cyclotron resonance in metals is possible only when the constant magnetic field **H** is strictly parallel to the metal surface. A small angle between the magnetic field and the surface makes the resonance disappear.

Cyclotron resonance has recently been observed in tin by Fawcett,[4] Bezuglyi and Galkin,[5] and Kip, and in bismuth by Dexter and Lax[6] and Aubrey and Chambers.[7] At the same time the variation of the high-frequency surface resistance of tin, copper, and bismuth in strong magnetic fields, predicted by us earlier,[8] has been confirmed.

We are concerned now with four problems.

(1) To compare the experimental results with theory.

(2) To extend the theory to arbitrary field-strengths.

(3) To find the temperature-dependence of the impedance.

(4) To explain how the field-dependence of the surface impedance gives information about the Fermi surface* $\epsilon(\mathbf{p}) = \zeta$.

In Section 4 we show that in principle the experiments enable us to determine completely the shape of the Fermi surface and the velocity of the electrons in it.

It is particularly interesting to find that the geometry of the Fermi surface (especially the main components of it) has a qualitative effect on the behavior of cyclotron resonance. This allows us to make some deductions about the Fermi surface from a rapid look at the experimental results, without any complicated analysis of the resonance curves.

2. FIELD AND TEMPERATURE DEPENDENCE OF THE SURFACE IMPEDANCE

The main objective of our work is to determine and study the complete surface impedance tensor of a metal in radio-frequency and constant magnetic fields both parallel to the surface. Anomalous skin-effect conditions are assumed $(\delta \ll r,\ \ell,\ v/\omega)$. The surface impedance tensor $Z_{\mu\nu}$ expresses the relation between the electric field $\mathbf{E}(0)$ at the metal surface $(z = 0)$ and the total current **I** following in the metal.

*Here ϵ is the energy and **p** the wave-vector of an electron, ζ is the chemical potential of the electron gas in the metal, and $\epsilon(\mathbf{p})$ is an even periodic function of **p** with the reciprocal lattice for period.

$$E_\mu(0)=\sum_{\nu=1}^{2} Z_{\mu\nu}I_\nu=\frac{ic^2}{4\pi\omega}\sum_{\nu=1}^{2} Z_{\mu\nu}E'_\nu(0),\ Z_{\mu\nu}=R_{\mu\nu}+iX_{\mu\nu}\ (\mu,\nu=x,y). \tag{2.1}$$

The problem involves four quantities with the dimensions of a length, the Larmor radius r, the effective electron mean free path $\ell_{eff} = vt_0/(1+\omega t_0)$, the actual mean free path $\ell = vt_0$ (v is the electron velocity and t_0 the mean collision time), and the skin-depth δ. Convenient asymptotic expressions for the surface impedance are obtainable only in the limits of very weak fields ($r \gg \ell^2/\delta$) and of fairly strong fields ($r \leq \ell$). We shall exhibit the final formulae for both cases, taking the results from Eq. (3.1) and (5.8) – (5.10) of Ref. 2.

1. Weak Fields

For simplicity we assume a quadratic dispersion law $\epsilon = p^2/2m$, where m is an effective mass. Then

$$Z(H)=Z^{(0)}\left(1+\frac{3}{128}\frac{\sigma^* l^* Z^{(0)}}{\gamma^2}\right),\ Z^{(0)}=\left(\frac{\sqrt{3}\pi\omega^2 l}{c^4\sigma}\right)^{1/3}(1+i\sqrt{3}), \tag{2.2}$$

where $Z^{(0)}$ is the surface impedance in zero field, $\sigma^* = \sigma/(1+i\omega t_0)$, σ is the static conductivity of the metal,

$$l^*=l/(1+i\omega t_0),\ \gamma=r/l^*=i\omega/\Omega+1/\Omega t_0=H_0/H,\ \Omega=eH/mc.$$

The small parameter, in which Eq. (2.2) is an expansion, is $[\sigma^*\ell^* Z^{(0)}/\gamma^2]$. The effect of the magnetic field becomes noticeable as soon as $H \geq H_1$, with $H, H_1 \ll |H_0|$. This is natural, since the value

$$H_1=|H_0/[\sigma^* l^* Z^{(0)}]^{1/2}|\ll|H_0|,$$

defines the field strength at which the arc length $s = \sqrt{r\delta_{eff}}$ of an electron path in the skin becomes comparable with the effective mean free path ℓ_{eff}. Apart from a numerical factor, Eq. (2.2) remains valid with any dispersion law and any direction of the magnetic field relative to the metal surface.

2. Strong Fields (either resonant or non-resonant)

$$Z_{\mu\nu}=2(\sqrt{3}\pi\omega^2/c^4)^{1/3}e^{i\pi/3}(A^{-1/3})_{\mu\nu},\quad Z_{xx,yy}=2(\sqrt{3}\pi\omega^2/c^4)^{1/3}e^{i\pi/3}[(\xi_1+\xi_2)\xi_1\xi_2+A_{xx,yy}]/\xi_1\xi_2(\xi_1^2+\xi_1\xi_2+\xi_2^2),$$
$$Z_{xy}=Z_{yx}=2(\sqrt{3}\pi\omega^2/c^4)^{1/3}e^{i\pi/3}A_{xy}/(\xi_1^2+\xi_1\xi_2+\xi_2^2), \tag{2.3}$$

with

$$\xi_{1,2}=\left\{\frac{1}{2}(A_{xx}+A_{yy})\pm\left[\left(\frac{A_{xx}-A_{yy}}{2}\right)^2+A_{xy}^2\right]^{1/2}\right\}^{1/3},\quad A_{\mu\nu}=\frac{8e^2}{3h^3}\int_0^{2\pi}\frac{n_\mu(\varphi)n_\nu(\varphi)}{K(\varphi)}\frac{d\varphi}{1-\exp(-2\pi i\omega/\Omega-2\pi/\Omega\tau_0)}, \tag{2.4}$$

The cube roots are to be taken with arguments in the interval $(-\pi/6, \pi/6)$; this is always possible since the radicands have positive real parts. The real part of the impedance is then necessarily positive. The notations are: e the electron charge, h Planck's constant, $\mathbf{v} = v\mathbf{n}$ the electron velocity, K the absolute value of the Gaussian curvature of the Fermi surface, $\Omega = eH/mc$ the cyclotron frequency, $m = (1/2\pi)\partial S/\partial\epsilon$ the effective mass of an electron, $S(\epsilon, p_x)$ the area of intersection of the surface $\epsilon(\mathbf{p}) = \epsilon$ by the plane† $p_x = \text{const}$. The integration in Eq. (2.4) is along the curve $v_z = 0$ on the surface $\epsilon(\mathbf{p}) = \zeta$, φ is the angle between the velocity (i.e., the normal to the Fermi surface) and the direction of the constant magnetic field $\mathbf{H}$, and $1/\tau_0 = \overline{(1/t_0)}$ is the average along the path of integration of the rate of collision of electrons with impurities, phonons, electrons, lattice defects, surface irregularities, etc.

In general, the complex tensors $A_{\mu\nu}$ and $Z_{\mu\nu}$ cannot be reduced to diagonal form by a rotation of axes. But if such a reduction is possible (for example, if $R_{\mu\nu}$ and $X_{\mu\nu}$ are proportional) then the principal values of Z_α are given by

† The x axis is taken parallel to $\mathbf{H}$, so that p_x is the component of the wave vector parallel to the constant magnetic field.

$$Z_\alpha = 2(\sqrt{3}\pi\omega^2/c^4)^{1/3} e^{i\pi/3} A_\alpha^{-1/3} \tag{2.5}$$

where A_α are the principal values of $A_{\mu\nu}$.

(a) Resonant Region. The dependence of the surface impedance on magnetic field and frequency in the resonant region has been already discussed by us in detail.[1,2] We have shown that the form of the dispersion law has an important effect on the shape of the resonance curve. Resonance occurs at values of the effective mass $(1/2\pi)(\partial S/\partial\epsilon)_{ext}$ which are extremal with respect to φ. An extremum of the effective mass will in any case occur at the central section $p_x = 0$, $\epsilon = \zeta$, and also at elliptical points of support of the Fermi surface* (i.e., points at which the electron velocity is parallel to the constant magnetic field; such points have $m = 1/v\sqrt{K}$).

It can be shown that a resonant behavior (i.e., a minimum of R and X), will be seen only in the $Z_{x'x'}$ component. Here x' means the direction of H when the resonance corresponds to the effective mass at an elliptical point of support, and x' is the direction of the velocity at the point $v_z = 0$ on a central section of the Fermi surface when the resonance corresponds to the effective mass on a central section.

The absorbed power P is given by

$$P = \frac{c^2}{8\pi}\{R_{xx}(E_x^{(i)})^2 + R_{yy}(E_y^{(i)})^2\} \tag{2.6}$$

($E^{(i)}$ is the field amplitude of the incident wave) and has a resonant behavior only when the incident wave is plane polarized in the plane of x'. More exactly, the angle between $E^{(i)}$ and x' should not exceed $\chi \sim [R^{res}/R^{(0)}]^{1/2}$. In the case of a maximum effective mass $\chi \lesssim (q^2/\omega\tau_0)^{1/12}$, and in case of a minimum $\chi \lesssim (q^2/\omega\tau_0)^{2/9}$. Here $q = \omega/\Omega_{res}$ is an integer.

If a resonance corresponds to an extremal effective mass which is neither equal to the mass at a central section nor to the mass at an elliptical point of support, then all components of the tensor $Z_{\mu\nu}$ have resonant behavior. Resonant absorption of power will then occur for any polarization of the electric field in the incident wave. The formulae for all components of the impedance which have resonant behavior have been exhibited previously[1,2] for frequencies close to resonance.

(b) Nonresonant Region. When the magnetic field is so strong that

$$\Omega \gg 2\pi/\tau_0,\ 2\pi\omega,\ \pi\omega\cdot\omega\tau_0 \tag{2.7}$$

the tensor $A_{\mu\nu}$ can be reduced to the principal axes (x'', y''), and its principal values are given by

$$A_\alpha = \frac{8e^2}{3h^3}(i\omega + 1/\tau_0)^{-1}\,\overline{(n_\alpha^2\Omega/K)} \tag{2.8}$$

where the bar indicates an average with respect to the angle φ. Hence Eq. (2.5) gives

$$Z_\alpha(H) = 2\left(\frac{\sqrt{3}\pi\omega^2}{c^4}\right)^{1/3} e^{i\pi/3}\left[\frac{8e^2}{3h^3}\overline{\left(\frac{n_\alpha^2\Omega}{K}\right)}\frac{\tau_0}{1+i\omega\tau_0}\right]^{-1/3} \sim H^{-1/3}\omega^{2/3}(1+\omega^2\tau_0^2)^{1/6}\exp\left[\frac{i}{3}(\pi + \tan^{-1}\omega\tau_0)\right] \tag{2.9}$$

One of us[8] obtained this formula earlier in the case of a quadratic dispersion law. In particular, when the frequency is low ($\omega\tau_0 \ll 1$)

$$Z_\alpha = \left(\frac{\sqrt{3}\pi\omega^2}{c^4}\right)^{1/3}\left[\frac{8e^2\tau_0}{3h^3}\overline{\left(\frac{n_\alpha^2\Omega}{K}\right)}\right]^{-1/3}(1+i\sqrt{3}) \sim \left(\frac{\omega^2}{H\tau_0}\right)^{1/3}(1+i\sqrt{3}), \tag{2.10}$$

while at high frequencies ($\omega\tau_0 \gg 1$)

$$Z_\alpha = 2\left(\frac{\sqrt{3}\pi}{c^4}\right)^{1/3}\left[\frac{8e^2}{3h^3}\overline{\left(\frac{n_\alpha^2\Omega}{K}\right)}\right]^{-1/3}\left(\frac{1}{3\tau_0} + i\omega\right),\quad R_\alpha \sim H^{-1/3}/\tau_0,\ X_\alpha \sim \omega H^{-1/3}. \tag{2.11}$$

In the case of high frequencies ($\omega\tau_0 \gg 1$), the formulae are still simple in the range

$$2\pi\omega \ll \Omega \ll \pi\omega\cdot\omega\tau_0. \tag{2.12}$$

*Resonance occurs in this case only when the frequency is not too high. The skin depth has to be small compared with the Larmor radius, which itself becomes small near to an elliptical point of support.

where the magnetic field is of intermediate strength. The result is then

$$A_{\mu\nu} = -i\frac{4e^2}{3\pi h^3\omega}\int_0^{2\pi}\frac{n_\mu(\varphi)\,n_\nu(\varphi)}{K(\varphi)}\Omega(\varphi)\,d\varphi + \frac{4e^2}{3h^3}\int_0^{2\pi}\frac{n_\mu(\varphi)\,n_\nu(\varphi)}{K(\varphi)}\,d\varphi, \tag{2.13}$$

and the impedance is independent of τ_0,

$$R_{\mu\nu}\sim\omega^2H^{-4/3},\quad X_{\mu\nu}\sim\omega H^{-2/3}. \tag{2.14}$$

The temperature dependence of the surface impedance can be obtained from the results of Refs. 1 and 2 and the present paper by substituting

$$1/\tau_0 = 1/\tau_0^{\mathrm{res}} + (1/\tau_0^{\mathrm{p}})(T/\Theta)^3 + \frac{1}{\tau_0^e}(T/\Theta_e)^2, \tag{2.15}$$

This behavior of τ_0 holds for $\hbar\omega \ll kT$; τ_0^{res} is the relaxation time corresponding to the residual resistivity, τ_0^{p} and τ_0^{e} the relaxation times for scattering by phonons and electrons, Θ is the Debye temperature, and Θ_e is the characteristic temperature for electron-electron collisions. Equation (2.15) is valid because of the possibility of defining a mean time of free passage $t_0(p)$ in the theory of the anomalous skin effect.[1,2]

In particular, when phonon collisions are preponderant, we have the following results:

(1) Quadratic dispersion law,

$$\begin{aligned} R_{\mathrm{res}} &\sim (T/\Theta)^2, & |\omega - q\,\Omega_{\mathrm{res}}| &\sim (T/\Theta)^{3/2},\\ X_{\mathrm{res}} &\sim T/\Theta, & |\omega - q\,\Omega_{\mathrm{res}}| &\sim (T/\Theta)^3,\\ (X/R)_{\mathrm{res}} &\sim (T/\Theta)^{-3/2}, & |\omega - q\,\Omega_{\mathrm{res}}| &\sim (T/\Theta)^{9/2}; \end{aligned} \tag{2.16}$$

(2) Nonquadratic dispersion law, $\partial S/\partial\epsilon$ having a minimum,

$$\begin{aligned} R_{\mathrm{res}} &\sim (T/\Theta)^{4/3}, & |\omega - q\,\Omega_{\mathrm{res}}| &\sim (T/\Theta)^2,\\ X_{\mathrm{res}} &\sim (T/\Theta)^{1/3}, & |\omega - q\,\Omega_{\mathrm{res}}| &\sim (T/\Theta)^3,\\ (X/R)_{\mathrm{res}} &\sim (T/\Theta)^{-1}, & |\omega - q\,\Omega_{\mathrm{res}}| &\sim (T/\Theta)^2; \end{aligned} \tag{2.17}$$

(3) Nonquadratic dispersion law, $\partial S/\partial\epsilon$ having a maximum,

$$Z_{\mathrm{res}}\sim(T/\Theta)^{1/2},\ |\omega - q\,\Omega_{\mathrm{res}}|\sim(T/\Theta)^3,\ \Omega_{\mathrm{res}} = eH_{\mathrm{res}}/cm_{\mathrm{ext}}\approx\omega/q \tag{2.18}$$

with q an integer.

(4) Strong field, ($\Omega \gg 2\pi\omega$, $\pi\omega^2\tau_0$, $2\pi/\tau_0$). For $\omega\tau_0 \ll 1$, Z_α is proportional to temperature. For $\omega\tau_0 \gg 1$, $R_\alpha \sim (T/\Theta)^3$ and X_α is independent of temperature.

(5) Intermediate field, ($2\pi\omega \ll \Omega \ll \pi\omega^2\tau_0$). At high temperatures the impedance is independent of temperature.

Naturally, the above results are valid only so long as the skin effect is anomalous, i.e., for $\delta \ll r$, $|l^*|$. This means that the frequencies are restricted by

$$c^2/2\pi\sigma l^2 \ll \omega \ll (v/c)\sqrt{2\pi ne^2/m}$$

and the fields by

$$H \ll v[2\pi nm\omega\tau_0/(1+\omega\tau_0)]^{1/2}$$

where n is the electron density in the metal. At low temperatures, taking $\tau_0 \sim 10^{-11}$ sec, $n \sim 10^{22}\ \mathrm{cm}^{-3}$, $m \sim 10^{-27}$ g, these limits become $10^7\ \mathrm{sec}^{-1} \ll \omega \ll 10^{13}\ \mathrm{sec}^{-1}$, $H \ll 10^6[\omega\tau_0/(1+\omega\tau_0)]^{1/2}$ oersted.

3. COMPARISON BETWEEN THEORY AND EXPERIMENT

In the introduction we mentioned that the variation of surface resistance with magnetic field has been observed in three metals, tin,[4,5] copper,[4] and bismuth[6,7] (see Figs. 1 and 2). In all cases there is qualitative agreement between experiment and theory. (1) A resonance is observed, except in the case of Fawcett's experiment with copper, in which the quantity $\omega\tau_0$ is of the order of unity and is not large enough for a resonance. (2) The surface resistance decreases monotonically in strong magnetic fields; the increase of surface resistance in bismuth for $H > 2000$ oersted is probably due to the onset of nor-

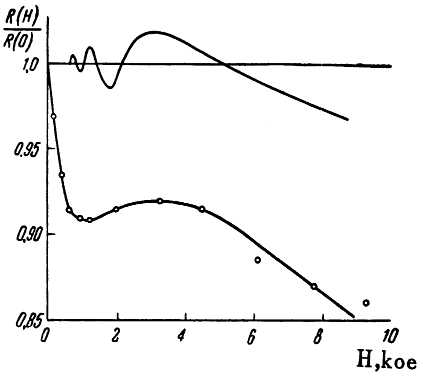

FIG. 1. Cyclotron resonance in tin according to Fawcett. Figure taken from Ref. 12. Experimental points for $\omega/2\pi = 24 \times 10^9$, $T = 4.2°$ K, $\omega\tau_0 \sim 27$ are marked O. The upper curve is theoretical.

mal skin-effect conditions when $r < \delta$. (3) R(H) is slightly decreasing in weak fields. (4) The resonant frequency is displaced as the temperature decreases,[5] because of the increase in ωt_0. (5) The surface resistance is anisotropic, even in the case of a single crystal of copper which possesses cubic symmetry, the axis of the asymmetry being provided by the direction of the constant magnetic field.

Nevertheless, the experimentally observed resonance is less deep, and the decrease of surface resistance in strong fields is smoother, than the theory predicts.

In the case of bismuth, the discrepancy between theory and experiment is apparently connected with the anomalously small density and effective mass of the electrons, which cause the basic assumption $\delta \ll r$ to be violated.

In the case of tin and copper, we believe that the cause of the discrepancy is as follows. The theoretical formulae apply only when two conditions are satisfied.

(1) For any value of the magnetic field, the angle between the field and the metal surface should satisfy

$$\Phi \ll (r/l)(\delta/r)^{2/3} = (\delta^2 r/l^3)^{1/3} \sim H^{-1/3}. \qquad (3.1)$$

The larger the field, the more strictly must the directions be parallel. When Eq. (3.1) is not satisfied, the effective distance which an electron can travel while remaining in an appreciable electric field increases more slowly with magnetic field than it does when $\Phi = 0$, (see Fig. 3). The variation of the impedance with magnetic field is therefore smoother when Eq. (3.1) does not hold, and for sufficiently high fields the impedance will saturate. Fawcett[4] observed just such a smoothed-out variation of R(H) for copper and tin, using an angle Φ of the order of one degree, whereas in a field $H \sim 10^4$ oersted with

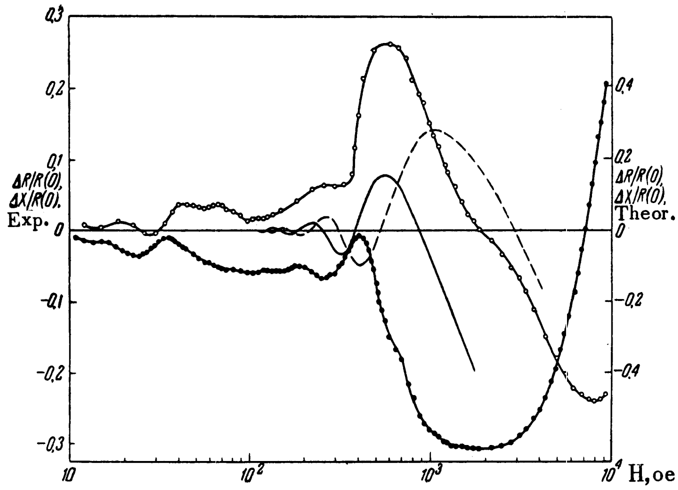

FIG. 2

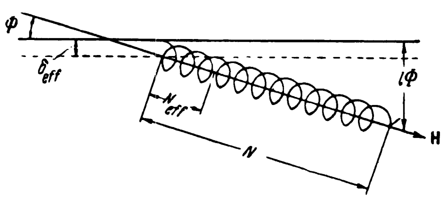

FIG. 3

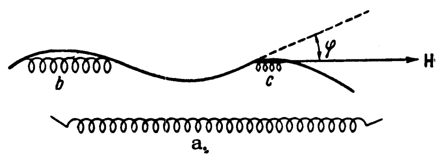

FIG. 4

FIG. 2. Cyclotron resonance in bismuth according to Aubrey and Chambers.[7] Full circles show $\Delta R/R(0)$ against H, open circles show $\Delta X/R(0)$ against H, for $\omega = 2\pi \times 9 \times 10^9$ cycles, $T = 4°$ K. The full curve gives theoretical values of $\Delta R/R(0)$, the dotted curve theoretical values of $\Delta X/R(0)$. $\Delta R = R(H) - R(0)$, $\Delta X = X(H) - X(0)$.

FIG. 3. Path of an electron in a magnetic field making an angle Φ with the metal surface. N_{eff} is the effective number of revolutions of an electron within the skin depth; N is the total number of revolutions between collisions.

FIG. 4. The "surface" mean free path of an electron. φ is the angular inclination of a surface irregularity. a shows an electron path in the interior of the metal, b and c are paths of electrons near the metal surface.

$\omega t_0 \sim 27$ the required upper limit for Φ is of the order of a few minutes. The inclination of the magnetic field will also produce similar effects in the resonance region.

(2) The second possible cause of departure of experimental results from theory is the imperfection of the metal surface. The collision rate $(1/\tau_0)$ which determines the depth of the resonance is an averaged rate of collisions for electrons close to the metal surface, since only these electrons contribute significantly to the current. The "surface" mean free time t_0^s may be quite different from the mean free time t_0^m in the interior of the metal which is calculated from the static conductivity (see Fig. 4). For t_0^s and t_0^m to be comparable, the angle of inclination φ of surface irregularities which are not small compared with the skin depth must also satisfy Eq. (3.1). Figure 4 shows how, if φ does not satisfy Eq. (3.1), δ_{eff}, and hence also t_0, will depend on H. In this case the dependence of the impedance on magnetic field is caused only by the fraction of the metal surface which is effectively parallel to the magnetic field, and this fraction in turn depends on H. Thus the values of ωt_0^m can be much larger than the values of ωt_0^s which determine the depth of the resonance. This is probably the explanation of the large width of the resonance in Fawcett's experiment,[4] in which $\omega t_0 \sim 27 \gg 1$.

Finally, it is possible that extremum values of m will occur only at points of support and at central sections of the Fermi surface. In this case one will observe sharp resonances only by using incident radiation with a definite polarization (see the preceding section).

4. THE POSSIBILITY OF DETERMINING THE FERMI SURFACE

The determination of the Fermi surface $\epsilon(\mathbf{p}) = \zeta$ and of the electron velocities in it is the basic problem of the electronic theory of metals. The problem is extremely difficult, partly because hitherto we have known nothing about the general shape of the surface except that it has central symmetry and the periodicity of the reciprocal lattice. The surface might be multiply connected, intersecting itself, or open. This makes difficult and partly ambiguous the theoretical interpretation even of those experiments[9] which allow in principle a determination of the form of the surface and of the velocities in it. It is therefore desirable to begin by exploring the topology of the surface, to find for example the number of nonsingular components into which it can be separated.

The results of Sec. 2 and of Refs. 1 and 2 show that the study of the anisotropy of cyclotron resonance leads to a number of conclusions about the geometry of the Fermi surface.

(1) If for every direction of the constant magnetic field and every polarization of the incident radiation we observe a resonance with the relative depth $(R_{res}/R^{(0)}) \sim (\omega\tau_0/2\pi q)^{-2/3}$, this resonance is evidently produced by ellipsoids. The number of ellipsoids is equal to the number of fundamental frequencies ($\omega \approx \Omega$) corresponding to resonances of this type.

(2) If, for some directions of the constant magnetic field and for some polarizations of the incident radiation, the relative depth of a resonance has the behavior $(R_{res}/R^{(0)}) \sim (\omega\tau_0/q^2)^{-4/9}$ or $(R_{res}/R^{(0)}) \sim (\omega\tau_0/q^2)^{-1/6}$, then one or more components of the surface $\epsilon(\mathbf{p}) = \zeta$ are not ellipsoids. The resonance is produced by an extremal value of the effective mass, which is a minimum in the first case and a maximum in the second.

(3) The number of fundamental resonant frequencies with the polarization parallel to the constant magnetic field is in general greater than the number appearing with any other adjacent direction of polarization. The difference between these two numbers of fundamental frequencies is equal to one half the number of elliptical points of support of the Fermi surface corresponding to the given direction H. As the direction of the constant magnetic field varies, the minimum number of elliptical points of support cannot be less than twice the number of closed convex components of the surface. In particular, if for any direction of H the number of elliptical points of support is zero, then the surface $\epsilon(\mathbf{p}) = \zeta$ has no closed convex component. It may happen that there are no points of support in this direction, so that the surfaces are all open, or it may be that there are points of support of hyperbolic of parabolic type. If one knows the number of elliptical points of support in every direction, one may be able to deduce the number of nonsingular components of the surface $\epsilon(\mathbf{p}) = \zeta$, and the number of open sections.

(4) If, for a certain direction of the constant magnetic field, the number of fundamental resonant frequencies is greater for one direction of plane polarization of the incident radiation (not parallel to the direction of H) than for other directions of polarization, then one component of the surface $\epsilon(\mathbf{p}) = \zeta$ is centrally symmetric. The number of centrally symmetric components is equal to the maximum number of such exceptional directions of polarization which appear as the direction of the field is varied.

If, for some direction of the constant magnetic field, the number of exceptional directions of polarization (not coinciding with the direction of H) which have the maximum number N of fundamental frequencies is less than the number M of centrally symmetric components of the surface, then the difference (N − M) gives the number of open central sections perpendicular to that direction of the constant field. By varying the direction of the field we may find all the open central sections.

(5) If, for all directions of H, resonance occurs only with particular polarizations of the incident radiation, the resonance arises only from central sections and elliptical points of support. The effective mass then never attains its extremum with respect to φ. The extremum with respect to p_x is attained only at central sections.

Such a study of the geometry of the Fermi surface will obviously require very laborious experiments. One needs to observe the anisotropy of the impedance with respect to both the direction of the constant field and the polarization of the incident radiation. However, these difficulties arise from the complicated geometry of the Fermi surface itself. We wish to emphasize that the study of cyclotron resonance allows us to elucidate the structure of the main components of the conduction band of the metal. The clues to the structure of the Fermi surface, which are obtained from observations of the anisotropy and relative depth of resonances, may greatly facilitate the solution of the next problem to which we now turn, the quantitative determination of the surface.*

A given resonant frequency corresponds in general to a value of the effective mass either on a single centrally symmetric surface or on a pair of surfaces which together have central symmetry. It is therefore sufficient to consider the determination of a single surface. If this surface turns out not to have central symmetry, a second surface can be obtained from it by reflection in the center. For simplicity we suppose that the surface is convex.

Measurement of the resonant frequencies and of the values of R and X at resonance determines the quantities

$$\partial S/\partial \varepsilon, \quad K^2 |\partial^3 S/\partial \varepsilon\, \partial \varphi^2|, \quad \tau_0$$

at points where $\partial S/\partial \epsilon$ is an extremum with respect to variations of φ. The anisotropy of the effect (see above) distinguishes the resonances arising from central sections and from elliptical points of support. In these two cases, the direction of polarization of the incident radiation for which resonance occurs determines the direction of the velocity at the corresponding point on the surface (see Sec. 2).

At an elliptical point of support $1/\tau_0 = 1/t_0(\mathbf{p})$, so that the collision rate at each point of the Fermi surface is directly determined, and $(\partial S/\partial \epsilon)_{ext} = 2\pi/v\sqrt{K}$. When we know $\partial S/\partial \epsilon$ and $\partial^3 S/\partial \epsilon\, \partial \varphi^2$, we can in many cases determine the form of the surface and the electron velocity in it. As an example we will discuss a convex surface with central symmetry. In this case, by observing the exceptional directions of polarization, not coinciding with the direction of H, for which resonance corresponding to a central section occurs, we determine the normal direction at every point on the surface. This allows us to fix the surface up to a similarity transformation. Knowing the shape of the surface, we deduce from $(\partial S/\partial \epsilon)_{ext}$ the magnitude of the velocities on the surface.[10] The scale of the similarity transformation can be determined, for example, from the value of X at resonance. This quantity fixes $K^2 |\partial^3 S/\partial \epsilon\, \partial \varphi^2|$, or in the case of an ellipsoid $\overline{(1/K)}$, once τ_0 is known. A check on the consistency of the construction of the surface is provided by our knowledge of the values of $v\sqrt{K}$ at every point on it.

Measurements of the surface impedance off resonance† can also be used for finding the form of the Fermi surface and the electron velocities in it. Such measurements are less convenient, since the impedance is then produced by the combined effect of all components of the surface $\epsilon(\mathbf{p}) = \zeta$. We therefore discuss only the case of a one-component surface.

As we showed earlier,[2] from a knowledge of the principal values Z_α of the surface impedance under anomalous skin-effect conditions in zero magnetic field[11] we can determine the Gaussian curvature K at every point of the surface. If the surface is convex, its shape is uniquely determined by its Gaussian curvature.‡

*The results stated here were in part published earlier.[2]

†Chambers[12] has recently arrived at similar conclusions by an independent method.

‡Note added in proof, (November 20, 1957). This requires the performance of experiments similar to those of Pippard.[13] After this paper was submitted for publication, we learned that Pippard has determined the Fermi surface of copper by this method (A. B. Pippard, Report to the 4th All-Union Conference on Low-Temperature Physics, Moscow, July 1957).

In addition, measurement of the impedance in strong magnetic fields ($\Omega \gg 2\pi/\tau_0, 2\pi\omega$) at low frequencies ($\omega\tau_0 \ll 1$) allows us to determine $\overline{(\Omega\tau_0/K)}$. If we know the form of the surface and the velocities in it we can deduce τ_0. The values of $\overline{(\Omega\tau_0/K)}$ are obtained from Eq. (2.10), without assuming τ_0 to be independent of φ, by taking the average of $n_\alpha^2\Omega\tau_0/K$. When τ_0 is known, a similar argument determines $1/t_0(\mathbf{p})$, so that the probability for electron transitions from one state to another is in principle determined. We must emphasize that the value of $t_0(\mathbf{p})$ obtained in this way will agree with the value appropriate to bulk metal only when the conditions discussed in Sec. 2 are fulfilled. Measurement of the imaginary part of the impedance at intermediate field strengths ($2\pi\omega \ll \Omega \ll \pi\omega^2\tau_0$) and high frequencies ($\omega\tau_0 \gg 1$) determines

$$X_\alpha = \left(\frac{\sqrt{3}\pi\omega^2}{c^4}\right)^{1/3}\left[\frac{8e^2}{3h^3\omega}\left(\overline{\frac{n_\alpha^2\Omega}{K}}\right)\right]^{-1/3}$$

This gives an independent determination of $\overline{\Omega/K}$ and consequently of Ω/K at every point.

Finally, one can introduce data on the de Haas–Van Alphen effect to help determine the Fermi surface. Measurement of the periods of the "high-frequency" oscillations of diamagnetic susceptibility in very strong magnetic fields ($H > 10^5$ oersted) determines the extremal areas S_{ext} of the surface $\epsilon(\mathbf{p}) = \zeta$. Experiments on cyclotron resonance in metals give values of $(\partial S/\partial\epsilon)_{ext}$, which are hard to obtain from the de Haas–Van Alphen effect. When we know both these quantities, we have a complete determination of the Fermi surface and of the velocities in it.

Unfortunately there is still a complete lack of the experimental data on cyclotron resonance which would enable us to study the dispersion law of conduction electrons.

CONCLUSIONS

(1) Cyclotron resonance occurs in metals at high frequencies ω and at low temperatures T, when the magnetic field is precisely parallel to the metal surface [$\Phi \ll (r/l)(\delta/r)^{2/3}$]. The resonance occurs at all the harmonic frequencies $\omega = \Omega, 2\Omega, \ldots$.

(2) The shape and width of the resonance, and the anisotropy of the effect, are strongly dependent on the shape of the Fermi surface $\epsilon(\mathbf{p}) = \zeta$. If the surface is ellipsoidal, the resonance must occur for all directions of the magnetic field and for all polarizations of the radiofrequency field. The relative depth of the resonance is then given by

$$R_{res}/R^{(0)} \sim (\omega\tau_0)^{-2/3}, \quad X_{res}/X^{(0)} \sim (\omega\tau_0)^{-1/3}.$$

(3) If the surface is not ellipsoidal, the resonant frequency corresponds to a value of the effective mass $(1/2\pi)(\partial S/\partial\epsilon)_{ext}$ which either is extremal with respect to variations of p_x or is associated with an elliptical point of support of the surface.

(4) For a given direction of the magnetic field H, a resonance corresponding to an elliptical point of support occurs when the radio-frequency field is polarized parallel to H.

(5) A resonance corresponding to a central section occurs only when the polarization is parallel to the electron velocity at the point $p_x = v_z = 0$ on the surface. Here the normal to the metal surface is taken as the z-axis.

(6) A resonance corresponding to an extremum of the effective mass, on a section p_x = const. of the surface $\epsilon(\mathbf{p}) = \zeta$ which is neither a central section nor a degenerate section at a point of support, occurs for all direction of polarization of the radio-frequency field.

(7) The relative depth of the resonance is different for maximum and minimum values of the effective mass. For a maximum it is given by

$$R_{res}/R^{(0)} \sim (\omega\tau_0)^{-1/6}, \quad X_{res}/X^{(0)} \sim (\omega\tau_0)^{-1/6}.$$

and for a minimum by

$$R_{res}/R^{(0)} \sim (\omega\tau_0)^{-4/6}, \quad X_{res}/X^{(0)} \sim (\omega\tau_0)^{-1/6}$$

(8) Away from resonance, the form of the dispersion law does not affect the main theoretical conclusions. The dependence of the surface impedance on H, in not too weak magnetic fields [$H \gtrsim (mc/e) \times (1/\tau_0 + \omega)$] strictly parallel to the metal surface, is described approximately by the same formula which holds for a quadratic dispersion law:

$$Z = Z^{(0)}[1 - \exp(-2\pi i\omega/\Omega - 2\pi/\Omega\tau_0)]^{1/3}.$$

In particular, Eq. (2.9) holds for any dispersion law in very strong magnetic fields ($\Omega \gg 2\pi/\tau_0$, $2\pi\omega$, $\pi\omega^2\tau_0$). At high frequencies ($\omega\tau_0 \gg 1$) the dependence of the impedance on frequency and magnetic field is given by

$$R \sim \omega^2 H^{-4/3}, \quad X \sim \omega H^{-1/3}.$$

also in the intermediate range of field-strengths ($2\pi\omega \ll H \ll \pi\omega^2\tau_0$).

(9) In weak magnetic fields ($\Omega \ll \omega + 1/\tau_0$), the impedance is constant to within a few per cent. In particular, if the field is very weak ($H \ll H_1$), the impedance varies according to Eq. (2.2).

(10) The temperature dependence of the impedance is determined by the quantity $1/\tau_0$ given by Eq. (2.15). Here τ_0^{res}, τ_0^{p}, and τ_0^{e} are independent of temperature.

(11) When plane-polarized electromagnetic waves are reflected from a metal surface in a magnetic field, there is always a considerable rotation of the plane of polarization, and this rotation may show a resonant behavior.

(12) Experimental study of cyclotron resonance can give abundant information about the form of the Fermi surface and about the mean collision time $t_0(\mathbf{p})$ of the electrons. It will make possible: (a) a determination of the number of ellipsoidal components of the surface $\epsilon(\mathbf{p}) = \zeta$, (b) a test for the existence of non-ellipsoidal components of the surface, (c) establishment of a lower bound to the number of closed convex components, (d) determination of the number and orientation of open central sections, (e) determination of the number of centrally-symmetric components, (f) in many cases, a complete specification of the Fermi surface and of the electron velocities and mean collision times at every point of it (see Sec. 4).

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Translated by F. J. Dyson