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NONLINEAR EFFECTS OF THE INTERACTION OF RESONANCE FIELDS IN THE MOLECULAR GENERATOR AND AMPLIFIER

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The position of the possible frequencies of generation and amplification, with saturation by an auxiliary field taken into account, is investigated on the basis of an analysis of the polarizability of a quantum system situated in two resonance fields. It is shown that an amplifier or a generator can operate at two frequencies which depend on the amplitude as well as on the frequency of the auxiliary field.

BASOV and Prokhorov[1] have considered a molecular generator and amplifier that does not utilize a molecular beam. In this case an auxiliary field of higher frequency is required for obtaining active molecules.

In contrast to the generator with a beam, where, with any kind of excitation (state selection), only the number of active molecules obtained plays a role, in generators and amplifiers with an auxiliary field the molecules must be situated simultaneously in two resonance fields with a common resonance level. In this case the auxiliary field will affect the shapes of the lines and the positions of the resonant frequencies of the transition used. Similar effects were observed by Burgess and Norberg[2] in the hyperfine spectrum of the $(SO_3)_2NO^{--}$ radical and are easily explained on the basis of the quantum theory of dispersion.

Let two resonance fields with frequencies ω_1 and ω_2 act simultaneously on a quantum system with a nonequidistant discrete spectrum; let these frequencies be close to the transition frequencies ω_{mn} and ω_{mq} between any levels of the system (these levels will be called resonance levels), under the condition that one of the resonance levels be common to both fields.

We shall give an expression for the Fourier component of the average dipole moment of a gas, in which τ is the average time between collisions of a molecule, obtained under the usual assumptions,[3] but in the presence of two fields with frequencies

$$|\omega_1 - \omega_{mn}| \lesssim 1/\tau, \quad |\omega_2 - \omega_{mq}| \lesssim 1/\tau$$

and a common resonance level m. The perturbed Hamiltonian is $\hat{H}(t) = \hat{H}_0 + \hbar \hat{V}(t)$, where the perturbation has the form

$$\hat{V}(t) = \hat{\Phi}^{\pm} e^{\mp i\omega_1 t} + \hat{\Psi}^{\pm} e^{\mp i\omega_2 t}$$

(summation over ±). The resonance part of the density matrix $\hat{D}$ is determined from the equation

$$\left(\frac{\partial}{\partial t} + i\omega_{ab} + \frac{1}{\tau}\right) D_{ab} + i \sum_k \left(V_{ak}(t) D_{kb} - D_{ak} V_{kb}(t)\right) = \frac{\rho_a^0 - \rho_b^0}{\omega_{ab}} \left(\frac{\partial \hat{V}}{\partial t}\right)_{ab}, \qquad (1)$$

where

$$\hat{\rho}^0 = \exp(-\hat{H}_0/kT) / \mathrm{Sp} \exp(-\hat{H}_0/kT)$$

is the equilibrium distribution function in the absence of a field. Near resonance with $|V| \gtrsim 1/\tau$ (strong field) it is necessary to take into account the second term in (1), because of which the equation for $\hat{D}$ becomes linear with variable coefficients, which leads to "nonlinear" effects. For definiteness let $m > n > q$. We seek a solution of the form

$$\hat{D} = \hat{R} + \hat{Q}^{\pm} e^{\mp i\omega_1 t} + \hat{S}^{\pm} e^{\mp i\omega_2 t} + \hat{T}^{\pm} e^{\mp i(\omega_2 - \omega_1)t},$$

in which we retain only the resonance matrix elements

$$R_{mm},\ R_{nn},\ R_{qq},\ Q_{mn}^{+},\ Q_{nm}^{-},\ S_{mq}^{+},\ S_{qm}^{-},\ T_{nq}^{+},\ T_{qn}^{-}.$$

The resonance part of the average dipole moment of the system is (omitting the indices of degeneracy)

$$\mathbf{d}(t) = 2\,\mathrm{Re}\,\{\mathbf{d}_{nm}Q_{mn}^{+}e^{-i\omega_1 t} + \mathbf{d}_{qm}S_{mq}^{+}e^{-i\omega_2 t} + \mathbf{d}_{qn}T_{nq}^{+}e^{-i(\omega_2-\omega_1)t}\}. \tag{2}$$

In the limiting case of a strong field Ψ and of a weak Φ

$$Q_{mn}^{+} = \Phi_{mn}^{+}\left\{\frac{A+Bi}{\Delta\omega_1 - \Delta\Omega_1 - i/\tau} + \frac{C+Di}{\Delta\omega_1 - \Delta\Omega_2 - i/\tau}\right\}, \tag{3}$$

where

$$\Delta\omega_1 = \omega_{mn} - \omega_1,\quad \Delta\omega_2 = \omega_{mq} - \omega_2,\quad \Delta\Omega_{1,2} = \Delta\omega_2/2 \mp \sqrt{(\Delta\omega_2/2)^2 + |\Psi|^2},$$

$$A = (\gamma_1\Delta\omega_2 - \gamma_2\Delta\Omega_1)/(\Delta\Omega_2 - \Delta\Omega_1),\quad A + C = \gamma_2,\quad B = (\gamma_3 - \gamma_2)/\tau(\Delta\Omega_2 - \Delta\Omega_1),\quad B + D = 0.$$

$$\gamma_k = \gamma_{mn} - k\gamma_{mq}\frac{|\Psi|^2}{\Delta\omega_2^2 + \tau^{-2} + 4|\Psi|^2},$$

and where $k = 1, 2, 3$; $|\Psi|^2 \equiv |\Psi_{mq}|^2$, and

$$\gamma_{mn} = (\omega_1/\omega_{mn})(\rho_m^0 - \rho_n^0),\quad \gamma_{mq} = (\omega_2/\omega_{mq})(\rho_m^0 - \rho_q^0).$$

From formula (3) it is evident that in general the absorption curve has two maxima.

Let $|\Psi|^2 \gg \Delta\omega_2^2,\ \tau^{-2}$. Then the positions of the two absorption maxima are given by the relations

$$(\Delta\omega_1)_1 = -|\Psi| + \Delta\omega_2/2 - \gamma_{mq}/8\gamma_2\tau^2|\Psi|,\quad (\Delta\omega_1)_2 = |\Psi| + \Delta\omega_2/2 + \gamma_{mq}/8\gamma_2\tau^2|\Psi|, \tag{4}$$

and the values of $I = Q_{mn}^{+}/\Phi_{mn}^{+}$ at these points are given by

$$I_1 = \tau\left\{\frac{\gamma_2}{2} + \frac{\Delta\omega_2}{4|\Psi|}\gamma_{mn}\right\},\quad I_2 = \tau\left\{\frac{\gamma_2}{2} - \frac{\Delta\omega_2}{4|\Psi|}\gamma_{mn}\right\}. \tag{5}$$

As is evident from (5), in this case the absorption curve has two approximately equal maxima, almost symmetrical relative to $\Delta\omega_1 = 0$.

If $|\Psi|^2,\ \tau^{-2} \ll \Delta\omega_2^2$, the positions of the maxima are

$$(\Delta\omega_1)_1 = -|\Psi|^2/\Delta\omega_2,\quad (\Delta\omega_1)_2 = \Delta\omega_2,$$

and their magnitude is given by

$$I_1 = \tau\left\{\gamma_{mn} - \frac{|\Psi|^2}{\Delta\omega_2^2}(\gamma_{mn} + \gamma_{mq})\right\},\quad I_2 = \tau\frac{|\Psi|^2}{\Delta\omega_2^2}(\gamma_{mn} - \gamma_{mq}).$$

In general the real part of the polarizability vanishes at three points. For $|\Psi|^2 \gg \Delta\omega_2^2,\ \tau^{-2}$, the positions of these points are

$$\Delta\omega_1 = \begin{cases} -|\Psi| + {}^{1}\!/_{2}\Delta\omega_2 + (2\gamma_3 - \gamma_2)/2\gamma_2\tau^2|\Psi| \\ \Delta\omega_2(\gamma_1/\gamma_2) \\ |\Psi| + {}^{1}\!/_{2}\Delta\omega_2 - (2\gamma_3 - \gamma_2)/2\gamma_2\tau^2|\Psi|. \end{cases}$$

The formulas obtained allow us to draw the following conclusions about the molecular generator and amplifier with auxiliary radiation:

1. If the auxiliary radiation is large, then the amplifier can operate at two frequencies.
2. The molecular generator is excited at those frequencies for which the real part of the polarizability is equal or close to zero. Therefore, for the type of generator considered, oscillations can arise at three frequencies, the oscillations being unstable for the central frequency.
3. The frequency of the oscillations of the generator will depend on both the frequency and amplitude of the auxiliary field.

Note added in proof (November 16, 1957). We note that the solution of (1) for both fields of arbitrary intensity is very cumbersome, but does not lead to results new in principle. Because of the transitions through the common level, which is an intermediate one, the absorption maximum is split for the frequency ω_1 as well as for ω_2.

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[1] N. G. Basov and A. M. Prokhorov, J. Exptl. Theoret. Phys. (U.S.S.R.) 28, 249 (1955), Soviet Phys. JETP 1, 184 (1955); Usp. Fiz. Nauk 57, 485 (1955).

[2] J. H. Burgess and R. E. Norberg, Phys. Rev. 100, 752 (1955).

[3] R. Karplus and J. Schwinger, Phys. Rev. 73, 1020 (1948).

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DETERMINATION OF PHASES OF MATRIX ELEMENTS OF THE S-MATRIX

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For reactions involving two or three channels and reactions with the formation of three low-energy particles, it is shown how the phases of the matrix elements of the S-matrix can be expressed in terms of measurable quantities by using the unitary property of the S-matrix and the invariance of the theory with respect to time reversal. The reactions $\tau^+ \rightarrow 2\pi^+ + \pi^-$ and $p + p \rightarrow D + \pi^+$ are considered.

In papers by Aizu, Fermi, and Watson[1,2] a relation has been established between the phases of the amplitudes for photoproduction of π mesons on a nucleon and the scattering phase shifts for scattering of π mesons by a nucleon; this relation follows from the unitary property of the S-matrix and the invariance of the theory with respect to time reversal. Subsequently analogous relations have been obtained in connection with a number of other reactions. But in all these cases the interaction in one of the channels was assumed to be weak. It seems that it may be of use to consider similar relations in the case of strong interactions.

The consequences of the unitary property of the S-matrix and of the invariance of the theory with respect to time reversal are usually discussed by means of the so-called K-matrix. That is, S is written in the form

$$S = (1 - iK)/(1 + iK)$$

and it is shown that in virtue of the properties of the S-matrix the matrix K is real and symmetric in a representation in which spin components are not included in the set of quantum numbers that specify the states of the system. If there are n channels, this means that the matrix elements of the S-matrix are expressed in terms of $n(n+1)/2$ independent real numbers. But the matrix elements of the matrix K are not directly measurable quantities. The quantities directly measured are the squares of the absolute values of matrix elements of the matrix $T = -i(S - 1)$.