THE INNER BREMSSTRAHLUNG OF A POLARIZED MU MESON AND THE NONCONSERVATION OF PARITY

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On the hypothesis of the conservation of the combined parity, formulas are obtained for the angular and energy distributions of the inner bremsstrahlung accompanying the decay of a polarized μ meson.

For an unpolarized μ meson decaying into a neutrino, an antineutrino, and an electron, the bremsstrahlung has been calculated by Lenard[1] and Skorniakov.[2] It has now become evident[3,4] that parity is not conserved in decay interactions. In this connection it has been pointed out[5] that the neutrino can be a longitudinal particle.* This hypothesis can be regarded as experimentally confirmed.[6] Because of the longitudinal character of the neutrino, only the vector and pseudovector types of coupling take part in the interaction for the decay of the μ meson.[5]

In connection with the nonconservation of parity in the weak interactions, the question arises of the asymmetry of the angular distribution of the inner bremsstrahlung of a polarized μ meson. That such an asymmetry must exist can be seen just from the fact that the radiation is mainly in the direction of motion of the electron, and the electrons have a preferred direction of emission in the decay.

We use the method for calculating the decay probability, developed by Lenard,[1] and the method of calculation by means of polarization projection operators, developed by Tolhoek, Fano, and Michel.[7]

The probability of radiative decay in a range of momenta of the electron and the quantum is[1,2]

$$\mathfrak{W}(\mathbf{p}, \mathbf{K})\, d^3K\, d^3p = (2\pi)^{-8}\iint d^3k\, d^3k' \frac{1}{2}\sum_s\sum_w |(f\,|R|\,i)|^2\, \delta(P-K-k-k'-p)\, d^3K d^3p. \quad (1)$$

Here and below we have used the notations of Ref. 1: M and m are the masses of the μ meson and electron; ϵ and W are the energies of the electron and photon; P, p, K, k$'$, and k are the four-momenta of the μ meson, electron, photon, neutrino, and antineutrino, and $\mathbf{p}$ and $\mathbf{K}$ are three-dimensional momenta. We set $\hbar = c = 1$ and $e^2/4\pi = 1/137$.

The matrix element for the transition is:

$$(f\,|R|\,i) = eg\,[\bar{\nu}(k)(1+\gamma_5)\gamma_\sigma(1-\gamma_5)\nu(k')]\left\{u(p)\left[\frac{w}{(2W)^{1/2}}\frac{\hat{q}+im}{(qq)+m^2}\gamma_\sigma + \gamma_\sigma\frac{\hat{Q}+iM}{(QQ)+M^2}\frac{\hat{w}}{(2W)^{1/2}}\right](1+\lambda\gamma_5)\,U(P)\right\}. \quad (2)$$

In Eq. (2) the nonconservation of parity has already been taken into account. u(p) and U(P) are the wave functions of the electron and μ meson; $q = p + K$ and $Q = P - K$ are the momenta of the electron and μ meson in the intermediate state. The probability is found by summing over the spin states of the electron and the polarizations of the quantum:

$$\mathfrak{W}(\mathbf{p}, \mathbf{K}) = \frac{1}{4}\frac{e^2g^2}{(2\pi)^8\,\varepsilon W}F;\quad F = \frac{\pi}{12}(GG)\,\mathrm{Sp}\left\{\hat{p}\left[\frac{\hat{w}}{pK}(\hat{p}+\hat{K})\gamma_\sigma - \gamma_\sigma(\hat{P}-\hat{K}+iM)\frac{\hat{w}}{PK}\right](1+\lambda\gamma_5)(1+\beta)(1+\mathbf{J}\gamma_5\beta\gamma)\right.$$
$$\times\left[\gamma_\sigma(\hat{p}+\hat{K})\frac{w}{pK} - \frac{\hat{w}}{PK}(\hat{P}-\hat{K}-iM)\right](1+\lambda\gamma_5)\Big\}\ \frac{\pi}{12}\mathrm{Sp}\left\{\hat{p}\left[\frac{\hat{w}}{pK}(\hat{p}+\hat{K})\hat{G} - \hat{G}(\hat{P}-\hat{K}+iM)\frac{\hat{w}}{PK}\right](1+\lambda\gamma_5)(1+\beta)\right.$$
$$\left.\times(1+\mathbf{J}\gamma_5\beta\gamma)\left[\hat{G}(\hat{p}+\hat{K})\frac{\hat{w}}{pK} - \frac{\hat{w}}{PK}(\hat{P}-\hat{K}-iM)(1+\lambda\gamma_5)\right\},$$

*Note (October 9, 1957): In connection with the latest data on the neutrino and antineutrino, it must be pointed out that since in the problem considered here the essential point is just the existence of the vector and pseudovector types of interaction, which can be regarded as experimentally established[7] by the measurements of the energy spectrum of the electrons, the hypothesis of the longitudinal character of the neutrino leads only to a renormalization of the interaction constant (the renormalization constant being 2).

$$G \equiv P - p - K; \quad \hat{P} \equiv (P\gamma) \text{ etc.} \tag{3}$$

Here $\mathbf{J}$ is a unit vector giving the direction of the spin of the μ meson. The calculations give a formula of the following form:

$$F = \frac{\pi}{3}\{f_1 + (\mathbf{pw}_1)^2 f_2 + (\mathbf{pJ})(\mathbf{pw}_1)^2 f_3 + (\mathbf{pJ}) f_4 + (\mathbf{KJ}) f_5 + (\mathbf{pw}_1)(\mathbf{p}, [\mathbf{J}\times\mathbf{w}_1]) f_6 + (\mathbf{pw}_1)^2 (\mathbf{KJ}) f_7 + (\mathbf{Jw}_1)(\mathbf{pw}_1) f_8. \tag{4}$$

Here

$$\mathbf{w}_1 = \mathbf{K}\times[\mathbf{p}\times\mathbf{K}]/|\mathbf{K}\times[\mathbf{p}\times\mathbf{K}]|, \qquad f_i = f_i(\varepsilon, M, W, (\mathbf{pK})).$$

The values of the functions f_i and the limiting cases for them are given in the appendix.

The terms in Eq. (1b) of the appendix that contain in their denominators the quantity $(\mathrm{pK} - \epsilon \mathrm{W})$ to only the zeroth or first degree are small, since in them integration over the angles does not give rise to the factor $\ln(2\epsilon/m)^2$. After neglecting such terms one gets the formula

$$\mathfrak{W}(\mathbf{K}, \mathbf{p})\, d^3p\, d^3K = \frac{1}{2}\frac{e^2}{(2\pi)^3}\frac{(\mathbf{pw}_1)^2 W^2}{(\mathbf{pK} - \varepsilon W)^2}\mathfrak{W}(\mathbf{p})\, d^3p d^3K. \tag{5}$$

Here $\mathfrak{W}(\mathbf{p})$ is the probability for the emission of the electron into a given range of angles and energy, as given by Landau's formula.[5] Equation (5) has just the same sort of form as Lenard's formula,[1] but now $\mathfrak{W}(\mathbf{p})$ stands for a different expression.

The precision of Eq. (5) is given by the ratio $1 : \ln(2\epsilon/m)^2$, or about $1/_{10}$.

In order to find the total probability for emitting a quantum one must integrate Eq. (3) with allowance for (1b).

Since we are confining ourselves to just the region of small frequencies and thus the condition $W \le M/2 - \epsilon$ is fulfilled everywhere except in a very small range of energies of the electron, we can ignore the fact that near the upper limit of the integration, given by $M/2$, not all angles between the momenta of the electron and the quantum are allowed. Integrating, we get

$$\mathfrak{W}(\mathbf{K})\, d^3K = \frac{e^2 g^2\, dW}{24\,(2\pi)^6\, W} d\Omega_\gamma \left\{64\left[(1+\lambda^2)\left(\frac{\varepsilon_m^5}{2}\ln\frac{2\varepsilon_m}{m} - \frac{5}{24}\varepsilon_m^5\right) - 2\lambda\frac{\mathbf{KJ}}{W}\varepsilon_m^5\left(\frac{1}{6}\ln\frac{2\varepsilon_m}{m} - \frac{1}{72}\right)\right]\right\}. \tag{6}$$

The ratio of the numbers of quanta emitted at small frequencies in directions with and against the spin (the numbers of quanta integrated over the hemispheres) is $1 : 1.5$ for $\lambda = 1$. Let us calculate the probability of emission of a quantum with energy larger than a certain value W_0. We confine ourselves to terms containing the factor $\ln(2\epsilon/m)^2$, and thus get an accuracy of about 10%.

In integrating the expression (3) over the angles, using Eq. (1c) one must note the fact that at large energies not all angles between the momenta of the electron and the quantum are allowed; the limits of integration are therefore as follows: $-1 \le \cos(\mathbf{Kp}) \le 1$ for $W \le M/2 - \epsilon$ and $-1 \le \cos(\mathbf{Kp}) \le a$ for $W \ge M/2 - \epsilon$. Here $a = M^2/2pW - M/W - M/p + 1$.

As the result of the integration we get

$$\begin{aligned}\mathfrak{W}\left(\frac{\mathbf{K}}{W}, W_0\right) d\Omega_\gamma = \frac{e^2 g^2}{(2\pi)^6\, 24}\Bigg[\Bigg\{&(3\varepsilon M^2 - 4\varepsilon^2 M)\int\limits_{M/2-\varepsilon}^{M/2}\frac{AC\ln W}{W(BW+C)}dW - (\varepsilon - M)(M - 6\varepsilon)M\ln\frac{(2\varepsilon/m)^2}{(2\varepsilon/M)+1}\\ &+\left(-(3\varepsilon M^2 - 4\varepsilon^2 M)\ln W_0 - (3M^2 - 8M\varepsilon)W_0 + 2MW_0^2\right)\ln\left(\frac{2\varepsilon}{m}\right)^2\Bigg](1+\lambda^2) + 2\lambda(\mathbf{pJ})\Bigg[\Bigg(-12M\varepsilon\ln W_0 - 46MW_0\\ &-\frac{3M^2}{\varepsilon^2}W_0^2 - \frac{8}{3}\frac{M}{\varepsilon^2}W_0^3\Bigg)2\ln\frac{2\varepsilon}{m} + 12M\varepsilon\int\limits_{\frac{M}{2}-\varepsilon}^{M/2}\frac{AC\ln W}{W(BW+C)}dW - \left(\frac{133}{3}M\varepsilon - 41M^2 - \frac{2M^2}{\varepsilon} + \frac{5M^4}{\varepsilon^2}\right)\ln\frac{(2\varepsilon/m)^2}{(2\varepsilon/M)+1}\Bigg]\Bigg\} d\Omega_\gamma. \end{aligned} \tag{7}$$

Here

$$A \equiv 1 + \varepsilon/|\mathbf{p}|; \qquad B \equiv (2\varepsilon - 2M)/|\mathbf{p}|; \qquad C \equiv (M - 2\varepsilon)M/|\mathbf{p}|.$$

The total energy radiated into the given range of angles, as obtained in the same approximation as the preceding formula, is given by

$$\frac{e^2g^2}{80\,(2\pi)^6} M^6\,[(1+\lambda^2) - 2\lambda\cos(\mathbf{KJ})]. \tag{8}$$

We observe that the radiation is symmetric with respect to the plane of p and J. I thank L. B. Okun' for the suggestion of the problem and K. A. Ter-Martirosian and I. S. Tsukerman for discussions.

APPENDIX

Exact values of the functions f_i:

$$f_1 = -\{3M^2W/(pK) - 4MW^2/(pK) - 4M\varepsilon W/(pK) + 4W + 3(pK)/W - 4(pK)/M - 4\varepsilon(pK)/M^2 + 4(pK)^2/M^2W + 2M\}(1+\lambda^2),$$

$$f_2 = [(1+\lambda^2)/M(pK)^2]\{3\varepsilon M^3 + 3M^3W - 3(pK)M^2 - 4\varepsilon^2M^2 - 8M^2\varepsilon W - 4M^2W^2 + 8MW(pK) + 8\varepsilon M(pK) - 4(pK)^2\},$$

$$f_3 = -2\lambda(pK)^{-2}\{-4M^2 + 16M\varepsilon + 16MW\},$$

$$f_4 = 2\lambda\{-3M^2/W^2 + 8M\varepsilon/W^2 + 10M/W - 2pK/W^2 - 8\varepsilon/W + 2(pK)/MW + 4MW/pK\},$$

$$f_5 = 2\lambda\{-6\varepsilon M/W^2 + 12\varepsilon^2/W^2 - 4\varepsilon(pK)/MW^2 + 6M^2/(pK) - 4M\varepsilon/(pK) - 8MW/(pK) + 20 - 2(pK)/W^2 + 4\varepsilon/W + 18(pK)/MW - 4(pK)^2/MW^2 - 6M^2\varepsilon/(pK)W + 14M\varepsilon^2/(pK)W + 2M/W\},$$

$$f_6 = -(1+\lambda^2)\{-8M/(pK) + 24\varepsilon/(pK) + 24W/(pK) + 4/W\},$$

$$f_7 = -2\lambda\left\{\frac{1}{(pK)W}(8\varepsilon - 4M) + \frac{24}{(pK)} - \frac{8}{MW} + \frac{4M^2}{(pK)^2}\right\},$$

$$f_8 = 2\lambda\left\{\frac{8M\varepsilon - 6M^2 - 16pK}{W^2} + \frac{4M}{(pK)}(M - 2W) - \frac{2M^2\varepsilon}{W(pK)} - \frac{14M}{W} + \frac{24\varepsilon}{W} + 54 - \frac{2M\varepsilon^2}{W(pK)} + \frac{2\varepsilon M}{(pK)} - \frac{24(pK)}{MW}\right\}. \tag{1a}$$

Here $(pK) = pK - \epsilon W$.

If we confine ourselves just to the case of small frequencies, i.e., if $W \ll \epsilon$, then the functions f_i go over into the following:

$$f_1 = f_5 = f_6 = f_7 = 0; \quad f_2 = \frac{1}{M(pK - \varepsilon M)^2}\{3\varepsilon M^3 - 4\varepsilon^2M^2\}(1+\lambda^2), \quad f_3 = -2\lambda(16M\varepsilon - 4M^2)/(pK - \varepsilon W)^2;$$

$$f_4 = 2\lambda(8M\varepsilon - 3M^2)/W^2, \quad f_8 = 2\lambda\{(-6M^2 + 8M\varepsilon)/W^2 - (2M^2\varepsilon + 2M\varepsilon^2)/W(pK - \varepsilon W)\}. \tag{1b}$$

If we include only those terms that give $\ln(2\epsilon/m)^2$ on integration, then the functions f_i go over into the following:

$$f_2 = \frac{(1+\lambda^2)}{M(pK - \varepsilon W)^2}(3\varepsilon M^2 - 4\varepsilon^2M^2 + 3M^3W - 8M^2\varepsilon W - 4M^2W^2),$$

$$f_3 = -2\lambda\{-4M^2 + 16M\varepsilon + 16MW\}/(pK - \varepsilon W)^2, \quad f_7 = -2\lambda\,4M^2/(pK - \varepsilon W)^2. \tag{1c}$$

Note added in proof (November 15, 1957). While the present paper was in press, papers were published by Kinoshita and Sirlin, in which effects of radiative corrections and inner bremsstrahlung on the asymmetry of the positrons are calculated.

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