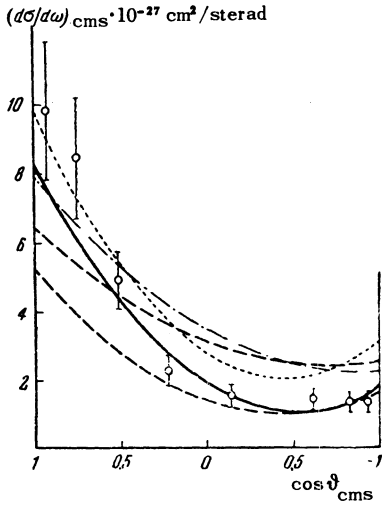

The solid curve represents $d\sigma/d\omega = 1.87 + 3.30\cos\vartheta + 3.14\cos^2\vartheta$. It is apparent that none of the computed γ distribution curves agree with the measured distribution, as was pointed out earlier.[1]

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[2] V. G. Zinov and C. M. Korenchenko, J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 301 (1958), Soviet Phys. JETP **7** (in press).

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EFFECT OF QUANTUM FLUCTUATIONS IN THE ELECTRON RADIATION OF THE SYNCHROTRON OSCILLATIONS

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THE problem of quantum fluctuations in the radiation of electrons in the synchrotron has been considered in a series of articles (see, for example, Refs. 1 — 4).

In this note we generalize the well known results of Sands,[3] namely, we take into account the damping of synchrotron oscillations caused by the increase in electron energy, and give several practical results.

Putting the damping coefficient

$$\rho = \frac{\dot{E}}{E} + \frac{3-4n}{1-n}\frac{2ce^2}{3R^2}\frac{\gamma^3}{1+\lambda}, \qquad (1)$$

where $\gamma = E/mc^2$, $\lambda = N\ell/2\pi R$, ℓ is the length of the straight section of the race track, N is the number of sections, into the phase equation of the synchrotron it is possible to obtain a formula for the stationary value of the mean square amplitude for synchrotron oscillations

$$\langle A_\varphi^2\rangle = \frac{55\sqrt{3}}{32}\frac{\hbar cq\,\cot\varphi}{e^2(1+\lambda)^2(3-4n)\gamma}F_1F_2. \qquad (2)$$

This expression differs from the result of Sands[3] by the presence of the factor F_1F_2, where

$$F_1 = \left(1+\alpha\frac{1-n}{3-4n}\frac{\dot{E}}{P}\right)^{-1}; \quad F_2 = \left(1+\frac{\dot{E}}{P}\right)^{-1}; \quad P = \frac{2ce^2}{3R^2}\frac{\gamma^4}{1+\lambda},$$

and α is a coefficient of order unity. At energies of several hundred Mev the factor F_1F_2 is important and, essentially, determines the energy dependence of $\langle A_\varphi^2\rangle$. Analysis of Eq. (2) shows that there is no danger of particle loss connected with a maximum of $\langle A_\varphi^2\rangle$ which, under the assumption $\cot\varphi = \text{const}$, occurs for $P = 7\dot{E}\alpha(1-n)/(3-4n) \approx 1$. The condition $\cot\varphi = \text{const}$ is, in fact, superfluous. Employing another law of increase for the accelerating voltage, it is easy to avoid this maximum.

At high energies where $F_1F_2 \to 1$, using results obtained by Sands,[3] one can find the excess of the amplitude of the accelerating voltage over the value of the amplitude, necessary to accelerate the elec-

trons and to compensate their energy loss by radiation

$$\frac{1}{\cos\varphi} > \frac{4}{\pi} + \frac{55\sqrt{3}}{64\pi^2}\,\frac{\hbar c}{e^2(1+\lambda)^2(3-4n)}\,\frac{\lambda_1^2}{\ln\dfrac{2}{\lambda_1 J_1(\lambda_1)}}\,\frac{q}{\gamma}. \qquad (3)$$

Here λ_1 is the first root of the Bessel function $J_0(x)$, q is the number of harmonics of the radio frequency employed in the synchrotron.

Satisfaction of the condition Eq. (3) is very difficult in accelerators with a large number of harmonics of the radio frequency and with weak focusing. Failure to satisfy this condition can lead to loss of a substantial fraction of the particles in the accelerator.

In conclusion, the author would like to thank M. S. Rabinovich for advice and suggestions. The author is grateful to V. I. Veksler for the interest he has shown in the present work.

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ON THE USE OF THE DISPERSION IN ENERGY AS A CRITERION FOR THE ACCURACY OF THE VARIATIONAL METHOD

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UP to now the accuracy of a solution of the Schrödinger equation by the variational method has been estimated by comparing the calculated energy value with its experimental value. However, this method is totally inapplicable when the experimental value of the level is unknown. Apart from this the error in the energy obtained in this way can reflect not only the inaccuracy of the trial function, but also the inaccuracy of the Hamiltonian. It is therefore of interest in a number of cases to estimate separately the magnitude of the error due to the inaccuracy of the trial function. We shall show below that this can be done by using the dispersion in the energy of the trial functions.

We consider a typical case, namely, the determination of the ground state of the system. This level is non-degenerate and its wave function is real. The deviation of the trial function Ψ from the eigenfunction Ψ_0 is thus equal to $\delta^2 = \|\Psi - \Psi_0\|$ (Ψ and Ψ_0 are normalized). We shall assume in the following that $\delta^2 \ll 1$. We expand the trial function for the complete system in terms of the normalized eigenfunctions of the Hamiltonian H,

$$\Psi = \sum_{k=0}^{\infty} C_k \Psi_k;\ \|\Psi\| = \sum_{k=0} |C_k|^2 = 1. \qquad (1)$$

For the error δ^2 we obtain the expression

$$\delta^2 = 2(1 - C_0) \approx 1 - C_0^2. \qquad (2)$$

The dispersion in energy in the state Ψ is equal to $D = (\Psi, H^2\Psi) - (\Psi, H\Psi)^2$. Using the expansion (1) we can obtain for D the expression