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NONLINEAR EFFECTS IN METALS AT LOW TEMPERATURES

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We investigate the possibility of observing deviations from Ohm's law in metals at low temperatures.

IN order to observe deviations from Ohm's law in all metals (apart from bismuth[1]) it is necessary to have huge current densities of the order of 10^8 to 10^9 amp/cm^2. This is connected with the fact that the energy gained by an electron between collisions is small. Since on decreasing the temperature the mean free path increases considerably, we can expect that the observation of nonlinear effects should, as a matter of course, be done at low temperatures. The present article is thus devoted to an analysis of this possibility.

1. At large current densities it is not possible to assume that the symmetrical part of the electron distribution function agrees with the Fermi equilibrium function. However, one can successfully approximate the symmetric part of the electron distribution function by the Fermi function, be it at a temperature different from the lattice temperature,[2,4] in those circumstances where the time between collisions between the electrons, which is given by the equation

$$\tau_{ee} = \frac{1}{q_e n v}\left(\frac{\zeta}{\Theta}\right)^2,$$

(where q_e is the cross section for electron collisions ($q_e \sim 10^{-15}$ cm^2), n is the number of electrons per unit volume, ζ the Fermi energy, v the velocity at the Fermi level, and Θ the temperature of the electron gas[2,3]), is considerably less than the time t_{rel} necessary to transfer energy from the electrons to the lattice. We calculate t_{rel} using the value of the heat transfer $\overline{U}$ found in Ref. 4.

$$\overline{U} \approx (ms^2 n / \tau_0 T_0^5)(\Theta^5 - T^5); \quad \Theta, T \ll T_0 \tag{1}$$

(here and henceforth we omit numerical factors of the order of unity; see Ref. 2). In Eq. (1) m is the electron mass, s the velocity of sound, T the lattice temperature, T_0 the Debye temperature, and τ_0 the relaxation time of the electrons at $T = T_0$ (putting $\Theta = T$).

The heat balance equations are of the form

$$C_e \partial\Theta / \partial t = -(ms^2 n / \tau_0 T_0^5)(\Theta^5 - T^5), \quad C_{\mathrm{ph}} \partial T / \partial t = (ms^2 n / \tau_0 T_0^5)(\Theta^5 - T^5), \tag{2}$$

where $C_e \approx n\Theta/\zeta$ is the electronic heat capacity and $C_{\mathrm{ph}} \approx n(T/T_0)^3$ the phonon heat capacity (we consider temperatures well below T_0).

Taking into account that $\Theta \approx T$ we have from equations (2)*

$$t_{\mathrm{rel}} \approx \begin{cases} \tau_0 T_0^2 / ms^2 T & (T \ll T_0 \sqrt{T_0/\zeta}), \\ \tau_0 T_0^5 / ms^2 \zeta T^3 & (T \gg T_0 \sqrt{T_0/\zeta}). \end{cases} \tag{3}$$

Comparing these expressions with τ_{ee}, we can satisfy ourselves that the time to produce the electronic Fermi distribution is less than t_{rel} down to temperatures of the order of 0.1 to 1°K (depending on the metal). This enables us to use as the electronic distribution function the quasi-equilibrium function $f_0(\Theta)$ $(\Theta \neq T)$ down to sufficiently low temperatures.

2. In calculating the resistance of a metal one usually assumes that $\Theta = T$. This is natural if one is not interested in nonlinear effects. If $\Theta \neq T$ the conductivity depends both on the phonon (lattice) tem-

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*In Ref. 4 we gave only the second equation of (3) and the region of applicability of this equation was not indicated.

perature and on the electron temperature. One can easily show that

$$\sigma = (ne^2\tau(\Theta, T)/m)\{1 - \pi^2\Theta^2/12\zeta^2\}, \tag{4}$$

where

$$1/\tau(\Theta, T) = 1/\tau_{\text{imp}} + 1/\tau_{\text{ep}}(T, \Theta) + 1/\tau_{ee}(\Theta). \tag{5}$$

In equation (5) τ_{imp}, τ_{ep} and τ_{ee} are the relaxation times referring respectively to collisions with impurities, with phonons, and with electrons.

The quantity τ_{imp} does not depend at all on the temperature, and τ_{ee} depends only on the electron temperature (see Sec. 1). The evaluation of the temperature dependence of τ_{ep} proceeds in the usual way (see, for instance, Ref. 5) with only one difference, namely that we assume the electron and phonon temperatures to be different $(\Theta \neq T)$. The result is

$$\frac{1}{\tau_{\text{ep}}(\Theta, T)} = \frac{1}{\tau_{\text{ep}}(T)}\left\{1 + \frac{4(\Theta^5 - T^5)}{5T^5}\right\}. \tag{6}$$

Here $\tau_{\text{ep}}(T)$ is the value of this relaxation time in the isothermal model.

To determine the electron temperature it is natural to use the heat balance equation

$$\rho j^2 = \overline{U}, \tag{7}$$

where ρ is the total resistivity which is practically equal to the impurity resistivity, $\rho \approx m/ne^2\tau_{\text{imp}}$. From Eqs. (7) and (1) it follows that

$$\Theta^5 = T^5 + (j/nes)^2(\tau_{\text{imp}}/\tau_0)T_0^5 \tag{8}$$

or

$$\frac{\Delta T}{T} \approx (j/nes)^2(\tau_{\text{imp}}/\tau_0)(T_0/T)^5. \tag{9}$$

Using expressions (4), (6), and (9), and the value of $\tau_{\text{ee}}(\Theta)$ we find the way the resistance depends on the current density (we assume that $\Delta T \ll T$),

$$\Delta\rho/\rho \approx (j/nes)^2\{1 + l_{\text{ep}}(T)/2l_{ee}(T)\}. \tag{10}$$

The ℓ's in this equation are connected to the τ through the relation $\ell = v\tau$. We see from this equation that nonlinear effects increase with the ratio $\ell_{\text{ep}}/\ell_{\text{ee}}$, which in turn increases as T^{-3} with decreasing temperature.

Our considerations are no longer correct at very low temperatures since the condition $\Delta T \ll T$ is no longer satisfied. We shall consider another limiting case, namely, $T = 0$. We have then from Eq. (8)

$$\Theta = T_0(\tau_{\text{imp}}/\tau_0)^{1/5}(j/nes)^{2/5},$$

and thus for $T = 0$

$$\Delta\rho/\rho \approx (j/nes)^2 + (\tau_{ee}^{2/5}\tau_{\text{imp}}^{3/5}/\tau_{0ee})(j/nes)^{4/5} \tag{11}$$

(where $\tau_{0\text{ee}} = \tau_{\text{ee}}(T_0)$) where the first term corresponds to the creation of phonons and the second one to electron-electron collisions.*

We notice that Eq. (11) is, strictly speaking, only justified for those current densities which produce an increase of the electron temperature to 0.1 to 1.0°K (see Sec. 1). If the current density is less, it is impossible to speak about two temperatures, and Eq. (11) must be assumed to give a purely qualitative description.

Finally we note that we must apparently look for nonlinear effects at low temperatures in those metals, for which a large part of the resistivity is due to electron-electron interactions.

If $\ell_{\text{ep}}/\ell_{\text{ee}} \sim 10^3$ to 10^4, we can expect an increase in the resistivity of one per cent for currents of the order of 10^5 to 10^6 amp/cm^2.

Apart from the paper by Ginzburg and Shabanskii, the papers by Shabanskii[6,7,8] are also devoted to a theoretical discussion of nonlinear effects in metals. In those papers the author has tried to extend the Fokker-Planck method to the case of a temperature well below the Debye temperature. It seems to us,

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*If we assume that the metal does not contain any impurities, we have at $T = 0$,

$$\rho = \rho_{ee}, \quad \rho_{ee}/\rho_{0ee} = (\tau_0/\tau_{0ee})^{2/3}(j/nes)^{4/3}.$$

however, that one can make serious objections to an application of this method to these cases. The trouble is that the terms which are neglected in the expansion of the collision integral for $T \ll T_0$ are of the same order of magnitude as the terms which are left in [see Ref. 6, Eq. (4)].

The inapplicability of the Fokker-Planck method to the problem under consideration can clearly be seen from the fact that the directly calculated coefficients A_1 and A_2 (see Ref. 6) do not agree with the values obtained by the author from the condition $S(f_0) = 0$. Apart from that, the author arrives in Ref. 7 at the paradoxical conclusion that the resistivity of a metal must go to infinity as $T \rightarrow 0$.

We shall not compare here all our results with the analogous equations of Refs. 6, 7, 8, although several of them agree in order of magnitude with the equations of those papers, in the case of small temperature differences ($\Delta T \ll T$).

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INVARIANT REPRESENTATIONS OF THE SCATTERING MATRIX

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The scattering matrix S for the reaction $a + b \rightarrow a' + b'$ is expressed in terms of a finite number of spin operators Q_i, each invariant under rotations and reflections. A method for constructing the Q_i is given, and their number is determined for a reaction with given initial and final spins. The restrictions placed upon the form and number of the Q_i by the condition that the scattering matrix be invariant under time reversal are considered. Examples are given in which S is represented by the Q_i for several reactions.

THE scattering matrix $S(\mathbf{k}', \mathbf{k})$ for the reaction $a + b \rightarrow a' + b'$ is an operator in the space defined by the spins of the incident and scattered particles, and is a function of their relative momenta $\mathbf{k}, \mathbf{k}'$. Since S is invariant under rotation, it can be written in the form

$$S(k', k) = \sum A_i (k'k)\, Q_i (k', k, T), \tag{1}$$

where the $Q_i(\mathbf{k}', \mathbf{k}, \mathbf{T})$ are invariant operators that depend on the vectors $\mathbf{k}', \mathbf{k}$ and on the matrices $\mathbf{T}$