where K_{max} is the maximum momentum of the emitted photon in units of m ($K_{max} \ll 1$).

2. The differential cross section of the Compton effect on a scalar particle has the following form

$$d\sigma / d\Omega = (\alpha / m)^2 (q_{20} / q_{10})^2 U, \tag{6}$$

where $d\Omega$ is the element of solid angle in the direction of the scattered photon and q_{10} and q_{20} are the energies of the incident and scattered photons in the laboratory system of coordinates (q_1 and q_2 are the photon four-momenta)

$$U = U^{(0)} + U^{(1)}(\varkappa, \tau) + U^{(1)}(\tau, \varkappa), \; U^{(0)} = (\varepsilon_1 \varepsilon_2)^2,$$

$$U^{(1)}(\varkappa, \tau) = -\frac{\alpha}{\pi}(\varepsilon_1\varepsilon_2)^2 \operatorname{Re}\left\{\frac{3}{4} + (1 - 2y\coth 2y)\ln\lambda + 2y\coth 2y\,(h(2y) - h(y)) - \frac{y^2}{\sigma} - \frac{\varkappa}{\varkappa - 1}\ln\varkappa \right.$$

$$\left. + \frac{\sigma - 2}{\sigma - \varkappa\tau}\left[2y\coth y\varkappa\,(h(y) - \ln\varkappa) + (\varkappa - 2)\left(y^2 + \frac{\varkappa}{2}C_0(\varkappa)\right)\right]\right\} - \frac{\alpha}{\pi}\frac{(\varepsilon_1\varepsilon_2)(\varepsilon_1 q_2)(\varepsilon_2 q_1)}{m^2}\operatorname{Re}\left\{\frac{1}{2\sigma} + \frac{2y^2}{\sigma^2} + \frac{2}{\varkappa - 1}\ln\varkappa \right. \tag{7}$$

$$\left. - \frac{2(\sigma - 2)}{(\sigma - \varkappa\tau)^2}\left[2y\coth y\varkappa\,(h(y) - \ln\varkappa) + (\varkappa - 2)\left(y^2 + \frac{\varkappa}{2}C_0(\varkappa)\right)\right] - \frac{2(\sigma - 2)}{\sigma - \varkappa\tau}\left[\frac{4\tau}{\sigma(\sigma - 4)}y\coth y\,(h(y) - \ln\varkappa) - \frac{\tau}{\sigma}\ln\varkappa\right]\right\},$$

$$m^2\varkappa = 2p_1q_1;\; m^2\tau = -2p_1q_2;\; \sigma = \varkappa + \tau,\; \sinh^2 y = -\sigma/4;\quad h(y) = y^{-1}\int_0^y \varphi\coth\varphi\, d\varphi;\; C_0(\varkappa) = -2\varkappa^{-1}\int_{1-\varkappa}^{1}\ln(1-u)\frac{du}{u}.$$

The photon polarizations are chosen to satisfy the relations $\epsilon_1\epsilon_1 = \epsilon_2\epsilon_2 = 1$, and $\epsilon_1 p_1 = \epsilon_2 p_1 = 0$.*

Formula (7) contains terms with $\ln\lambda$. These terms cancel each other out in the total cross section of the ordinary Compton effect and in the Compton effect accompanied by emission of one soft photon (the so-called double Compton effect).† Let us note that in the nonrelativistic approximation the formula for the total cross section of the above processes is the same as obtained in Ref. 5.

In conclusion, the authors are grateful to Professor A. I. Akhiezer for supervising the work and to P. I. Fomin for help in evaluating the integrals.

*The properties of the functions $h(y)$ and $C_0(\kappa)$ are treated in detail by Brown and Feynman.[4]

† The connection between the cross sections of the double and ordinary Compton effects is given by Eq. (7).

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MAGNETIC RESONANCE OF NUCLEI OF PARAMAGNETIC ATOMS

K. A. VALIEV

Kazan' State University

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WE have carried out a detailed theoretical investigation of the possibility of observing magnetic resonance in ionic crystals of nuclei of paramagnetic atoms of the iron group and of the rare earth group. The effect under consideration is intermediate between the well known phenomena of paramagnetic electron resonance and the usual nuclear resonance of nuclei of diamagnetic atoms with respect to the inten-

sity of the absorption lines and to the frequency of the rf field at which the resonance should be observed. As is well known, the intensity of the resonance line is proportional to the square of the frequency of the rf field and to the probability of magnetic dipole transitions between the energy levels. At the ordinarily employed values of the intensity of the external magnetic field ($\sim 10^3$ oersted) the nuclear resonance frequency of paramagnetic atoms ($10^8 - 10^9$ cps) will be 10 to 100 times greater than the resonance frequency in the case of diamagnetic atoms ($10^6 - 10^7$ cps). Further, owing to the influence of the magnetic moment of the electron shell of the ion, the probabilities of nuclear magnetic dipole transitions are also considerably increased — approximately by a factor 10^4 for ions with an odd number of electrons and by a factor of several fold for ions with an even number of electrons. From this it may be seen that nuclear resonance absorption in the case of paramagnetic atoms will be so strong that it should be easily observable with already existing apparatus. But the possibility of observing resonance absorption depends also on the line width: the line must be sufficiently narrow. The line width of magnetic resonance is determined by dipole-dipole and spin-lattice interactions of the magnetic particles. Magnetic interactions between particles can make a large contribution to the line width in undiluted crystals, but their role rapidly becomes less important as the crystal is magnetically diluted. In dilute crystals the line width will be determined only by the spin-lattice interactions of the nuclei.

We have calculated the probabilities of nuclear relaxation transitions for the Cr^{3+} ion (odd number of electrons) and for the V^{3+} ion (even number of electrons). From the calculations it follows that in chrome alums at liquid air temperatures the line width of nuclear resonance does not exceed a few oersted and the absorption may be observed by the usual method of modulating a constant magnetic field. In salts of the vanadium ion the spin-lattice nuclear relaxation time will be short and it will be possible to observe nuclear resonance only at low temperatures. A situation favorable for experiments on nuclear resonance exists in crystals of Tutton salts of the ion Ni^{2+} enriched in the isotope of mass 61 (nuclear spin $5/2$).

We have also calculated the probabilities of nuclear relaxation transitions in crystals of ethyl sulphates of rare earth elements. We have investigated electron levels two-fold degenerate in the crystalline field and possessing electron magnetism. The probabilities calculated for the nuclear spins were compared with the probabilities of relaxation transitions between electron levels. For most ions with an odd number of electrons the probabilities of nuclear transitions are approximately 100 times smaller than the probabilities of electronic transitions, while for ions with an even number of electrons they are smaller by a factor 10^5. One may expect that it should be possible to observe nuclear resonance in the case of ions with an even number of electrons at liquid air temperatures, and for ions with an odd number of electrons at liquid hydrogen temperatures.

By using the method of magnetic resonance of nuclei of paramagnetic ions (by means of measuring the resonance transition frequencies) one can determine quite accurately the magnetic and quadrupole coupling constants of the nuclei of paramagnetic atoms. But the most interesting application of nuclear resonance in the case of paramagnetic atoms will evidently be investigations of spin-lattice interaction in crystals, particularly in those cases when direct measurements of the electronic relaxation time by the method of parallel fields or by the method of saturation are impossible due to very short relaxation times (in salts of the rare earth elements and in some salts of ions of the iron group). It should be possible to determine the nuclear spin relaxation time from the line width of nuclear resonance. On the other hand, the ratio of the probabilities of electronic and nuclear relaxation transitions may be calculated with suffient accuracy. Thus, for the Cr^{3+} and the V^{3+} ions, it has the form:

$$A(m,\ m-1)/A(M-1,\ M) = {}^1/_4 (S+M)(S-M+1)(I+m)(I-m+1)(A/D)^2,$$

where S and I are the electron and the nuclear spin of the ion, A is the coupling constant between the nuclear and the electron spin of the ion, and D is the energy interval between neighboring energy levels of the electron spin. We have also calculated similar ratios for the neodymium and praseodymium ions.

The results of these calculations will be published in greater detail in the journal "The Physics of Metals and Metal Research" and in the Scientific Notes of the Kazan' University.

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