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RADIATION CORRECTIONS TO PARTICLE SCATTERING IN EXTERNAL FIELD AND TO COMPTON EFFECT IN SCALAR QUANTUM ELECTRODYNAMICS

V. F. ALEKSIN and D. V. VOLKOV

Physico-Technical Institute, Academy of Sciences, Ukrainian S.S.R.

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THE expression for the single-photon mass operator of a scalar particle, obtained by one of the authors,[1] was used to calculate the radiation corrections to the scattering of a scalar particle in an external electromagnetic field and to the Compton effect. The following expressions were obtained thereby for the differential cross sections.

1. The differential scattering cross section (in the first Born approximation) has the following form

$$d\sigma/d\Omega = (d\sigma/d\Omega)_0 + (d\sigma/d\Omega)_{\Delta M} + (d\sigma/d\Omega)_{A'}, \tag{1}$$

where $(d\sigma/d\Omega)_0$ is the differential scattering cross section without allowance for radiation corrections, and the indices ΔM and A' distinguish the radiation corrections from the mass operator and from the polarization of vacuum, respectively.[2,3] For the first correction in (1) we have the following formula

$$(d\sigma/d\Omega)_{\Delta M} = -(2\alpha/\pi)(d\sigma/d\Omega)_0\,[2y\coth 2y\,(h(2y)-h(y)) - y\tanh y + \ln\lambda\,(1-2y\coth 2y)],$$

$$h(y) = y^{-1}\int_0^y \varphi\coth\varphi\, d\varphi;\quad \sinh^2 y = (p_1-p_2)^2/4m^2, \tag{2}$$

where λ is the photon mass in units of m; p_1 and p_2 are four-dimensional particle momenta before and after scattering, and $\alpha = e^2/4$. (We use a system of units in which $\hbar = c = 1$.)

For the second correction we have

$$(d\sigma/d\Omega)_{A'_0} = \frac{\alpha}{\pi}\left(\frac{d\sigma}{d\Omega}\right)_0\left[\frac{4m^2+(p_1-p_2)^2}{3(p_1-p_2)^2}(1-y\coth y)+\frac{1}{9}\right] \tag{3}$$

if the vacuum polarization is due to particles with zero spin, and

$$(d\sigma/d\Omega)_{A'_{1/2}} = -\frac{2\alpha}{\pi}\left(\frac{d\sigma}{d\Omega}\right)_0\left[\frac{4m_{1/2}^2-2(p_1-p_2)^2}{3(p_1-p_2)^2}(1-y_{1/2}\coth y_{1/2})+\frac{1}{9}\right],\quad \sinh^2 y_{1/2} = (p_1-p_2)^2/4m_{1/2}^2, \tag{4}$$

if the vacuum polarization is due to particles with spin $1/2$.

When λ approaches zero, formula (2) diverges logarithmically. This divergence is offset by an analogous divergence in the inelastic-scattering differential cross section of the particle with emission of a single soft photon.

$$(d\sigma/d\Omega)_{\text{inel}} = -\frac{2\alpha}{\pi}(d\sigma/d\Omega)_0\left\{(1-2y\coth 2y)\left[\ln\left(\frac{K_{\max}}{\lambda}\right)-\frac{1}{2}\right]+4y\coth 2y\,(h(2y)-1)\right\}, \tag{5}$$

where K_{max} is the maximum momentum of the emitted photon in units of m ($K_{max} \ll 1$).

2. The differential cross section of the Compton effect on a scalar particle has the following form

$$d\sigma / d\Omega = (\alpha / m)^2 (q_{20} / q_{10})^2 U, \tag{6}$$

where $d\Omega$ is the element of solid angle in the direction of the scattered photon and q_{10} and q_{20} are the energies of the incident and scattered photons in the laboratory system of coordinates (q_1 and q_2 are the photon four-momenta)

$$U = U^{(0)} + U^{(1)}(\varkappa, \tau) + U^{(1)}(\tau, \varkappa), \quad U^{(0)} = (\varepsilon_1 \varepsilon_2)^2,$$

$$\begin{aligned} U^{(1)}(\varkappa, \tau) = & -\frac{\alpha}{\pi}(\varepsilon_1\varepsilon_2)^2 \operatorname{Re}\Big\{\frac{3}{4} + (1 - 2y\coth 2y)\ln\lambda + 2y\coth 2y\,(h(2y) - h(y)) - \frac{y^2}{\sigma} - \frac{\varkappa}{\varkappa - 1}\ln\varkappa \\ & + \frac{\sigma - 2}{\sigma - \varkappa\tau}\Big[2y\coth y\varkappa\,(h(y) - \ln\varkappa) + (\varkappa - 2)\Big(y^2 + \frac{\varkappa}{2}C_0(\varkappa)\Big)\Big]\Big\} - \frac{\alpha}{\pi}\frac{(\varepsilon_1\varepsilon_2)(\varepsilon_1 q_2)(\varepsilon_2 q_1)}{m^2}\operatorname{Re}\Big\{\frac{1}{2\sigma} + \frac{2y^2}{\sigma^2} + \frac{2}{\varkappa - 1}\ln\varkappa \\ & - \frac{2(\sigma - 2)}{(\sigma - \varkappa\tau)^2}\Big[2y\coth y\varkappa\,(h(y) - \ln\varkappa) + (\varkappa - 2)\Big(y^2 + \frac{\varkappa}{2}C_0(\varkappa)\Big)\Big] - \frac{2(\sigma - 2)}{\sigma - \varkappa\tau}\Big[\frac{4\tau}{\sigma(\sigma - 4)}y\coth y\,(h(y) - \ln\varkappa) - \frac{\tau}{\sigma}\ln\varkappa\Big]\Big\}, \end{aligned} \tag{7}$$

$$m^2\varkappa = 2p_1q_1; \quad m^2\tau = -2p_1q_2; \quad \sigma = \varkappa + \tau, \quad \sinh^2 y = -\sigma/4; \quad h(y) = y^{-1}\int_0^y \varphi\coth\varphi\, d\varphi; \quad C_0(\varkappa) = -2\varkappa^{-1}\int_{1-\varkappa}^{1}\ln(1 - u)\frac{du}{u}.$$

The photon polarizations are chosen to satisfy the relations $\epsilon_1\epsilon_1 = \epsilon_2\epsilon_2 = 1$, and $\epsilon_1 p_1 = \epsilon_2 p_1 = 0$.*

Formula (7) contains terms with $\ln\lambda$. These terms cancel each other out in the total cross section of the ordinary Compton effect and in the Compton effect accompanied by emission of one soft photon (the so-called double Compton effect).† Let us note that in the nonrelativistic approximation the formula for the total cross section of the above processes is the same as obtained in Ref. 5.

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*The properties of the functions $h(y)$ and $C_0(\kappa)$ are treated in detail by Brown and Feynman.[4]

† The connection between the cross sections of the double and ordinary Compton effects is given by Eq. (7).

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MAGNETIC RESONANCE OF NUCLEI OF PARAMAGNETIC ATOMS

K. A. VALIEV

Kazan' State University

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WE have carried out a detailed theoretical investigation of the possibility of observing magnetic resonance in ionic crystals of nuclei of paramagnetic atoms of the iron group and of the rare earth group. The effect under consideration is intermediate between the well known phenomena of paramagnetic electron resonance and the usual nuclear resonance of nuclei of diamagnetic atoms with respect to the inten-