An approximate solution of the first two equations of this set in the case of a short range force $[\Phi(|\mathbf{q}-\mathbf{q}'|) = \nu(0)\,\delta(\mathbf{q}-\mathbf{q}')]$ enables us to obtain the transport equation used by Landau in his theory of Fermi-liquids.[17]

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NUCLEAR REACTIONS IN SUPER-DENSE COLD HYDROGEN

IA. B. ZEL'DOVICH

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It is shown that nuclear reactions occurring in cold hydrogen at densities of $10^4 - 10^6$ g/cc via barrier penetration proceed with a probability which is quite noticeable on an astrophysical scale. This fact puts a limit on the possible compression of cold hydrogen, since a celestial body cannot last more than 10^8 years at a density of 0.7 g/cc. Such a density is reached in cold hydrogen under the action of gravitation for a mass close to that of the sun.

It is known[1-3] that the thermonuclear reactions $p + p = D + e^+ + \tilde{\nu}$, $p + D = He^3 + \gamma$ occur in stars; in addition, at high temperature we have the reaction $He^3 + He^3 = He^4 + p + p$, and a high density the reactions $He^3 + e^- = T + \nu$, $T + p = He^4 + \gamma$.

Schatzman[4] first noted that at high densities and low temperatures deviations from the usual expressions for the rate of thermonuclear reactions should begin to appear, and gave general expressions for the case of a degenerate nuclear gas.

Let us consider reactions occurring in hydrogen compressed to a density of $10^4 - 10^6$ g/cc, at a temperature below 10^6 degrees. Under these conditions hydrogen is a solid, so that the protons are located at the lattice points of a crystal; thermal motion and thermonuclear reactions can be neglected.

It is well known that the Coulomb repulsion of the nuclei, which hinders nuclear reaction, can be overcome by virtue of the quantum-mechanical phenomenon of barrier penetration (tunnel effect). The probability of tunneling is exponentially small, and the exponent depends more strongly on the width than on the height of the barrier.

The separation at which nuclear reaction begins is of order 3×10^{-13}; at this distance the Coulomb energy of two protons is 0.5 Mev. At a hydrogen density of $10^4 - 10^6$ g/cc, the distance between protons is $6.5 \times 10^{-10} - 1.35 \times 10^{-10}$ cm (100 times as great as nuclear distances and only 10 — 50 times smaller than the distance between the nuclei in a hydrogen molecule). At this distance, the Coulomb energy of two protons is 400 — 2000 ev.

Thus a compression to such densities hardly lowers the height of the barrier; however, the wide low energy region, which is most difficult to overcome, is gotten rid of through the internal pressure, and the rate of the nuclear reaction $p + p = D + e^+ + \tilde{\nu}$ turns out to be entirely perceptible on an astronomical scale. To calculate the reaction rate at zero temperature and a given density, we find the equilibrium distance r_0 between nuclei under conditions of close packing. We approximate the dependence of the energy on distance between nuclei by the expression $e^2/r + e^2/(2r_0 - r)$. The initial energy is $2e^2/r_0 + E_0$, where $E_0 = \hbar\omega/2 = e\hbar/\sqrt{Mr_0^3}$ is the zero point vibration energy, and M is the proton mass.

The exponential factor for barrier penetration is

$$B = \exp\{-\varphi(4e/\hbar)\sqrt{2M_r r_0}\},$$

where M_r is the reduced mass, equal to $M/2$, and φ is determined by numerical integration. For small ϵ,

$$\varphi = 0.70 - 0.17\,\varepsilon\ln(1/\varepsilon), \quad \varepsilon = E_0 r_0/e^2.$$

We determine the probability $\psi^2(0)$, for finding two protons at the same point in the absence of nuclear interaction, approximately, by assuming that we have a diatomic molecule[5] with equilibrium separation r_0 and zero point energy E_0

$$\psi^2(0) = BE_0 M_r^2 e^2/\hbar^4.$$

Finally we determine the reaction rate by using Salpeter's[2] calculations: the reaction probability is $p(\mathrm{sec}^{-1}) = w\psi^2(0)$. The constant w is related to the reaction cross section σ (for the case where the reaction occurs in the laboratory, using a beam of particles), by the formula (for singly-charged ions),

$$\sigma = \frac{w}{v}\frac{2\pi e^2}{\hbar v}\exp\left(-\frac{2\pi e^2}{\hbar v}\right) = w\frac{\pi M e^2}{\hbar}\frac{B}{E} = \frac{S}{E}B,$$

where v is the velocity, E the energy, and S is given in Ref. 2, formula (7). Comparing formulas (9) and (39) of Ref. 2, we find $S = 4 \times 10^{-19}$ barn-ev, so that $w = 5 \times 10^{-40}$ cc/sec. After the reaction of formation of the deuteron, the formation of He^3 should follow extremely rapidly. At the densities we are considering, the maximum energy of electrons E_e (in the degenerate Fermi gas) is greater than the tritium decay energy; therefore the reaction $He^3 + e^- = T + \nu$ also goes rapidly,[2] and after it the reaction $p + T = He^4 + \gamma$. Thus each initial reaction of two protons soon leads to the liberation of ~ 25 Mev of energy. A summary of the results for two densities is given in the table:

ρ (g/cc)	r_0 (cm)	E_0, (ev)	$-\log_{10} B$	p (sec^{-1})	H (erg/g-sec)	E (ev)	P (dyne/cm^2)	T (ev)
$5\cdot10^5$	$1.65\cdot10^{-10}$	125	8.5	$4\cdot10^{-17}$	1000	$2.6\cdot10^5$	$5\cdot10^{22}$	400
$0.7\cdot10^5$	$3.1\cdot10^{-10}$	45	11.6	10^{-20}	0.25	$0.7\cdot10^5$	$0.2\cdot10^{22}$	150

Here H is the rate of energy liberation, P is the pressure of the degenerate electron gas, T is that temperature at which the same reaction rate would be reached through the thermonuclear mechanism.[2]

The considerations presented

above impose a limit on the possible compression of cold hydrogen. In fact, even at a density of 0.7×10^5 (the lower line in the table), with zero initial temperature we will reach a temperature of 150 v after 6×10^7 years, and the speed of the reaction will roughly double, so that a celestial body consisting of cold hydrogen at a density of 0.7×10^5 cannot last more than 10^8 years.

From the well known formula of Emden (cf. Ref. 6), the density of cold hydrogen reaches 0.7×10^5 g/cc at a pressure of 2×10^{21} dyne/cm^2 under the action of gravitation, at the center of a body whose mass is 1.5×10^{33} g, i.e., 0.75 times the mass of the sun.

In 1938, Landau[7] pointed out that at high density a "neutron condensation" occurs. For hydrogen, the limiting energy of the reaction $e^- + p = n + \tilde{\nu}$ is 0.75 Mev. The corresponding density is 10^7 g/cc.

The tunneling reaction sets in much earlier. Consequently, the neutron condensation begins only after the conversion of the hydrogen to helium (or to heavier nuclei), and correspondingly at an even greater density than that considered in Ref. 6.

The reactions $p + D$, $p + T$, $D + D$, $D + T$ can also occur via the cold mechanism and require lower pressures. However, this process will never be of practical importance: the required pressures are so high that, under terrestrial conditions, they can be realized only in a non-stationary state, in an extremely small volume and for extremely short times. For equal expenditure of energy or equal pressure, the tunnel reaction is far inferior to the thermonuclear reaction in heated matter.

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