A NEW POSSIBILITY IN NEUTRINO THEORY

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In addition to the usual description of electrons and other fermions by means of spinors of the second kind, it is suggested that for the neutrino, use can be made of Cartan spinors of the first kind with other transformation laws under reflection. Despite the conservation of parity, a number of conclusions of such a theory are analogous to the Yang-Lee theory.

In connection with the recent discovery of a number of phenomena[1] which are interpreted most naturally as a violation of the law of conservation of parity (decay of τ and θ mesons, $\pi-\mu$ mesons and the β-decay of oriented nuclei), it is necessary to investigate one more possible description of the neutrino, which may possibly permit the explanation of part of the abovementioned phenomena, and at the same time preserve all the conservation laws.

Let us assume that the transformation laws of the neutrino wave functions under reflection differ in a definite way from those of the usual spinors, such as electrons, for example. We will call spinors with special transformation properties pseudospinors. Cartan[2] pointed out the possibility of the existence in a world with an even number of dimensions, of pseudospinors, forming a special representation of the complete Lorentz group ("spinors of the first kind"*).

We will use the following notation:

$$ab = a_\mu b_\mu = a_i b_i - a_0 b_0;\ \gamma_\alpha\gamma_\beta + \gamma_\beta\gamma_\alpha = -2\delta_{\alpha\beta}(\delta_{00} = -1,\ \delta_{11} = \delta_{22} = \delta_{33} = 1).$$
$$\gamma_0 = \tilde{\gamma}_0 = \overset{*}{\gamma}_0 = \overset{+}{\gamma}_0$$

(the symbols next to the matrices represent successively the transpose, the complex conjugate and the Hermite conjugate);

$$\gamma_i^+ = -\gamma_i;\quad \gamma_2^* = -\gamma_2,$$

all the remaining matrices are real;

$$\gamma_5 = -\gamma_5^+ = \gamma_0\gamma_1\gamma_2\gamma_3;\ \gamma_5^2 = -1;$$

the charge conjugated spinor is $\psi^c = \gamma_2\psi^*$. Let the usual spinors corresponding to the electron-positron field (Cartan's spinors of the second kind) transform under reflection with respect to a hyperplane perpendicular to the unit vector $a_\mu\,(a_\mu^2 = \pm 1)$, as

$$\psi \to \psi' = \pm L(a_\mu)\psi,\quad L(a_\mu) = a_\mu\gamma_5\gamma_\mu;\quad L^2 = -a_\mu^2.$$

This choice is unique to within i with the requirement that the relation connecting ψ and ψ^c be invariant.[3] Here, pseudospinors will be transformed by the matrix

$$L'(a_\mu) = a_\mu\gamma_\mu.$$

The properties of the usual spinors and the pseudospinors are identical under rotations. If a vector and pseudovector are constructed of pseudospinors (with respect to spatial reflections† the structure

*The properties of pseudospinors in connection with β-decay were investigated in detail by Brodskii at one of the lectures at the Moscow State University (1951).

†We recall that with regard to reflections with a change in the sign of time, the known invariant combinations behave as reciprocal pseudoquantities, for example $\psi^+\gamma_0\psi$ as a pseudoscalar and $\psi^+\gamma_0\gamma_5\psi$ as a scalar, because of the condition $-1 = a_\mu^2$ for a timelike a_μ.

will be the same as in the usual case), it is easy to verify that the matrix γ_5 is added to the scalar, pseudoscalar and tensor constructed of pseudospinors. In particular, the Dirac equation for a free pseudospinor field is replaced by the modified equation

$$(-i\gamma_\mu \partial / \partial x^\mu + i\gamma_5 (im_0))\psi = 0, \tag{1}$$

where the factor i is placed in front of the real m to conserve the correct relation between the energy-momentum and the mass (when the Hermitian nature of the Lagrangian is violated). For $m = 0$, Eq. (1) is outwardly indistinguishable from the corresponding Dirac equation. Now let the neutrino field ψ_ν be a pseudospinor. The β-decay interaction is described, as is well known, in the form of products of a scalar, vector, etc., constructed from nucleon functions multiplying respectively a scalar, vector, etc., constructed from the functions ψ_e and ψ_ν.

Let us define the latter quantities in our case, limiting ourselves for the time being to invariance under spatial reflections ($a_\mu^2 = 1$). As follows from elementary calculations, the mixed scalar will be $\psi_e^+\gamma_0(1 + \gamma_5)\psi_\nu$, the pseudoscalar $\psi_e^+\gamma_0(1 - \gamma_5)\psi_\nu$, the vector $\psi_e^+\gamma_0\gamma_\mu(1 + \gamma_5)\psi_\nu$, the pseudovector $\psi_e^+\gamma_0\gamma_\mu(1 - \gamma_5)\psi_\nu$, and the tensor $\psi_e^+\gamma_0\gamma_\mu\gamma_\nu(1 + \gamma_5)\psi_\nu$. Indeed, for example,

$$(L\psi_e)^+\gamma_0(1 + \gamma_5)(L'\psi_\nu) = a_\mu a_\nu \psi_e^+\gamma_\nu^+\gamma_5^+\gamma_0(1 + \gamma_5)\gamma_\mu\psi_\nu = -a_\mu a_\nu \psi_e^+\gamma_0(1 + \gamma_5)\gamma_\mu\gamma_\nu\psi_\nu = a_\mu^2\psi_e^+\gamma_0(1 + \gamma_5)\psi_\nu = \psi_e^+\gamma_0(1 + \gamma_5)\psi_\nu.$$

In all of these expressions, ψ_e and ψ_ν may be replaced simultaneously by anti-particle (charge conjugated) quantities.*

Let us now investigate the invariance with respect to time inversion and anti-particle (charge) conjugation. Just as in the case of the usual invariants, invariants composed of pseudospinors, or spinors and pseudospinors, will behave as reciprocal pseudoquantities under time inversion (i.e., a scalar will become a pseudoscalar, etc.). In distinction from other types of interaction, this circumstance does not impose any kind of compulsory supplementary conditions on the coefficients of the various terms in the β-decay interaction Lagrangian L_{int}, because all the quantities in L_{int} composed of ψ_ν and ψ_e enter in the form of a product multiplying the corresponding nucleon combination, which also change their character in the transition from space to time inversions. As regards symmetry with respect to anti-particle conjugation, in all the usual interactions, with the exception of β-decay, it is a necessary consequence of the invariance with respect to time and space reflections and therefore need not necessarily be considered a compulsory condition (see the Luders-Pauli theorem). If, nevertheless, conservation of symmetry with respect to anti-particle (charge) conjugation is demanded, then in the scalar case, for example, we must take the half-sum†

$$\tfrac{1}{2}(\psi_N^+\gamma_0\psi_N \cdot \psi_e^+\gamma_0(1 + \gamma_5)\psi_\nu + \psi_N^{c+}\gamma_0\psi_N^c \cdot \psi_{\bar{e}}^+\gamma_0(1 + \gamma_5)\psi_{\bar{\nu}}). \tag{2}$$

The investigation may be carried further as in the work of Lee and Yang.[1] However, since in the present approach we assume conservation of parity, the coefficient of asymmetry in β-decay will be determined only by the Coulomb correction, proportional to Ze^2. This also results in the present case from the relations between the coefficients $C_S' = iC_S$, $C_P' = -iC_P$, etc. If experiment confirms the presence in the β-decay of oriented nuclei of an asymmetry due to nonconservation of parity, i.e., to a non-Coulomb term, the possibility suggested herein of using pseudospinors must be rejected, but it will be desirable to base this circumstance on general arguments.

On the other hand, the asymmetry observed by Lederman and co-workers in the successive decay of pions and muons with the emission of a neutrino, is also preserved with the use of pseudospinors.

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*A special situation arises if we choose $L^2 = +1$ instead of $L^2 = -1$ as is done above. In this case, as is well known, the antineutrino may be identified with the neutrino $\psi_\nu = \psi_\nu^c = \gamma_2\gamma_\nu^*$ (Majorana variant) with corresponding consequences for the invariants.

†In the case in which L_{int} is symmetrized only with respect to electron-positrons and nucleon-antinucleons, instead of Eq. (2) we obtain

$$\tfrac{1}{2}(\psi_N^+\gamma_0\psi_N \cdot \psi_e^+\gamma_0(1 + \gamma_5)\psi_\nu + \psi_N^{c+}\gamma_0\psi_N^c\psi_{\bar{e}}^+\gamma_0(1 - \gamma_5)\psi_\nu),$$

which is outwardly equivalent to invariance with respect to the simultaneous transformations of space inversion and charge conjugation, with violation of each of them separately, if the special character of the transformations of the pseudospinors ψ_ν is not taken into account.

We also note that the pseudospinor treatment of the neutrino leads to the Salam condition[5] of invariance with respect to the transformation $\psi'_\nu \rightarrow \gamma_5\psi_\nu$, introduced by him as a postulate. Also, the possibility of applying pseudospinors to other fermions is not excluded.

Note added in proof (Sept. 18, 1957). We should mention the interesting possibility of mixed spinors of the first kind with respect to space (time) reflections and of the second kind with respect to time (space) reflections. We also note that in the case of nonconservation of parity with invariance with respect to the Salam transformation,[5] a new law of conservation of "neutrino charge" holds, with a current density defined by a pseudovector.

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[2] E. Cartan, Leçons sur la theorie des spineurs, Paris, 1938.

[3] I. S. Shapiro, Usp. Fiz. Nauk **53**, 14 (1957).

[4] L. Lederman et al., Phys. Rev. **105**, 1415 (1957). Castagnoli, Frazinetti, and Manfredini, Proc. Avogadro Congress, Turin, 1956.

[5] A. Salam, Nuovo cimento **4**, 1 (1957).

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NONLINEAR THEORY OF PHASE OSCILLATIONS INDUCED IN ELECTRON SYNCHROTRONS BY QUANTUM RADIATION FLUCTUATIONS

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A nonlinear theory of phase oscillations induced by radiation fluctuations in electron synchrotrons is considered. It is shown that the nonlinear theory gives essential correction to results of the linear theory. The nonlinear theory predicts greater electron losses and imposes upon synchrotron parameters stronger restrictions than does a linear theory.

It is well known[1] that the quantum nature of radiation causes phase oscillations of electrons in synchrotrons. The mechanism of exciting these oscillations is similar to that described by Sokolov and Ternov[2,3] for exciting betatron oscillations.

Sands[1] calculated the mean square deviation of the electron phase oscillations induced by radiation for a weak-focusing synchrotron. The calculation was performed in the linear approximation with a neglect of the existence of limits to the phase oscillations. Such a calculation is valid only for small oscillations. These results have been generalized in the same approximation to the case of strong-focusing synchrotrons.[4-6]

The linear theory of phase vibrations treated by Sands,[1] Kolomenskii,[4] and the present author[5,6] is valid only for small deviations of the electron phase from its equilibrium value. In general, however, this assumption does not resolve the fundamental problem, that of determining the loss of electrons caused by these oscillations. In order that an electron be removed from further acceleration, it is necessary that its phase of oscillation be outside the allowable limits. As is well known, the left limit is φ_S, the negative of the equilibrium phase. The boundary φ_1 on the right is given by

$$\sin\varphi_1 + \sin\varphi_s - (\varphi_1 + \varphi_s)\cos\varphi_s = 0. \tag{1}$$