$$[p_\lambda, h]_- = 0, \quad [\Gamma_\sigma^2, h]_- = 0, \quad [\Gamma_3, h]_- = 0. \tag{52}$$

Then the metric matrix h, which commutes with all the operators of the complete set, must also be diagonal.

It may appear strange that, for example, the operators p_λ and h commute in one but not in another of two equivalent representations. This occurs because the transformation for h is different from that for all the other operators:

$$\Omega = V\Omega', \ \Omega^* = \Omega'^* V^*, \qquad \langle \Omega^* h p_\lambda \Omega \rangle = \langle \Omega'^* V^* h V V^{-1} p_\lambda V \Omega' \rangle = \langle \Omega'^* h' p'_\lambda \Omega \rangle; \qquad h' = V^* h V; \ p'_\lambda = V^{-1} p_\lambda V. \tag{53}$$

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ELECTRICAL CONDUCTIVITY OF FERROMAGNETIC SEMICONDUCTORS (FERRITES)

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A ferrite is considered as a lattice of classical magnetic dipoles submerged in a dielectric continuum. An analysis of the electron conductivity of such a model shows that the line ln $\lambda \sim T^{-1}$ (λ is the electrical conductivity) must have a break at the Curie point, in agreement with experiment, as the activation energy in the ferromagnetic region decreases.

KOMAR and Kliushin[1] detected a break in the line $\ln \lambda = f(T^{-1})$ (λ is electrical conductivity) for ferrites in the transition through the Curie point, where the activation energy in the ferromagnetic region is less than in the paramagnetic region. The fact that the break is observed at precisely the Curie point indicates a connection between this phenomenon and the presence of spontaneous magnetization. We show that the existence of this break finds a simple explanation on the basis of a theory that takes into account the interaction of the conduction electrons and the electrons of the unoccupied shells of the magnetic ions.[2]

Ferrites have an electron conductivity due to the stoichiometric excess of metal[3] or to the presence of impurities.[4] The problem of electron motion in a lattice of nonmagnetic ionic crystals has already been solved under the assumption that the ion lattice can be replaced by a dielectric continuum (polaron theory, Refs. 5 and 6). It is natural to use this method for ferrites, which essentially are also ionic crystals.

In order to take their magnetic properties into account, let us assume that spins, located in a certain spatial lattice so that they can be separated into two magnetic sublattices, are "impregnated" in the continuum. The sublattice magnetizations are anti-parallel and cancel each other incompletely. As in the Néel[7] theory, let us assume that there is only one kind of magnetic ion, i.e., one kind of spin.

The Hamiltonian of a ferrite containing one "excess" conduction electron is

$$H = p^2/2m + H_l + H_i, \tag{1}$$

where H_ℓ is the lattice Hamiltonian and H_i the Hamiltonian of electron interaction with the lattice.

To the accuracy of terms of the same order as the anharmonic terms, H_ℓ is composed additively of the Hamiltonian of the optical vibrations H_K and of the mutual spin energy of the lattice H_C. Acoustic vibrations cannot be taken into account since it is known that the electrons in ionic crystals are scattered principally by optical vibrations. In nonmagnetic ionic crystals, H_i will be composed of the periodic potential $V_0(\mathbf{r})$ and of the interaction with the inertial polarization H_{iK} (Refs. 5 and 6) ($\mathbf{r}$ is the electron coordinate). Interaction of the electron with the spin of the system is added thereto in ferrites.

Choosing the separated magnetic axis as the z axis, let us assume as the periodic part of this interaction

$$U(\mathbf{r}) = 2s_z V_1, \quad V_1 = \eta_1 W(\mathbf{r}) - \eta_2 W(\mathbf{r} - \mathbf{h}), \tag{2}$$

where s_z is the z component of the electron spin operator, η_1 and η_2 are the absolute values of the relative magnetizations of the first and second sublattices; $W(\mathbf{r})$ is a periodic function.

The Hamiltonian (1) becomes

$$H = p^2/2m + V_0(\mathbf{r}) + 2s_z V_1(\mathbf{r}) + H_\varkappa + H_{i\varkappa}. \tag{3}$$

H_C can be discarded as a constant. The explicit forms of H_K and H_{iK} are not required. The expression (3) differs from the Hamiltonian of the polaron problem only in the addition of $2s_z V_1(\mathbf{r})$ to the electron periodic potential.

Let us be limited to the case of weak electron coupling to the polarization vibrations and, consequently, let us consider H_{iK} as a perturbation. Separating the optical variables, we arrive at the problem of an electron in a periodic field with the Hamiltonian

$$H_e = p^2/2m + V_0(\mathbf{r}) + 2\sigma V_1. \tag{4}$$

The spin variable is evidently separated out, which would lead to the replacement of s_z by the electron spin quantum number σ ($\sigma = \pm 1/2$). Let us consider the electron energy spectrum. In order for the crystal to have ferrite properties, i.e., for it to have non-equivalent magnetic sublattices, it must be assumed that the sites $\mathbf{R}$ and $\mathbf{R} + \mathbf{h}$ of the first and second sublattices are crystallographically non-equivalent. (For example, this is attained in the spinel lattice because of the dissimilar coordinations of these sites by the oxygen ions). The non-equivalence is retained even if the added term $2\sigma V_1$ is neglected. Consequently, V_0 and V_1 have the same periodicity. Therefore, the energy spectrum will always have a band structure. The presence of the added term only introduces a small quantitative increment.* Actually, the estimate assumed for V_1 in Ref. 2 for metals can be retained: $V_1 \sim 0.2 - 0.5$ ev. The width of the permitted band ΔE is of the order of several ev. Consequently, the increments will be of the order of $V_1/\Delta E$. Let us expand the electron energy in this quantity:

$$E_\sigma(\mathbf{K}) = E^{(0)}(\mathbf{K}) + 2\sigma E^{(1)}(\mathbf{K}) + \ldots \tag{5}$$

($\mathbf{K}$ is the quasi-wave vector). Using also the effective-mass method, we finally obtain

$$E_\sigma(\mathbf{K}) = 2\sigma\varepsilon + \hbar^2 K^2/2\mu, \quad \mu = \hbar^2/(d^2E^{(0)}/dK^2)_0, \tag{6}$$

*The sublattice sites in antiferromagnetics must be crystallographically equivalent so that their magnetizations cancel. The magnetic ions located therein will create identical force fields for an electron if H_{ic} is not taken into account. The force fields of the magnetic ions of different sublattices are different when H_{ic} is taken into account. Consequently, $V_0 + 2\sigma V_1$ will have "twice" the period of V_0. This leads to the splitting of each of the permitted bands into two with a transition from the paramagnetic region into a region below the Curie point, as Slater[8] already has remarked.

where μ is the effective mass; $\epsilon = E^{(1)}(0)$. For simplicity, it is assumed that $K = 0$ corresponds to the minimum energy in the conduction band and that the effective mass is isotropic; the energy is measured from the position of the bottom of the conduction band in the paramagnetic region ($V_1 = 0$), in this connection, the constant $E^{(0)}(0)$ is discarded; the term proportional to $d^2E^{(1)}/dK^2$ will be of higher order since $h^2K^2/2\mu\Delta E$ is also small. Without loss of generality, it can be considered that $\epsilon > 0$. In the opposite case, the parts of the "right" and "left" conduction electrons must be interchanged.

As is seen from (6), the bottom of the "right" electron band is raised by $\epsilon \sim 0.2$ ev and the "left" is lowered by the same quantity when the transition is made from the paramagnetic into the ferromagnetic region.

It should be noted that we at once obtain the energy spectrum (6) in the Sommerfeld approximation, where

$$V_0 \to \Omega_0^{-1}\int V_0 d\mathbf{r}, \quad V_1 \to \Omega_0^{-1}\int V_1 d\mathbf{r}, \quad \varepsilon = (\eta_1 - \eta_2)\,\Omega_0^{-1}\int W d\mathbf{r},$$

and μ is replaced by the actual mass m (the integrals are taken over the volume Ω_0 of the elementary cell).

In order to determine the chemical potential of the electron gas ζ, let us assume that the conductivity is caused by a univalent metal impurity. We will have the donor level at a distance $\Delta E'$ below the bottom of the conduction band. The intrinsic conductivity can be considered insignificant. The potential ζ is determined according to Ref. 9 from the equation (n_0 is donor concentration)

$$n_0 = n_0\left[\exp\frac{-\Delta E' - \zeta}{kT} + 1\right]^{-1} + \frac{1}{2\pi^2}\int \exp\left(-\frac{\hbar^2K^2}{2\mu kT}\right)K^2 dK \sum_\sigma \exp\left(\frac{\zeta - 2\sigma\varepsilon}{kT}\right),$$

from which we obtain

$$\exp\frac{\zeta}{kT} = n_0^{1/2}(2\hbar)^{3/2}\left(\frac{\pi}{2\mu kT}\right)^{3/4}\left(\sum_\sigma \exp\frac{2\sigma\varepsilon}{kT}\right)^{-1/2}\exp\left(-\frac{\Delta E'}{2kT}\right).$$

For the ferromagnetic region $\epsilon \gg kT$ and, consequently, $\exp(-\epsilon/kT)$ can be neglected in comparison with $\exp(\epsilon/kT)$. We then obtain for the electron concentration $n = n_r + n_l$ (the subscripts r and l refer to "right" and "left" electrons, respectively):

$$n = n_r + n_l = \sqrt{2}n_0^{1/2}(2\hbar)^{-3/2}(2\mu kT/\pi)^{3/4}\exp(-\Delta E'/2kT) \tag{7}$$

in the paramagnetic region ($\epsilon = 0$),

$$n = n_l = n_0^{1/2}(2\hbar)^{-3/2}\left(\frac{2\mu kT}{\pi}\right)^{3/4}\exp\left(-\frac{\Delta E' - \varepsilon}{2kT}\right), \quad n_r \approx 0 \tag{8}$$

below the Curie point. The electron mobilities above and below the Curie point will be approximately identical since we do not take the scattering caused by the term ($H_{ic} - U$) into account.

We see from (7) and (8) that the transition from the paramagnetic into the ferromagnetic region will be accompanied by a break* in the line $\ln\lambda \sim T^{-1}$, where the slope in the second case will be smaller (independently of the sign of ϵ). Hence, the reason for the break is the lowering of the bottom of the band for conduction electrons of one of the two spin orientations. Experimental values of the activation energy usually do not exceed 0.5 ev. The slope is halved, roughly, for $\epsilon \sim 0.2$ ev, in full agreement with experiment.

The theory leads to a curious consequence. We obtain for the ratio of the concentrations n_r and n_l

$$(n_r/n_l) = \exp(-2\varepsilon/kT),$$

i.e., electrons of just one spin orientation will exist almost exclusively in the conduction band below the Curie point.

For antiferromagnetics ($\eta_1 = \eta_2$) $\epsilon = 0$, i.e., if the reason cited for the break is unique, the break should be absent in this case, at least, for the model chosen.

In conclusion, I take this opportunity to express my thanks to Prof. E. I. Kondorskii for reviewing the work and for discussions.

*The mobilities in ionic crystals depend exponentially on $\hbar\omega_0/kT$, where ω_0 is the limiting frequency. This factor can be considered included in $\exp(-\Delta E'/2kT)$.

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PROPAGATION OF CASCADES IN A MULTI-LAYER MEDIUM

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The propagation of a cascade consisting of several types of particles in a medium composed of homogeneous layers R^λ is considered. The layer boundaries may be mobile. The cascade particles collide with the particles of the medium and in the process are absorbed, scattered, and produce new particles. The functions W^λ describing the distribution of particles of each type according to position and velocity are found under the assumption that the particle distribution functions V^λ in the layer R^λ when the layer occupies the whole space are known. The functions W^λ are represented as infinite series of integrals which, in general, show a good convergence. The integrands are certain products of the functions V^λ.

IN the present article we shall make use of the notation and results of Ref. 1, in which a method of calculating the functions $V_{ij}(s, \mathbf{q}, \mathbf{u}, t, \mathbf{r}, \mathbf{v})$ was given. The functions V_{ij} determine the probabilities $V_{ij}\,d\mathbf{r}d\mathbf{v}$ that a particle of a given type A_j will be found at the time t to possess radius vector between $\mathbf{r}$ and $\mathbf{r} + d\mathbf{r}$ and velocity between $\mathbf{v}$ and $\mathbf{v} + d\mathbf{v}$ if the cascade is initiated by a single particle appearing with the velocity $\mathbf{u}$ at the time s and the point $\mathbf{q}$. It has been assumed that the medium is filling the whole space and that its properties are independent of time and place. In Ref. 1 it has been assumed that all new particles of the cascade are produced at the moment of collision. In Ref. 2, an analogous function V describing a neutron cascade — a cascade consisting of particles of a single type which may be produced with a delay — was found by means of the same method. It is easy to combine the two cases and to consider a cascade consisting of n types of particles which may be produced with a delay. In that case, too, we shall assume the functions V_{ij} to be known for the case of an infinite homogeneous medium. In the present, a method of solution of this problem will be given for the case of a multi-layer medium, i.e., a medium consisting of different homogeneous layers occupying adjoining regions $R^\lambda (\lambda = 1, 2 \ldots)$. Let E be the space of variables t, $\mathbf{r}$, $\mathbf{v}$, j. It consists of n 7-dimensional spaces $E_j (j = 1, 2 \ldots n)$. We shall assume that the boundaries are varying with time t. Furthermore, for the sake of symmetry, we shall assume that they may depend also on j and $\mathbf{v}$, i.e, that the R^λ are arbitrary regions in E.

The solution of this problem is important, for example, for the study of transient cosmic ray effects,[3,4] in the theory of nuclear reactors,[5] in calculations of radiation shielding,[6] etc. The problem amounts to solving the Boltzmann linear integral equation of the type (1.15) or (1.16)* in a multi-layer medium and represents a generalization of an analogous problem in the theory of parabolic differential equations.[7,8]

*Here and in the following (1.15) denotes the formula (15) of Ref. 1, etc.