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OPTICAL DETERMINATION OF THE DENSITY OF He^3

V. P. PESHKOV

Institute of Physical Problems, Academy of Sciences, U.S.S.R.

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An interference method is described for determining the density of gaseous and liquid He^3. It is found that for He^3 the molar polarization $A = 0.123$ cm^3/mole. It is shown that at temperatures and pressures below the critical values the equation of state for gaseous He^3 is described by the formula $p/\rho = 27.35\,T - 2.3 \times 10^3\rho + 1.8 \times 10^4\rho^2$, where p/ρ is expressed in atm cm^3/g. The compressibility curves for liquid He^3 are determined. For the critical point the values $T_c = 3.38 \pm 0.03$°K, $p_c = 930 \pm 20$ mm Hg, and $\rho_c = 0.041 \pm 0.001$ g/cm^3 are obtained.

FOR the determination of the density of He^3, in both the liquid and the gaseous phase, one can employ the formula for the molar polarization

$$\frac{3}{4\pi}\,\frac{n^2-1}{n^2+2}\,\frac{M}{\rho} = A = \text{const.}$$

In accordance with the measurements of the Cuthbertsons,[1] the value of the molar polarization for gaseous He^4 under normal conditions is 0.124 cm^3/mole. From the measurements of Johns and Wilhelm[2] this constant is $A = 0.117$ cm^3/mole for liquid He^4 in the vicinity of 2°K at $\lambda = 5461$ A. It could be assumed that for He^3 as well the molar polarization would to within a few percent be constant and equal to the molar polarization of He^4. Since for helium n differs only very slightly from 1, we obtain, by setting $n = 1 + \Delta$ and expanding A in a series in Δ, the expression

$$A = \frac{M}{2\pi\rho}\,\Delta\left(1 - \frac{\Delta}{6} + \cdots\right).$$

For liquid He^3, Δ is less than 0.02; therefore to within 0.4% the density $\rho = M\Delta/2\pi A$. Using this relation it is possible to determine the density of He^3 by measuring Δ from the number of interference fringes passing as an optical wedge is filled with He^3. Fig. 1 represents an apparatus constructed for this purpose.

A Dewar vessel 1 filled with liquid He^4 served as a cryostat, within which was placed a cylindrical glass tube 2. At its top the tube terminated in a flange, and was sealed off, both from the Dewar vessel and the atmosphere, with the aid of a rubber ring and a plane glass window. At its lower end the tube had an internal flange ground accurately perpendicular to the axis of the tube. The cell 3, a cross-section of which is illustrated on a larger scale at the bottom of Fig. 1, was attached to the ground portion with BF-4 cement. The cell was assembled, also with BF-4 cement, from a plane-parallel glass spacer 4, two plane-parallel glass flats 5, and a platinum foil 6. The spacer was 2.9 mm thick, with an aperture 4 mm wide and 17 mm long. The glass flats, 4 mm thick and 17 mm wide, were cemented to the top and bottom

of the spacer. The upper flat was made 5 mm longer than the spacer and lower flat, in order that the interference at the very edge of the cell could be observed after it was cemented to the tube. The bottom face of the cell was frosted and blackened, so that reflection from the lower face would not affect the interference within. The opening at the end of the cell was closed with a bent piece of platinum foil 0.2 mm thick. This platinum foil insured thermal contact between the interior and the surrounding helium, at least in the immediate vicinity of the foil. Into the rear portion of the spacer was sealed a platinum tube 7, through which He^3 was admitted to the cell. A German silver tube 8, 1.4 × 2 mm in diameter, was soldered to the platinum tube. The pressure of the He^3 within the cell was measured with a mercury manometer 9. With the aid of a telescope 10 and a low-pressure mercury lamp 11, fringes of equal optical thickness were observed between the plane-parallel plates 5. Thirteen fringes were included within the length of the cell. The lamp produced its most intense illumination at a wave length $\lambda = 5461$ A. The clearest interference pattern was obtained for a light-source distance of 4 — 5 m, and remained adequately clear for a lens diaphragm opening of 1 cm.

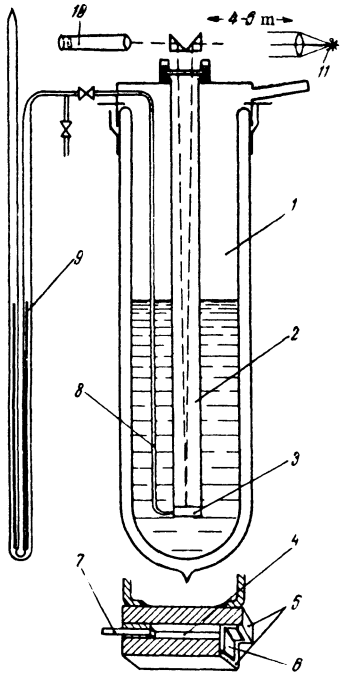

FIG. 1. Apparatus for determining the index of refraction of He^3.

A displacement of k interference fringes within the cell corresponded to a change in the index of refraction $\Delta = k\lambda/2d$, where d is the distance between the plane-parallel plates at the point at which the fringes are observed; the density of the helium can consequently be determined from the formula $\rho = (M\lambda/4\pi dA)k$. The distance between the flats near the foil was found after assembly to be d = 2.93 mm; the density of liquid He^3 in equilibrium with its vapor at T = 2°K, from the measurements of Kerr,[3] is $\rho = 0.078$ g/cm^3, which corresponded to the passage of a number of fringes k = 216. From this we obtain A = 0.123 cm^3/mole, and $\rho = 3.62 \times 10^{-4}$k g/cm^3. Using this formula, determinations of the density were carried out for both the liquid and the gaseous phase of He^3.

The experiments for the determination of the density of the gaseous phase were conducted at constant temperature. The pressure of the He^3 within the cell was progressively raised or lowered, and the reading of the manometer 9 was recorded each time a full fringe passed across a mark immediately adjacent to the platinum foil. As the condensation pressure was reached the motion of the fringes stopped, and liquid appeared within the cell. If observations on the compressibility were made in the vicinity of the critical point, then in addition to an increase in the rapidity of motion of the fringes marked fluctuations in the density were noted, the fringes behaving in an animated fashion.

The accuracy of the counting was limited primarily by the error in determining the moment at which the fringes passed, and corresponded to 0.2 — 0.3 of a fringe, or to $\Delta\rho \approx 10^{-4}$ g/cm^3. Since for He^3 at extremely low densities deviations from the ideal gas law are hardly to be expected, the procession of the fringes at the foil was observed in the most convenient position for counting. p/ρ was then plotted graphically as a function of ρ, a fraction of the value of k being chosen such that extrapolation of the curve to zero density gave a value for p/ρ corresponding to that for an ideal gas.

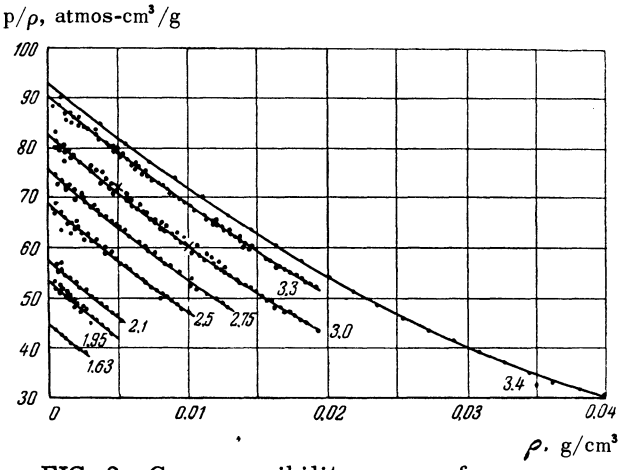

FIG. 2. Compressibility curves for gaseous He^3. The temperature is indicated on the curves in °K.

The compressibility curves for gaseous He^3 are presented in Fig. 2. The points indicate the experimental data, while the solid curves correspond to the equation

$$p/\rho = 27.35T - 2.3 \cdot 10^3\rho + 1.8 \cdot 10^4\rho^2,$$

in which p/ρ is expressed in atm-cm^3/g. For comparison, the crosses in the graph represent some of the data of Keller,[4] who determined the density of He^3 by the customary method of measuring volumes, pres-

sures and temperatures. Other data by Keller are not included, since his measurements terminated at densities below 0.01 g/cm^3 and were carried out at temperatures differing from those on the curves of Fig. 2, and when recomputed agree with the latter to within the accuracy of the measurements. As is evident from Fig. 2, the state curve for gaseous He^3 at temperatures and pressures below the critical values may to a sufficient degree of accuracy be described by an equation having three constant coefficients, the first of which corresponds to the ideal gas coefficient.

The density curve for gaseous helium in equilibrium with the liquid phase was determined both by fixing the moment at which the procession of fringes ceased during compression at various given temperatures and by observation of the variation of the equilibrium density with temperature. In the latter case, the displacement of the fringes was observed in the immediate vicinity of the foil as the temperature was lowered, under the condition that the liquid phase be visible within the cell at some distance from the foil. For this purpose, the apparatus was inclined laterally through 9°. The results of the measurements are presented in Fig. 3 (lower curve). The points correspond to the measurements described above, the crosses are taken from the data of Kerr.[3]

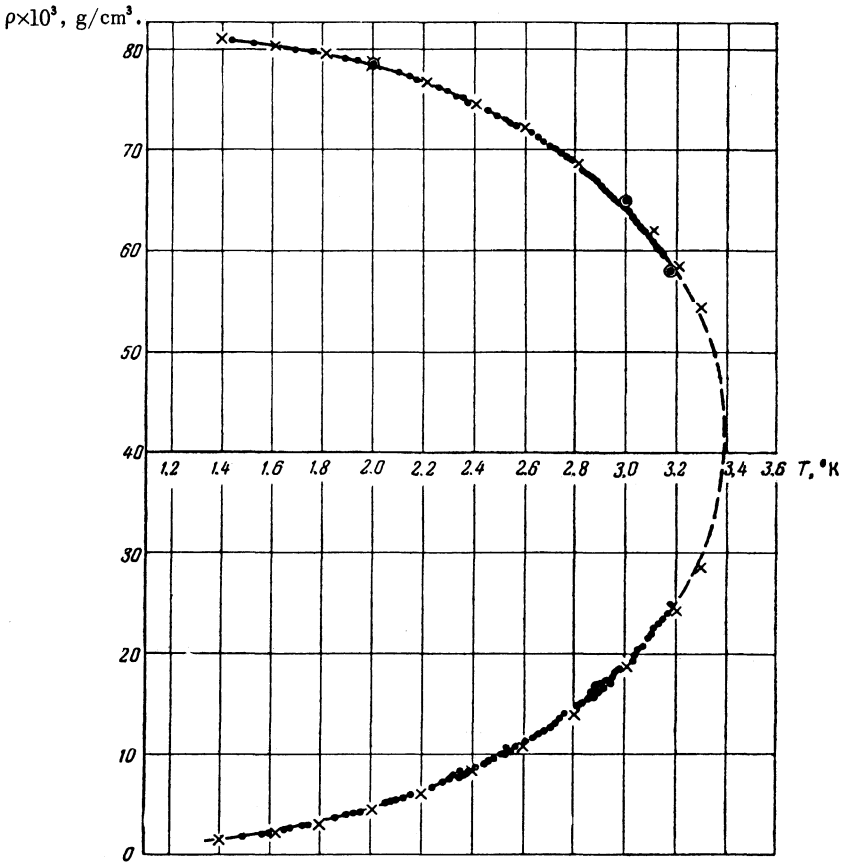

FIG. 3. Dependence of the equilibrium densities of liquid and gaseous He^3 upon temperature.

In order to determine the density of liquid He^3 in equilibrium with the gaseous phase, an uninterrupted count of the fringes was maintained during the following operations. First, the cell was filled at a temperature of 3.37°K to a pressure above the critical value (996 mm Hg); then the temperature was lowered to 2.00°K and at this temperature the pressure within the cell was reduced to the equilibrium value for the vapor, which was observed by the appearance of bubbles. In this case the apparatus was tilted in such a way that at the foil the cell was completely filled with liquid, while further away the boundary between the liquid and gaseous phases was visible. During the process described above the equilibrium density of liquid He^3 at 2.00°K, which, as indicated above, was taken to be 0.078 g/cm^3, corresponded to the passage of 216 fringes. All of the remaining liquid He^3 density measurements were referred to this point. A curve showing the dependence of the density of liquid He^3 in equilibrium with its vapor was obtained by slowly reducing the temperature (about 2 hr), under the condition that the boundary between the liquid and gaseous phases should always be visible within the cell. The results of these measurements are shown in Fig. 3 (points along the upper curve); the crosses are from the data of Kerr.[3] The large circles on the curve represent points measured during the course of the above-described continuous transition from the gaseous phase to the liquid through the critical point.

The most careful measurements correspond to $k = 216$, $\rho = 0.078$ g/cm^3, and $T = 2.00$°K; the remaining two points to $k = 180$, $\rho = 0.065$ g/cm^3, and $T = 3.01$°K, and to $k = 161$, $\rho = 0.058$ g/cm^3, and $T = 3.16$°K.

For the critical point the values $T_c = 3.38 \pm 0.03$°K, $p_c = 930$ mm Hg, and $\rho_c = 0.041 \pm 0.001$ g/cm^3 are obtained.

Using this same apparatus, measurements of the compressibility of liquid He^3 were carried out at various temperatures. In these experiments the temperature of the bath was held constant and the dis-

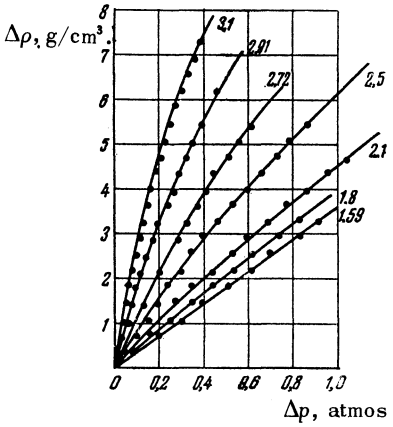

FIG. 4. Compressibility curves for liquid He^3. Numbers on the curves give T in °K.

placement of the fringes in the immediate vicinity of the foil was observed as the pressure within the cell was increased. The results of these measurements are presented in Fig. 4. From these data on the compressibility the velocity of sound in He^3 should vary from 170 m/sec at 1.6°K to 80 m/sec at 3°K.

The state diagram for He^3 in the coordinates ρ and p is given in Fig. 5. The points represent data from individual measurements. The solid lines in the diagram show the course of the variations in the density of liquid He^3 as the temperature was varied from 2.1° and from 3.22°K at a rate of approximately 1° per hour. During this time there was no pumping on the helium vapor, the tube extending upwards was not cooled by evaporating helium, the heat input increased, and a non-equilibrium situation was observed. With pumping and the establishment of a constant temperature, the excess heat was gradually removed. It should be noted that with the cell half filled with liquid the He^3 vapor pressure corresponded to a temperature 0.02° higher than the bath temperature; it is possible, therefore, that there existed during the experiments a systematic error lowering the temperature by approximately 0.01°.

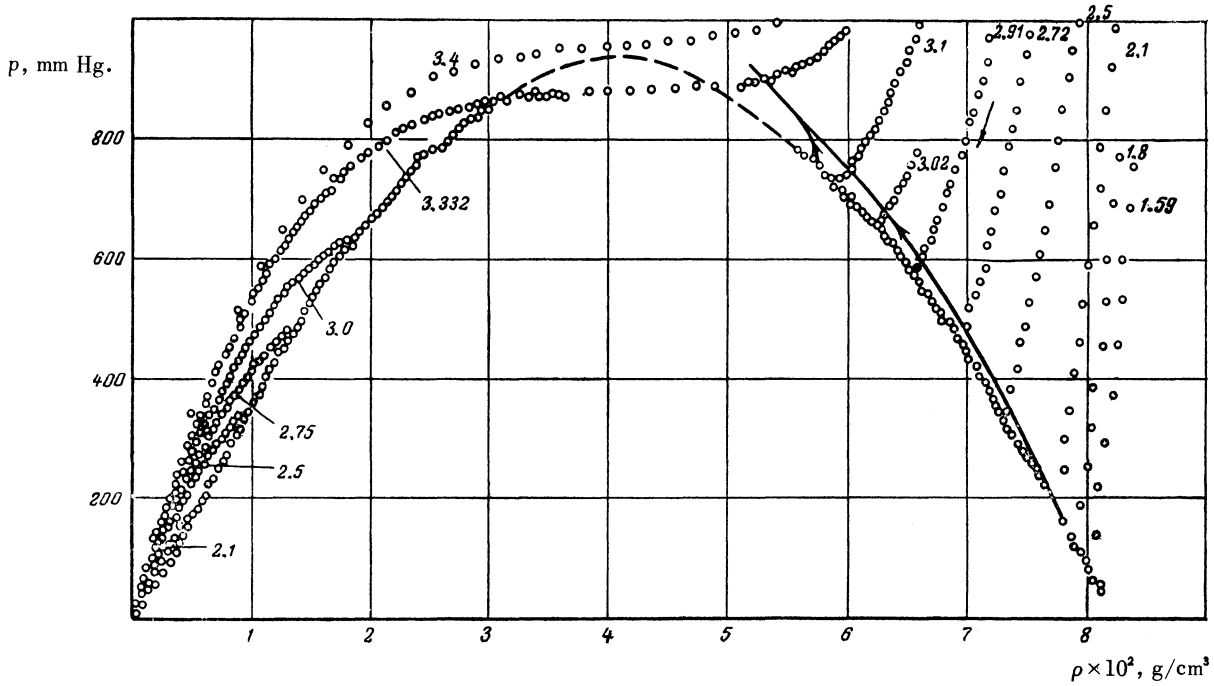

FIG. 5. State diagram for He^3 in coordinates ρ and p.

The optical method thus described may, after adaptation for measurements at higher pressures, be used successfully for measurements of the densities of other gases and liquids.

In conclusion, I take this opportunity to express my gratitude to N. I. Kondrat'ev, A. I. Filimonov, and I. A. Uryutov, who assisted me in the performance of these experiments.

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