EFFECT OF PROTON STRUCTURE OF THE LAMB SHIFT

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AFTER taking into account mass corrections and electromagnetic processes of the fourth and in part of the fifth order, a discrepancy of 0.6 Mc/sec remains between the experimental and theoretical values for the Lamb shift in hydrogen; it can be partially attributed to the structure of the proton (smearing of the proton charge). This has already been pointed out by several authors.[1-3] The estimates of Salpeter yield a total correction $\Delta E = 0.025$ Mc/sec; a preliminary estimate by Ivanenko and Kolesnikov[3] leads to $\Delta E = 0.11$ Mc/sec.

For computing the displacement of the levels in hydrogen, we use the method of the more accurate wave functions,[4] known from the theory of the isotope shift, in which the level shift is determined by the effective change of the principal quantum number

$$\Delta E = \alpha^2 mc^2 n^{-3} \Delta n, \quad \alpha = e^2/\hbar c, \tag{1}$$

and Δn is determined from the condition for joining the solutions for the inner and outer regions. In particular, for $S_{1/2}$ levels,

$$\Delta n = 2\alpha^4 (R/r_0)^2 (1 + 2f/\alpha g), \quad r_0 = e^2/mc^2, \tag{2}$$

where R is the radius of joining, and f and g are the components of the Dirac wave function at the point of joining. The shift $P_{1/2}$ is of order α^2 relative to the shift of the $S_{1/2}$ level and does not have to be considered now.

For determining f and g one may confine oneself to the approximation

$$g = a, \; f = \frac{a}{\hbar c} r^2 \int_0^r V(r) r^2 dr \tag{3}$$

which is correct to terms of second order in α in the region of order r_0. In the inner region, the potential of the electric field of the proton has the form

$$\varphi(r) = e\left[\int_r^R \frac{dy}{y^2} \int_0^y \rho(x) x^2 dx\right]\left[\int_0^R \rho(x) x^2 dx\right]^{-1} + \frac{e}{R} = e\left[\frac{1}{r}\int_0^r \rho(x) x^2 dx + \int_r^R \rho(x)\, x dx\right]\left[\int_0^R \rho(x) x^2 dx\right]^{-1} \tag{4}$$

Substituting in (3) and then in (2), we obtain

$$\Delta n = \frac{2}{3}\frac{\alpha^4}{r_0^2}\left[\int_0^R \rho(x) x^4 dx\right]\left(\int_0^R \rho(x) x^2 dx\right)^{-1}. \tag{5}$$

Having taken the radius of joining sufficiently large in comparison with the dimensions of the region of the charge distribution, we obtain a general formula for Δn which depends on the charge distribution only through the rms radius

$$\Delta n = {}^2/_3 \alpha^4 \langle r^2 \rangle / r_0^2, \tag{6}$$

$$\Delta E \text{ (Mcs)} = (\alpha^6 c/3\pi\lambda n^3) \langle r^2 \rangle / r_0^2 (\lambda = \hbar/mc). \tag{7}$$

An analogous formula for the deuteron is obtained by Salpeter[1] with the aid of perturbation theory and was used recently by Kolesnikov to calculate the effect of proton structure on the energy levels of deuterium.

Using the experimental values for the rms radius of the proton $(7.7 \pm 1.0) \times 10^{-14}$ cm, obtained from scattering electrons on protons by Hofstadter's group,[5] we arrive at $\Delta E = (0.117 \pm 0.030)$ Mc/sec, which exceeds the result of Salpeter,[1] but coincides by a curious manner with a previous estimate.[3] Perturbation theory, however, does not afford a possibility of determining the correction to the energy of interaction with the vac-

uum due to the distortion of the wave function. This correction is determined by means of the formula

$$\Delta E_L = -\frac{E_{0L}\alpha^2}{3}\left[4\frac{\langle r^2\rangle}{r_0^2} + \frac{(4\pi)^2}{2}\int_0^\infty \rho(y)\,dy\int_0^y \rho(x)\,x^4 dx - \frac{(4\pi)^3}{12}\int_0^\infty \rho(y)\frac{dy}{y^2}\left(\int_0^y \rho(x)\,x^4\,dx\right)^2\right], \quad (8)$$

where E_{0L} is the electromagnetic part of the Lamb shift.

In formula (8) the first term takes into account the change in the normalization $\Delta N/N = -\Delta n/\alpha^2$ and yields -0.006 Mc/sec; the second and third terms take into account the distortion of the wave function. The last part depends essentially on the form of the charge distribution inside the proton; its magnitude, however, does not exceed $\Delta E = -0.012$ Mc/sec (for the charged sphere) and, consequently, lies beyond the range of the present experiment.[6]

Thus taking into account the volume of the proton reduces the Lamb-shift discrepany between theory and experiment from 0.6 to 0.5 Mc/sec. It will be possible to draw further conclusions about the effect of the structure of the elementary particles on the Lamb shift after calculating electromagnetic processes of the fifth order and improving the accuracy of experiment. We are going to apply an analogous method to the calculation of the correction to the hyperfine structure due to the volume of the proton.

In conclusion I consider it my duty to express gratitude to Professor D. D. Ivanenko for constant attention to the work and for a discussion of the results.

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PARTICLE ANGULAR DISTRIBUTION FUNCTION AT THE CASCADE SHOWER MAXIMUM

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BELEN'KII[1] calculated the distribution function of particles with respect to energy and angle at cascade shower maximum without assuming small deflection angles. The scattering, however, was considered to be mutliple. In the following we shall drop this limitation. We can then write the equation for $P(E, \vartheta)$, integrated over the depth of shower development with boundary conditions corresponding to a single primary electron of energy E_0 incident vertically upon the boundary of the material layer at $t = 0$, in the following form:

$$-\frac{1}{2\pi}\delta(E_0 - E)\,\delta(1-\cos\vartheta)\cos\vartheta = q\frac{N(E_0, E, \vartheta)}{E} + q\frac{\partial N(E_0, E, \vartheta)}{\partial E}$$

$$+ n\int_0^{\chi_{\max}}\int_0^{2\pi} P(E_0, E, \vartheta') - P(E_0, E, \vartheta)]\,f\left(2\sin\frac{\chi}{2}\right)\sin\chi\,d\chi\,d\varphi, \qquad N(E_0, E, \vartheta) = \int_E^\infty P(E_0, E, \vartheta)\,dE. \quad (1)$$

Here $q = 2.29$ and $P(E, \vartheta)$ is the required distribution function of electrons with respect to energy E and angle ϑ. The angle ϑ' is determined by the equation