RESONANCE SCATTERING OF γ-RAYS

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Submitted to JETP editor February 15, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 655-659 (September, 1957)

Some experiments are suggested which may be used to determine the width of excited levels as well as to determine the correlation constant λ between the electron and the neutrino (if the experiments are of sufficent duration). The experiments consequently can be used to ascertain the choice of type of forces in β-decay.

1. The observation of nuclear resonance scattering is made difficult by the Doppler shift which the γ-rays experience due to nuclear recoil both in emission and absorption, while at the same time the nuclear levels are very narrow. However, in radioactive substances the emission of a γ-ray in a ground state transition is usually preceded by a β-decay and, sometimes, by another γ-quantum. The nuclear recoil due to this proceding radiation can be utilized to compensate for the Doppler shift. In Refs. 1 and 2 the conditions necessary for such a compensation have been treated if the preceding radiation is a γ-ray. The present paper is devoted to the case when the preceding radiation is a β-decay, or a β-decay with a γ-ray cascade.

2. We analyze the following experimental setup (Fig. 1). The source S emits β-particles and γ-quanta. The decay of the source material takes place according to Fig. 2. We assume

$$E_{k\beta\ \max} \geqslant \sqrt{E_\gamma^2 + m_0^2c^4} - m_0c^2. \tag{1}$$

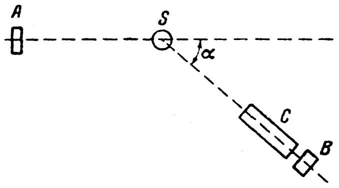

FIG. 1

Here $E_{k\beta\,\max}$ is the maximum kinetic energy of the electron in the β-decay, E_γ — the energy of the photon and m_0c^2 — the rest energy of the electron. In Fig. 1, A detects β-particles of a given energy and B is a γ-ray counter. The scatterer C is placed in the path of the γ-rays and comprises the nuclei obtained after the β-decay. The transparency of the specimen C to gamma rays from the source is determined, also as a function of the angle α.

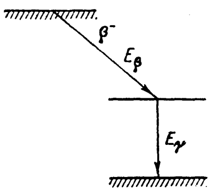

FIG. 2

Resonance scattering will lead to increased γ-ray absorption in the scatterer C. We shall now determine the cross section of resonance scattering in terms of the above experiment. We shall assume that the radioactive source material is given in gaseous form so that one can assume the recoil nucleus to be free.

3. We start from Eq. (10) of Ref. 2. The cross section for resonance scattering of γ-rays is

$$\sigma = \sigma_0 (\sqrt{\pi}\,\Gamma / 2\Delta) \exp\{-(E_{\gamma 1} - E_r)^2 / \Delta^2\}. \tag{2}$$

with the following abbreviations:

$$\sigma_0 = 4\pi\lambda\!\!\!^{-}{}^2 (2j+1)/(2i+1) \tag{3}$$

i.e., the peak resonance scattering cross section (see, for example, Ref. 3); j is the angular momentum of the excited state of the daughter nucleus, i is angular momentum of the ground state of the daughter nucleus $\lambda\!\!\!^{-} = c\hbar/E_{\gamma_1}$; Γ — intrinsic width of the excited level Δ — Doppler width, due to the thermal motion of the nuclei both in the source and in the scatterer, given by (see, for example, Ref. 4)

$$\Delta = E_\gamma \sqrt{2k(T_1 + T_2)/m_1c^2}\,, \tag{4}$$

where T_1 is the absolute temperature of the source, T_2 the absolute temperature of the scatterer, k is Boltzmann's constant, m_1 the nuclear mass, and E_r the energy required to excite the nucleus:

$$E_r = E_{\gamma 1}(1 + E_{\gamma 1}/m_1c^2); \tag{5}$$

E_{γ_1} is energy of the γ-quantum emitted in the direction of the γ-counter after emission of the β-particle. The energy E_{γ_1} depends on the angle α between the directions of the electron and the γ-quantum, and also on the direction of emission of the neutrino if the momentum of the electron is not close to the maximum. Since one cannot observe the neutrino, one has to average (2) over all directions of the outgoing neutrinos.

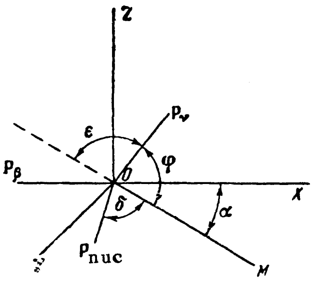

FIG. 3

4. We introduce the coordinate system shown in Fig. 3. The source is placed at the origin of the coordinate system. The electron is emitted in the direction of the negative X axis, and OM is the direction of the γ-ray counter. We denote the angles between OM and the momentum $\mathbf{p}_{nuc}$ of the nucleus by δ, between the momenta of the β-particle $\mathbf{p}_\beta$ and of the neutrino $\mathbf{p}_\nu$ by β, and between OM and $\mathbf{p}_\nu$ by φ; $\epsilon = \pi - \varphi$. Then

$$E_{\gamma 1} = E_\gamma (1 - E_\gamma / 2m_1c^2 + p_{nuc} \cos\delta / m_1 c). \tag{6}$$

The mean value of the cross section is

$$\bar{\sigma} = \frac{1}{4\pi} \int \sigma \left(1 + \lambda \frac{v_\beta}{c} \cos\beta\right) d\Omega. \tag{7}$$

Here σ is given by (2), $v\beta$ is the velocity of the electron λ the electron-neutrino correlation constant for β-decay (see, for example, Ref. 5). Utilizing the conservation laws

$$E_\beta + E_\nu = E_{\beta\,max}, \quad \mathbf{p}_\beta + \mathbf{p}_\nu + \mathbf{p}_{nuc} = 0, \tag{8}$$

where E_β is the energy of the detected electron, E_ν the energy of the neutrino, $E_{\beta max}$ the energy of the β-decay, and taking into account

$$p_{nuc} \cos\delta = p_\beta \cos\alpha + p_\nu \cos\varepsilon, \quad \cos\beta = \cos\varepsilon \cos\alpha + \sin\varepsilon \sin\alpha \cos\psi, \tag{9}$$

and that $d\Omega = \sin\epsilon\, d\epsilon\, d\psi$ (ψ — azimuthal angle in a plane perpendicular to OM), one can evaluate the integral (7).

The mean cross section σ depends not only on Γ and Δ but also on the quantity λ which characterizes the interaction of the β-decay. Defining

$$\rho = \Delta m_1 c / p_\nu E_\gamma; \quad \tau = \Delta m_1 c / E_\gamma p_\beta; \quad \omega = \Delta m_1 c^2 / E_\gamma^2, \tag{10}$$

we have

$$\bar{\sigma}(\alpha, p_\beta, \lambda) = \sigma_0 \frac{\sqrt{\pi}}{2} \frac{\Gamma}{\Delta} \frac{1}{2} \left\{ \left(1 + \lambda \frac{cp_\beta}{E_\beta} \frac{\rho}{\omega} \cos\alpha - \lambda \frac{cp_\beta}{E_\beta} \frac{\rho}{\tau} \cos^2\alpha \right) \frac{\sqrt{\pi}}{2} \rho \left[\operatorname{erf}\left(\frac{1}{\rho} + \frac{\cos\alpha}{\tau} - \frac{1}{\omega}\right) + \operatorname{erf}\left(\frac{1}{\rho} - \frac{\cos\alpha}{\tau} + \frac{1}{\omega}\right)\right] + \lambda \frac{cp_\beta}{E_\beta} \frac{\rho^2}{2} \cos\alpha \left[\exp\left\{-\left(\frac{\cos\alpha}{\tau} - \frac{1}{\omega} - \frac{1}{\rho}\right)^2\right\} - \exp\left\{-\left(\frac{\cos\alpha}{\tau} - \frac{1}{\omega} + \frac{1}{\rho}\right)^2\right\}\right]\right\}. \tag{11}$$

The cross sections for the β^+ decay of As^{74} are plotted in Fig. 4. The following values have been used: $E_{k\beta\,max} = 0.92$ Mev, $E_\gamma = 0.596$ Mev, $E_\beta = 0.8\, E_{\beta\,max}$. The shapes of the curves depend markedly on λ. The position of the maximum for $\lambda = -1$ and $\lambda = 1/3$ differs by 28°. In order to calculate the absolute value of the maximum one needs to know the width of the excited state of Ge^{74}. The mean life of this state has been determined by Metzger[6] by the method of resonance scattering and was found to be $(1.9 \pm 0.3) \times 10^{-11}$ sec.

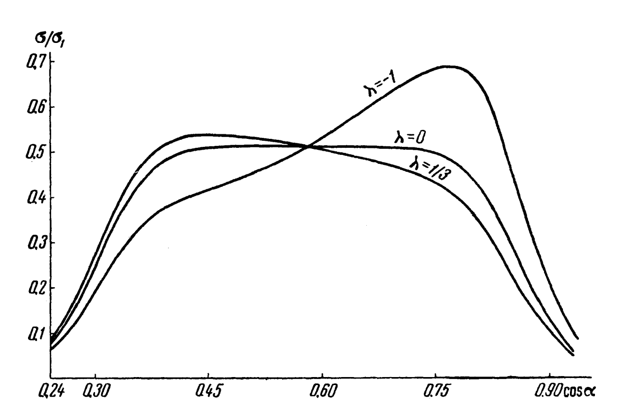

FIG. 4

The cross section at the maximum then turns out to be 6.9×10^{-24} cm^2. Such a cross section can be measured (it is ~ 80% of the cross section of the Compton effect and the photo effect).

5. We now turn to the case where the decay scheme is given by Fig. 5. In the experimental arrangement of Fig. 1 counter A now detects γ-rays of energy E_{γ_1} and counter B γ-rays of energy E_{γ_2}. C is a scatterer consisting of nuclei with atomic number Z + 1. In Ref. 2

this case was treated neglecting the influence of the preceding β-decay. We shall here again consider the recoiling nuclei to be free (gas source). We now shall determine the cross section for resonance scattering of γ-quanta with energy E_{γ_2} on nuclei of the scatterer C if γ-coincidences are counted.

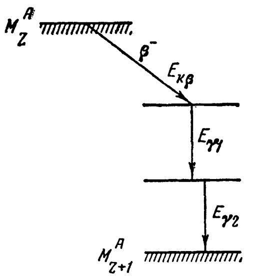

FIG. 5

6. Let the β-decay-induced momentum of the nucleus prior to the emission of the first γ-quantum be p_{nuc}, making an angle θ with the direction of the second γ-quantum. Then the energy E_{γ_2} of the second γ-quantum will be

$$E_{\gamma 2} = E^0_{\gamma 2}\left(1 - \frac{E^0_{\gamma 2}}{2m_1c^2}\right)\left(1 + \frac{p_{nuc}}{m_1c}\cos\theta + \frac{E_{\gamma 1}}{m_1c^2}\cos\alpha\right). \tag{12}$$

The energy needed to excite the scattering nucleus is, as before,

$$E_r = E^0_{\gamma 2}(1 + E^0_{\gamma 2}/2m_1c^2), \tag{13}$$

and the cross section will be given by (2) where one has to replace $E_{\gamma_1} - E_r$ by

$$E_{\gamma 2} - E_r = E^0_{\gamma 2}\left(\frac{E_{\gamma 1}}{m_1c^2}\cos\alpha - \frac{E_{\gamma 2}}{m_1c^2} + \frac{p_{nuc}}{m_1c}\cos\theta\right). \tag{14}$$

Since the β-decay is no longer recorded one has now to average over the angle θ and over all momenta of the recoil nuclei. We shall assume an allowed β-decay so that there are no $\beta - \gamma -$ correlations. Denoting by $f(p_{nuc})\,dp_{nuc}$ the distribution of recoil momenta normalized to unity we have

$$\bar{\sigma} = \sigma_0 \frac{\sqrt{\pi}}{2}\frac{\Gamma}{\Delta}\frac{1}{2}\int\limits_0^{p_{nuc\,lim}} f(p_{nuc})\,dp_{nuc}\int\limits_0^{\pi}\exp\left\{-\frac{E^2_{\gamma 2}}{\Delta^2}\left(\frac{E_{\gamma 1}}{m_1c^2}\cos\alpha - \frac{E_{\gamma 2}}{m_1c^2} + \frac{p_{nuc}}{m_1c}\cos\theta\right)^2\right\}\sin\theta\,d\theta.$$

Since $E_{\gamma_2}p_{nuc}/\Delta m_1c \gg 1$ one can evaluate the integral over θ by approximating its argument by a δ-function. Thus we obtain

$$\bar{\sigma} = \sigma_0 \frac{\pi}{4}\frac{\Gamma}{E_{\gamma 2}}\int\limits_{|E_{\gamma 1}\cos\alpha - E_{\gamma 2}c|-1}^{p_{nuc\,lim}} \frac{m_1c}{p_{nuc}} f(p_{nuc})\,dp_{nuc}. \tag{15}$$

7. The recoil-nuclei distribution function can be obtained numerically for each concrete case. From (8) follows

$$p^2_{nuc} = p^2_\beta + p^2_\nu + 2p_\beta p_\nu\cos\beta, \tag{16}$$

where β is as above the angle between the electron and neutrino directions. The number of decays with this angle between β and $\beta + d\beta$ is given by

$$\frac{1}{2}\left(1 + \lambda\frac{cp_\beta}{E_\beta}\cos\beta\right)\sin\beta\,d\beta.$$

For a given momentum p_β the distribution function of the recoil nuclei is thus

$$\frac{1}{2}\left(1 + \lambda\frac{cp_\beta}{E_\beta}\frac{p^2_{nuc} - p^2_\beta - p^2_\nu}{2p_\beta p_\nu}\right)\frac{p_{nuc}\,dp_{nuc}}{p_\beta p_\nu}, \tag{17}$$

if

$$p_\beta + p_\nu > p_{nuc} > |p_\beta - p_\nu|, \tag{18}$$

and equal to zero if (18) is not satisfied. By virtue of (18) the momentum p_β for a given p_{nuc} has to lie between $A_2(p_{nuc}) > p_\beta > A_1(p_{nuc})$, where

$$A_1(p_{nuc}) = \frac{1}{c}\left|\frac{(E_{\beta\,max} - cp_{nuc})^2 - m_0^2c^4}{2(E_{\beta\,max} - cp_{nuc})}\right|;\quad A_2(p_{nuc}) = \frac{1}{c}\frac{(E_{\beta\,max} + cp_{nuc})^2 + m_0^2c^4}{2(E_{\beta\,max} + cp_{nuc})}. \tag{19}$$

Assuming an allowed β-spectrum, the momentum distribution function of the recoil nuclei will have the form

$$f(p_{\text{nuc}}) = cp_{\text{nuc}} \int_{A_1}^{A_2} \left(1 + \lambda \frac{cp_\beta}{E_\beta} \frac{p^2_{\text{nuc}} - p_\beta^2 - p_\nu^2}{2p_\beta p_\nu}\right) F(Z, E_\beta)\, p_\beta p_\nu\, dp_\beta. \quad (20)$$

Here $F(Z, E_\beta)$ is the well known Fermi function describing the influence of the nuclear Coulomb field on the β-decay. The integral (20) can be evaluated numerically. The distribution function depends on λ.

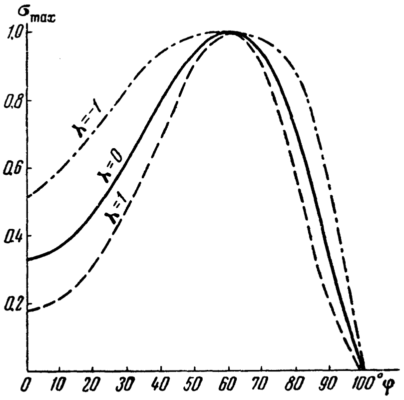

FIG. 6

8. The numerical integration was performed for the decay of Na^{24} which has a decay scheme similar to that of Fig. 5. The following values have been assumed:

$$E_{\beta\,\max} = 1.911, \quad E_{\gamma 1} = 2.76, \quad E_\gamma = 1.38\,\text{Mev}.$$

One sees easily that the maximum of the cross section occurs when

$$\cos\alpha = E_{\gamma 2} / E_{\gamma 1}.$$

The integral in (15) which determines the cross section equals $0.448\ m_1/m_0$ for $\lambda = 0$, $0.378\ m_1/m_0$ for $\lambda = +1$, and $0.530\ m_1/m_0$ for $\lambda = -1$ (m_1 — mass of the nucleus, m_0 — electron mass).

The dependence of the cross section on the angle α has been plotted in Fig. 6. All curves have been normalized at the point $\bar{\sigma} = \sigma_{\max}$. The maximum cross section (assuming $\Gamma = 3.4 \times 10^{-3}$ ev, lifetime 2×10^{-12} sec) will be $3 \times 10^{-25}\ cm^2$, $2.4 \times 10^{-25}\ cm^2$, and $2 \times 10^{-25}\ cm^2$ for $\lambda = -1.0$, and $+1$ respectively. The variation of the cross section with the angle is quite dependent on λ.

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Translated by M. Danos
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