follows that the energy of primary particles should be $> 6 \times 10^9$ev.

By the method described in Ref. 7, using our data on the number of penetrating particles and their angular distribution, we obtained a value of the order of 30 Bev for the mean energy of the star-producing particles. The mean number of prongs was found to be 7.6, the mean number of secondary particles capable of penetrating 10 and 20 cm of lead is 5.6 and 2.7 respectively. The above results, however, are not very accurate because of the limitations of the hodoscope and the value obtained for the mean energy of the primary particles is only approximate. In addition, this value would be correct only for isotropic distribution of secondary particles in the center-of-mass system.

In conclusion, the authors wish to express their gratitude to A. I. Alikhanian and A. V. Khrimian for their help and discussion, to V. Sh. Kamalian and A. M. Gal'per for taking part in the measurements and to V. M. Krishchian for help in reducing data.

[1] A. V. Khrimian, Izv. Akad. Nauk SSSR, Ser Fiz. **19**, 700 (1955).
[2] Froehlich, Harth and Sitte, Phys. Rev. **87**, 504 (1952).
[3] R. L. Cool and O. Piccioni, Phys. Rev. **87**, 531 (1952).
[4] A. J. Hartzler, Phys. Rev. **82**, 359 (1951).
[5] Whetten, Preston, Adams, Walker and Kraybill, Phys. Rev. **93**, 1356 (1954).
[6] K. H. Barker and C. C. Butler, Proc. Phys. Soc. **A64**, 4 (1951).
[7] N. M. Duller and W. D. Walker, Phys. Rev. **93**, 215 (1954).

Translated by H. Kasha
114

SOVIET PHYSICS JETP VOLUME 6 (33), NUMBER 3 MARCH, 1958

ON THE HALL EFFECT IN FERROMAGNETICS

N. V. BAZHANOVA

Moscow Institute of Transportation Engineers

Submitted to JETP editor March 6, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 567-570 (September, 1957)

The Hall effect has been studied in Fe—Ni alloys of the invar group near the Curie temperature. It has been established that in ferromagnetics above technical saturation, the Hall emf continues to increase linearly with increasing true magnetization; this confirms the validity of formula (2), which was proposed by Volkov for describing the Hall effect in ferromagnetics. A relation between the Hall emf and the magnetic field, obtained on the basis of Ginzburg's equation for the true magnetization of ferromagnetics near the Curie temperature, is satisfactorily confirmed by experiment.

NUMEROUS studies of the Hall effect in ferromagnetic materials have led to the establishment of a number of laws. Basic among these—as has been shown by the systematic investigations of Kundt,[1] Pugh,[2] and Kikoin[3]—is a linear relation between the Hall electromotive force and the magnetization of the material in the region of technical magnetization of ferromagnetics. As is shown by experiment, after a ferromagnetic has attained magnetic saturation, the Hall effect in it continues to increase with increase of the magnetic field.

On this basis, Pugh and his coworkers[2,4] proposed the following formula for describing the Hall effect in ferromagnetics:

$$E = R_0 H + R_t I_t, \quad (1)$$

where E is the Hall electromotive force for unit current density and for unit distance between the Hall electrodes, H is the magnetic field, I_t is the magnetization of the ferromagnetic, R_t is the constant of the so-called extraordinary Hall effect, and R_0 is the constant of the ordinary effect and is a measure of the number of conduction electrons per unit volume. The term $R_t T_t$ describes the Hall-effect behavior in weak fields; the term $R_0 H$ describes the behavior in strong fields, above technical saturation.

However, after analyzing the experimental material that led Pugh to formula (1) and after considering also the work of Kikoin,[5] Volkov[6] reached the conclusion that the increase of the Hall emf in ferromagnetics in strong fields (above technical saturation) is determined not by the magnetic field H but by the true magnetization I_i. Then for ferromagnetics the Hall emf should be expressed in the following form:

$$E = R_t I_t + R_i I_i, \quad (2)$$

where R_i is a constant of the material, which describes the Hall effect in those magnetic fields in which it is determined by the true magnetization. It must be expected that in general $R_i \neq R_t$. Experiment in fact confirms this expectation.

The aim of the present work was to study the Hall effect in the region of true magnetization of ferromagnetics (the region of the paraprocess). Chosen for study were iron-nickel alloys of the invar group; in these, as was shown by the systematic investigations of Belov,[7] the paraprocess is appreciable in comparatively weak magnetic fields.

The method that we used for measuring the Hall effect was similar to that used earlier by Pugh and by Kikoin. The specimens had the form of rectangular parallelopipeds, with dimensions 6 × 12 × 150 mm, with electrodes arranged on the middle part of the specimen. The magnetization of the ferromagnetic was measured by a ballistic method, with use of a remote coil mounted outside the solenoid and connected in series with the test coil. The remote coil was so arranged as to compensate the effect of the magnetic field of the solenoid on the test coil wound on the specimen. The measurements of the Hall emf and of the magnetization were made simultaneously.

As follows from formula (2), the dependence of the Hall emf on the magnetic field H above technical saturation will exhibit a linear character only when the true magnetization I_i increases linearly with the magnetic field intensity. For ferromagnetic materials this will be approximately the case far from the Curie point, where the paraprocess susceptibility is independent of the magnetic field H. Near the Curie temperature, however, this proportionality will not be observed, because at these temperatures the true magnetization I_i depends in a complicated way on the magnetic field H. The investigation of the Hall effect that we have conducted on the alloys mentioned has shown that at temperatures near the Curie tem-

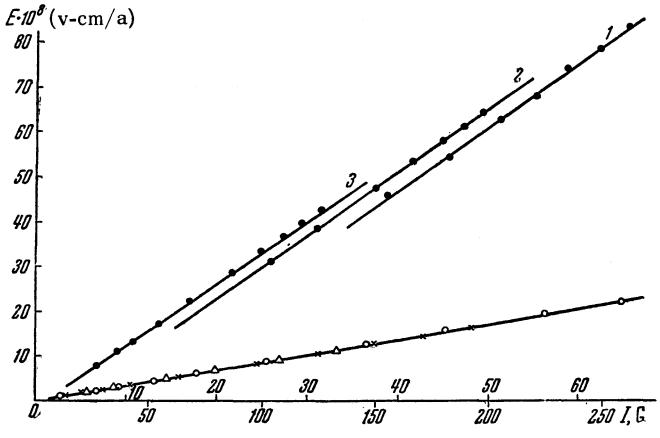

FIG. 1. Dependence of Hall emf on magnetization in the range of the paraprocess for an alloy containing 32% Ni, remainder Fe. The lower scale on the axis of abscissas corresponds to temperatures: Curve 1, 84°; 2, 100.5°; 3, 121°C. The upper scale corresponds to: ○, 150°C; ×, 159.5°C; Δ, 174°C.

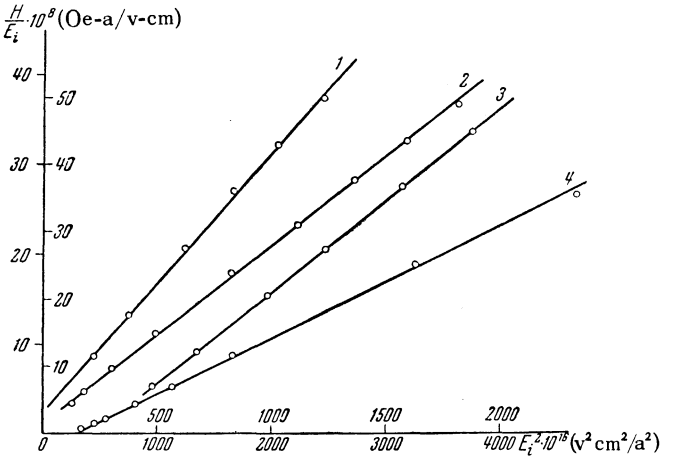

FIG. 2. Dependence of H/E_i on E_i^2 for alloys: 1, 30% Ni, 70% Fe, t = 71.5°C; 2, 32% Ni, 68% Fe, t = 121°C; 3, 34% Ni, 66% Fe, t = 178°C; 4, 37% Ni, 63% Fe, t = 235°C (1, 2, and 4, inner scale; 3, outer).

peratures, there actually is a nonlinear type of dependence of the Hall emf on magnetic field H above technical saturation in a ferromagnetic. Furthermore, in the region of the para-process—as is shown, for example, by Fig. 1—there indisputably exists a linear relation between the Hall emf and the true magnetization I_i of the ferromagnetic.

The establishment of a linear relation between the Hall emf and the magnetization in the paraprocess region makes it possible to obtain, in explicit form, the dependence of the Hall emf on the magnetic field H. By treating the Curie-point transition of the material from the ferromagnetic state to the paramagnetic as a phase transition of the second kind, Ginzburg[8] obtained the following equation for the true magnetization curve for ferromagnetics near the Curie temperature:

$$H = \alpha I_i + \beta I_i^3, \tag{3}$$

where α and β are constants, dependent on the temperature and on the pressure. Near the Curie temperature, the Hall effect due to technical magnetization, $E_S = R_t I_S$, is small in comparison with the size of the effect evoked by the paraprocess; clearly, therefore, we may expect that in this case the Hall effect will be described by the formula $E_i = R_i I_i$. Upon substituting the value of I_i from this formula in (3), we get the desired dependence of the Hall emf in the region of true magnetization upon the magnetic field:

$$H = aE_i + bE_i^3, \tag{4}$$

where

$$a = \alpha / R_i, \quad b = \beta / R_i^3. \tag{5}$$

Our experiments with the alloys mentioned show that near the Curie temperature the Hall effect satisfies the relation (4) over a wide range of values of H. This is evident, for example, from Fig. 2, which shows—in accordance with the relation (4)—the dependence of H/E_i on E_i^2 for a series of alloys of the invar group at temperatures in the immediate vicinity of the Curie point. The Curie points of these alloys were determined by the method suggested by Belov ($T = \Theta$ if $\alpha_T = 0$). The straight lines drawn in this figure have intercepts on the ordinate axis whose numerical values are evidently equal to the coefficients a in Eq. (4) for the corresponding alloys, and the slopes of the lines give the values of the coefficients b. It is possible to compare the graphically determined a and b with the values calculated from the known α, β, and R_i. The numerical values of α and β in Eq. (3) were determined from the graphs of H/I_i as a function of I_i^2 for the corresponding alloys; the coefficient R_i was determined, for example, from Fig. 1 for the alloy with 32% Ni and the rest Fe, and from similar figures for the other alloys. The comparison gives completely satisfactory results. For example, for the alloy with 32% Ni and the rest Fe ($t = 121°$), the values determined from Fig. 2 are $a = 1.3 \times 10^8$ (amp gauss/volt cm), $b = 2.7 \times 10^{22}$ (amp^3 gauss/volt^3 cm^2); the values calculated with formula (5) are $a = 0.9 \times 10^8$, $b = 2.75 \times 10^{22}$. For the alloy with 34% Ni and the rest Fe ($t = 178°C$), the values determined from Fig. 2 are $a = -4.3 \times 10^8$, $b = 10.16 \times 10^{21}$; the values calculated with the formula are $a = -6.7 \times 10^8$, $b = 9.68 \times 10^{21}$.

In view of the fact that similar results were found for the range of true magnetization in other Fe-Ni alloys as well, it may be concluded that the Hall effect in ferromagnetics above technical saturation is directly determined not by the magnetic field but by the true magnetization of the material, and should be described by formula (2).

In the most general case it is apparently necessary to take account also of the influence of the external magnetic field on the conduction electrons; but in ferromagnetics this effect, as is shown by experiment, is quite small in comparison with the effect produced by technical and true magnetization.

In closing I express my deep gratitude to D. I. Volkov, who proposed this problem, for his guidance of the work reported.

[1] A. Kundt, Wied. Ann. **49**, 257 (1893).
[2] E. M. Pugh, Phys. Rev. **36**, 1503 (1930); E. M. Pugh and T. W. Lippert, Phys. Rev. **42**, 709 (1932).
[3] I. K. Kikoin, Physik. Z. Sowjetunion **9**, 1 (1936).
[4] Pugh, Rostoker, and Schindler, Phys. Rev. **80**, 688 (1950).
[5] I. K. Kikoin, J. Exptl. Theoret. Phys. (U.S.S.R.) **10**, 11 and 1242 (1940).
[6] D. I. Volkov, Вестник Моск. ун-та (Bulletin of Moscow Univ.) **3**, 65 (1954).

[7] K. P. Belov, Упругие, тепловые и электрические явления в ферромагнитных металлах (Elastic, Thermal, and Electrical Phenomena in Ferromagnetic Metals), GITTL, 1951.
[8] V. L. Ginzburg, J. Exptl. Theoret. Phys. (U.S.S.R.) 17, 833 (1947).

Translated by W. F. Brown Jr.
115

SOVIET PHYSICS JETP VOLUME 6 (33), NUMBER 3 MARCH, 1958

INVESTIGATION OF THE CHARACTERISTIC ENERGY LOSSES OF ELECTRONS AND THE SECONDARY ELECTRON EMISSION FROM GeO_2

N. B. GORNYI and A. IU. REITSAKAS

Submitted to JETP editor March 1, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) 33, 571-575 (September, 1957)

The characteristic energy losses of electrons reflected from the surfaces of two germanium plates of the n and p types covered with GeO_2 are investigated. The spectra of the characteristic losses for both types of plates are similar. If the difference in the lattice constants is taken into account, the spectra of the characteristic energy losses for GeO_2 and MoO_2, which have identical (tetragonal) lattices, are found to be similar. Secondary electron emission has been investigated for the indicated GeO_2 samples. The secondary electron yield from GeO_2 is almost twice as large as the yield from germanium.

CHARACTERISTIC ENERGY LOSSES OF ELECTRONS

In an earlier work on the characteristic energy losses in reflection from a MoO_2 surface[1] one of the authors has showed that these losses can be determined by the formula

$$W = (h^2 / 8m)(n/d)^2.$$

For MoO_2 which crystallizes in a tetragonal lattice $(n/d)^2 + (h^2 + k^2)/a^2 = (l/c)^2$ where h, k, and l are the Miller indices, and a and c are the lattice constants. Thus, the spectra of the characteristic energy losses of electrons must depend on the structure and size of the crystal lattice of a given material. Therefore, different substances with the same crystal structure must have similar spectra. This conclusion was arrived at in one of the recent works of Marton and co-workers.[2]

To test the above conclusion we undertook to investigate the characteristic energy losses of electrons in GeO_2, which has the same crystal lattice as MoO_2. We treated two thin plates of germanium (thickness 0.5 mm), one type n and the other type p, with a solution of nitric acid; this, as is known, forms a layer of GeO_2 on the surface of germanium. After treating with nitric acid the sample was washed in distilled water, then treated in a solution of NaOH, and once again washed in distilled water and in pure alcohol. As a result there remained on the surface of the germanium a layer of GeO_2, "insoluble" in water, and crystallized in a tetragonal lattice.[3] The lattice constants of GeO_2 are a = 4.39 A and c = 2.859 A.

The method of electrical differentiation in a spherical capacitor[4,5] circuit was used to investigate the characteristic energy losses of electrons reflected from the surfaces of the above two GeO_2-coated germanium plates. The vacuum apparatus was as that used in Ref. 6. The two targets were arranged on a cylindrical table made of tantalum inside the spherical vacuum apparatus. By means of a magnet outside the vacuum it was possible to turn the table so that either one of the specimens could be placed in the beam. After long adjustment and degassing of the apparatus the pressure, measured with an ionization