ON SOME ELECTROMAGNETIC EFFECTS INVOLVING STRONGLY INTERACTING PARTICLES

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A number of electromagnetic effects involving strongly interacting particles are discussed. The calculations are carried out within the framework of quantum electrodynamics by introducing form factors. It is shown that by studying the considered effects it is possible to obtain information on the structure and the characteristics of strongly interacting particles.

It is well known that the absence of a theory describing strong interactions make it impossible to interpret experiments involving π-mesons and nucleons in a unique way. In particular, it seems not to be possible on the basis of such experiments to substantiate the current ideas of the electromagnetic structure of strongly interacting particles. Therefore it is of interest to investigate such electromagnetic effects involving strongly interacting particles which can be interpreted within the framework of quantum electrodynamics.

The experimental investigation of effects of this nature could give direct information on the structure and the electromagnetic characteristics of nucleons and π-mesons. However, at present it is unfortunately rather difficult to perform such experiments.

The present paper will treat the creation and annihilation of π-mesons and nucleon-antinucleon pairs in "pure" electromagnetic processes, i.e., in the case where atomic nuclei or similar strongly interacting particles are absent in the initial state. In addition, electromagnetic π-π scattering will be treated.

1. CREATION OF π-MESON AND NUCLEON-ANTINUCLEON PAIRS IN THE ANNIHILATION OF HIGH ENERGY POSITRONS

We now consider the creation of π-meson and nucleon-antinucleon pairs in positron annihilation. We shall assume that quantum electrodynamics is valid up to an energy in the center of mass system $E_{cr} \leq M$ where M is the nucleon mass[1] (the units are such that $\hbar = c = 1$). This assumption strictly speaking does not allow to discuss antinucleon creation, while the treatment of π-meson pair creation particularly at not too high energies seems to be fully justified. Therefore the possible deviations from the results obtained within the framework of quantum electrodynamics can be ascribed to the "anomalous" electromagnetic properties of the strongly interacting particles. Thus, the interaction of π-mesons with virtual nucleons will lead to a finite electromagnetic size of the particle,[2] i.e., to the form factor of the mesons. For nucleons the interaction with the real vacuum will lead in addition to the "smearing out" of the particle also to the anomalous magnetic moment. Another reason for such deviations would be a strong interaction between the particles of the created pair; this would also show up as a form factor.[3] However, as will be shown below there exists an energy region where the nature of the form factor can be uniquely established.

The smallest energy of a positron (electron) necessary for the creation of a meson pair equals μ in in the c.m.s. where μ is the mass of the π-meson. In the laboratory system this corresponds to a positron energy $E_p = 2\mu^2/m = 76$ Bev. This large difference of the energy in the two systems evidently shows that the only feasible way of performing experiments of this nature is by means of colliding beams of fast particles. The differential cross section for creation of a $\pi^{\pm}$ pair in the c.m.s. is given by

$$d\sigma = (\pi/16)\, r_0^2 (\mu/E)^2 (1 - \mu^2/E^2)^{3/2} \sin^2\theta\, |F|^2\, d(\cos\theta), \qquad (1)$$

where E is the energy of the particle in the c.m.s. and θ is the angle between the momentum of the positron and the π^+-meson. The form factor F which is a relativistically invariant function of the energy and the momenta does not depend on the angles in the c.m.s.[3] Therefore, after integration over the angles we obtain

$$\sigma = (\pi/12)\, r_0^2 (\mu / E)^2 (1 - \mu^2 / E^2)^{3/2} \,|\, F(2E^2 / M_0^2) \,|^2. \tag{2}$$

The quantity M_0 in the argument of the form factor corresponds to a certain reciprocal length, characteristic for a π-meson (evidently $\mu \leq M_0 \leq M$). For $x = 0$ we have $F(x) \approx 1$. Therefore it is essential to calculate the form factor for $E \gtrsim M_0$.

At the threshold of the reaction, for E close to μ, we have

$$\sigma = (\pi/12)\, r_0^2 (1 - \mu^2 / E^2)^{3/4} \,|\, F \,|^2. \tag{3}$$

We note that the annihilation of an electron-positron pair in singlet states into a $\pi^\pm$ pair is forbidden for reasons of conservation of parity and angular momentum. (See also Ref. 4.) On the other hand, the creation of the meson pair in an S-state is excluded since the transition matrix element is proportional to the meson momentum.[5] Therefore the smallest possible angular momentum of the meson pair is $\ell = 1$ (P-wave). This circumstance renders the interaction of the mesons in the pair unimportant at small energies in view of the short range of the interaction. It therefore follows that for small energies the form factor is uniquely given by the electromagnetic dimensions of the π-mesons.

In the relativistic case $E \gg \mu$ we have

$$\sigma = (\pi/12)\, r_0^2 (\mu / E)^2 \,|\, F \,|^2. \tag{4}$$

Neglecting the form factor the cross section (2) has its maximum at $E = 1.58\mu$. There it has a value $\sigma = 0.5 \times 10^{-31}$ cm^2. The form factor which decreases with increasing energy will shift this maximum to a smaller energy.

Equations (1) — (4) are also valid for the creation of K-meson pairs if one substitutes for μ and F respectively the mass and form factor of the K-meson.

The minimum energy of the positron (electron) in the c.m.s. for creation of a nucleon-antinucleon pair equals the nucleon mass M. In the laboratory system this corresponds to a minimum positron energy $M_p = 3.4 \times 10^3$ Bev. Here one has to take into account also the "anomalous" magnetic moment of the nucleon which can have a magnitude different from the static case. In an analogous way the cross section for creation of nucleon-antinucleon pairs can be obtained. In the c.m.s. it is given by

$$\sigma = \frac{\pi}{3} \frac{e^2}{M} \left(\frac{M}{E}\right)^2 \left(1 - \frac{M^2}{E^2}\right)^{1/2} \left\{\left(1 + \frac{M^2}{2E^2}\right) \frac{e^2}{M} - 6e\mu_1 + 2\mu_1^2 \frac{E^2}{M}\left(1 + \frac{2M^2}{E^2}\right)\right\} |\, F \,|^2, \tag{5}$$

where E is the positron (electron) energy, and the full magnetic moment equals $(e/2M) + \mu_1$.

In the nonrelativistic approximation $E \gtrsim M$ Eq. (5) becomes

$$\sigma = (2\pi e^2 / M)(1 - M^2 / E^2)^{1/2} \{e^2/4M - \mu_1 e + \mu_1^2 M\} \,|\, F \,|^2. \tag{6}$$

Putting $e = 0$ in the curly brackets of (5) and (6) we obtain the cross section for creation of a neutron-antineutron pair. As seen from these equations the anomalous magnetic moment contributes the main part to the cross section for $\mu_1 > 1$. Then the form factor is determined mainly by the basic strong interaction between the particle and the antiparticle of the pair and evidently depends much less on the electromagnetic size of the particles in contrast to the above treated case of π- and K-mesons.

2. ON THE ELECTROMAGNETIC INTERACTIONS OF π-MESONS

At present there exists considerable interest on the problem of a specific meson-meson interaction. Definite data on this interaction obviously can be obtained only in an experiment involving two colliding meson beams. It is therefore of interest to split off the purely electromagnetic part of the meson-meson interaction. To this end we shall investigate the scattering of mesons by mesons and the annihilation of a π^+ and a π^- with emission of two γ-quanta or a pair of light fermions.

The differential cross section for scattering of π-mesons of the same sign of the charge in the c.m.s. is given by

$$d\sigma = \frac{1}{16} \frac{e^4}{(pv)^2} \left[\frac{1 + v^2 \cos^2(\theta/2)}{\sin^2(\theta/2)} + \frac{1 + v^2 \sin^2(\theta/2)}{\cos^2(\theta/2)}\right]^2 \left| F\left(\frac{2p^2}{M_0^2} \sin^2 \frac{\theta}{2}\right)\right|^2 d\Omega \tag{7}$$

where p and v are the momentum and the velocity of the mesons respectively and θ is the scattering angle. The form factor F describes the electromagnetic size of the π-mesons. Since the electromagnetic interactions are important only for small angle scattering, i.e., for large impact parameters, we shall not take into account the influence of the short range specific meson-meson interaction on the form factor.

In the limiting cases of small and large energies we have

$$d\sigma_{\text{nonrel}} = e^4 (pv)^{-2} \sin^{-4}\theta \mid F \mid^2 d\Omega, \quad d\sigma_{\text{rel}} = \frac{e^4}{p^2}\left[1 + \sin^4\frac{\theta}{2} + \cos^4\frac{\theta}{2}\right]^2 \mid F \mid^2 \frac{d\Omega}{\sin^4\theta}.$$

In the nonrelativistic case (7) is identical with the first Born approximation to Mott's formula[6] derived for the scattering of slow α-particles in helium.

For mesons of opposite charge the scattering cross section is given in the c.m.s. by

$$d\sigma = \frac{1}{16}\frac{e^4}{(pv)^2}\left[\frac{1 + v^2\cos^2(\theta/2)}{\sin^2(\theta/2)} - v^4\cos\theta\right]^2 \mid F \mid^2 d\Omega. \tag{8}$$

The first term in (8) corresponds to scattering of spin-zero particles while the second term is due to the possibility of meson annihilation.

In analogy to above we have

$$d\sigma_{\text{nonrel}} = \frac{1}{16}\frac{e^4}{(pv)^2}\frac{1}{\sin^4(\theta/2)} \mid F \mid^2 d\Omega, \quad d\sigma_{\text{rel}} = \frac{1}{16}\frac{e^4}{p^2}\left[1 + \sin^4\frac{\theta}{2} + \cos^4\frac{\theta}{2}\right]^2 \mid F \mid^2 \frac{d\Omega}{\sin^4(\theta/2)}.$$

Comparing (7) and (8) one sees that the differential cross sections for small angles are the same for both processes.

The differential cross sections for annihilation of two $\pi^{\pm}$-mesons into two γ-quanta is given in the c.m.s. by

$$d\sigma = r_0^2 \frac{1}{v_{12}}\left(\frac{\mu}{E}\right)^2\left\{1/2 - \frac{\mu^2}{E^2 - p^2\cos^2\theta} + \frac{\mu^4}{(E^2 - p^2\cos^2\theta)^2}\right\} \mid F \mid^2 d\Omega,$$

where E is the energy of the π-mesons, v_{12} the relative velocity of the annihilating particles, and μ the π-meson mass. The form factor F is here essential because in annihilation small distances are important where the influence of the short range forces may be great. The total cross section in the c.m.s. neglecting the form factor is given by

$$\sigma = \frac{2\pi}{v_{12}} r_0^2 \left(\frac{\mu}{E}\right)^2\left\{1 + \frac{\mu^2}{E^2} - \frac{\mu^2}{2E\sqrt{E^2 - \mu^2}}\left(2 - \frac{\mu^2}{E^2}\right)\ln\frac{E + \sqrt{E^2 - \mu^2}}{E - \sqrt{E^2 - \mu^2}}\right\}. \tag{9}$$

In the nonrelativistic approximation $E \approx \mu$ and

$$\sigma = 2\pi r_0^2 / v_{12}. \tag{10}$$

At large energies we have

$$\sigma = \pi r_0^2 (\mu / E)^2.$$

The total cross section for annihilation of mesons with creation of a pair of charged fermions of mass m and anomalous magnetic moment $\mu_1 = \mu_0 - e/2m$ is

$$\sigma = \frac{2\pi}{3}\left(\frac{e^2}{E}\right)^2\left(1 - \frac{\mu^2}{E^2}\right)^{1/2}\left(1 - \frac{m^2}{E^2}\right)^{1/2}\left\{\frac{1}{2}\left(1 + \frac{m^2}{2E^2}\right) - \frac{3\mu_1}{e}\mu + \left(\frac{\mu_1}{e}\right)^2 E^2\left(1 + \frac{m^2}{E^2}\right)\right\} \mid F \mid^2. \tag{11}$$

Here the form factor F as shown above at not too large energies is given essentially by the electromagnetic size of the π-mesons and does not depend on the angles. Putting $\mu_1 = 0$ we have for the annihilation of $\pi^{\pm}$-mesons into an electron-positron pair

$$\sigma = (\pi e^4 / 3E^2)(1 - \mu^2 / E^2)^{1/2}. \tag{12}$$

In the nonrelativistic approximation this becomes

$$\sigma = \pi r_0^2 v_{12}/6. \tag{13}$$

For relativistic energies of the annihilating particles we have

$$\sigma = (\pi r_0^2/3)(\mu/E)^2.$$

We note that Brown and Peshkin[7] have given formulae for the electromagnetic annihilation of antinucleons.

Finally we shall discuss the properties of an atomic system comprising π^+ and π^--mesons (π-mesonium). Such a system might appear in nuclear reactions producing a pair of π-mesons. Near the threshold the ratio of the number of created π-mesonium atoms to the number of free π-meson pairs is given by

$$\sigma_{\text{bound}} / \sigma_{\text{free}} \approx \alpha^3 (\mu/\Delta)^{3/2},$$

where $\alpha = 1/137$, and Δ ($\Delta \ll \mu$) is the excess energy above the reaction threshold.

From (10) one can estimate the lifetime of π-mesonium in the 1S state with respect to decay to two γ-quanta:

$$\tau_{2\gamma} = \{(\sigma v_{12})_{v_{12}\to 0} |\Psi(0)|^2\}^{-1} \approx \frac{4}{\alpha^5\mu} \approx 0.9 \times 10^{-12}\ \text{sec} \tag{14}$$

We note that in the annihilation of π-mesonium in the 1S state the polarization of the produced photons is parallel, in contradistinction to the annihilation of parapositronium where the polarization of the photons is perpendicular.

The mean life of π-mesonium against decay into an electron-positron pair can not be obtained from (14) since in the approximation used no S-waves participate in the process (see Sec. 1). In the next approximation the mean life to π-mesonium in the 1S state against decay into an electron-positron pair is of the order

$$\tau(e^+ + e^-) \sim (137)^2 \tau_{2\gamma}.$$

We also note that

$$\tau(e^+ + e^-) \sim \tau(2(e^+ + e^-)),$$

where the right hand side is the mean life of π-mesonium in the 1S state against decay into two electron-positron pairs.

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