MAGNETIC SUSCEPTIBILITY OF A SEMICONDUCTOR WITH AN IMPURITY BAND IN A STRONG MAGNETIC FIELD

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The magnetic susceptibility of semiconductors with impurity bands in strong magnetic fields is considered. It is established that for a given dispersion law the susceptibility has a term which oscillates with increasing field strength. Since these oscillations depend substantially on the properties of the model accepted for the semiconductor with an impurity band, their experimental observation can yield a partial confirmation of the correctness of the proposed model.

1. INTRODUCTION

CONSIDERABLE attention has been devoted recently to experimental and theoretical investigation of the properties of semiconductors with impurity bands, especially semiconductors of the Ge type.[1–3]

In spite of this, very little is known about the nature of the impurity band, the dispersion law, and the distribution of the impurity atoms in the host lattice. One way to investigate these questions is to invent a specific reasonable model for semiconductors of this type. One would then consider those parameters whose behavior depends fundamentally on the assumed model. By comparing theory with experiment, one can then determine whether the model is correct.

The parameters to be considered, probably, can not be the electrical conductivity or the Hall coefficient, whose qualitative behavior does not basically depend on the particular properties of models. It will be shown in the course of this paper, that a suitable property for investigation is the magnetic susceptibility in strong magnetic fields **H**.

For this purpose, we shall limit the discussion to the simplest kind of semiconductor, namely, one which has an energy spectrum that contains a conduction band and a fundamental impurity s-band (for a p-type semiconductor it would contain a valence band and an acceptor impurity band). The impurity to be considered will have only one valence and its concentration per cc will be denoted by n_0. The forbidden energy gap, $\Delta\epsilon$, between the bottom of the conduction band and "the top" of the impurity band will be considered positive.

If the impurity band is narrow, i.e., its width is $D \lesssim kT$, one must take into account how far the relation between the energy of the electron, $\epsilon(\mathbf{k})$, departs from a quadratic dependence on the wave vector. We shall assume that $\epsilon(\mathbf{k})$ is nearly periodic in the fundamental impurity s-band. That this assumption is valid will become clear as we progress. For simplicity, we shall assume that

$$\varepsilon(\mathbf{k}) = -\Delta[\cos k_x a + \cos k_y a + \cos k_z a], \tag{1}$$

i.e., it is as if the impurity atoms were distributed in a simple cubic lattice with a lattice constant, a, given by: $a = n_0^{-1/3}$. In Eq. (1) the following relations were used:

$$-\pi \leqslant k_x a,\ k_y a,\ k_z a \leqslant \pi,\quad \Delta = D/6 \tag{2}$$

and the zero-energy value was assigned to the center of the impurity band.

It is in this model that we shall now consider the magnetic susceptibility in a strong magnetic field, **H**.

2. MAGNETIC SUSCEPTIBILITY OF ELECTRONS IN THE IMPURITY BAND

Calculation of the magnetic susceptibility of electrons in the impurity band presupposes a solution of the corresponding statistical sum:

$$Z_H = \mathrm{Sp}\exp\left(-\hat{\mathscr{H}}/kT\right), \tag{3}$$

where to the particular degree of approximation used here[4]

$$\hat{\mathscr{H}} = -\Delta\left[\cos k_x a + \cos\left(k_y a + \frac{eH}{\hbar c}ax\right) + \cos k_z a\right] + s\mu_B H, \tag{4}$$

if the field, **H**, is applied along the Z-axis and the corresponding vector potential is chosen; $s = \pm 1$; μ_B is the Bohr magnetron, and $\Delta = \Delta(H)$. This yields a closed expression for Z_H for any non-vanishing H and $\Delta/kT = b_0$. However, one must take it into account that impurity bands, generally speaking, are narrow and their width D is less than or of the order of kT. Consequently, it is assumed that $b_0 < 1$, and we proceed to calculate Z_H under this assumption.

To calculate Z_H, it is expedient to transfer to the k-representation, i.e. to write $x = i\partial/\partial k_x$. The value of Z_H can be calculated to terms of the order of b_0^4 inclusively and for any H (in the case of an impurity band); therefore, since the part of $\hat{\mathscr{H}}$, which depends on k_z and on the spin of the electron, enters additively, one has the following expression for Z_H

$$Z_H = \frac{1}{2\pi a^3}\int_{-\pi}^{\pi} e^{b_0\cos t}dt\left(e^{\mu_B H/kT} + e^{-\mu_B H/kT}\right)\mathrm{Sp}\exp\left[b_0\cos X + b_0\cos\left(Y + i\alpha\frac{\partial}{\partial X}\right)\right],$$

$$-\pi \leqslant X = k_x a \leqslant \pi;\quad -\pi \leqslant Y = k_y a \leqslant \pi;\quad \alpha = eHa^2/\hbar c,$$

from which

$$Z_H = \frac{2}{a^3} I_0(b_0)\cosh\frac{\mu_B H}{kT}\,\mathrm{Sp}\exp\left(\hat{A} + \hat{B}\right), \tag{5}$$

where

$$\hat{A} = b_0\cos X,\quad \hat{B} = b_0\cos\left(Y + i\alpha\partial/\partial X\right),$$

and I_0 is Bessel's function of zero order with imaginary argument. Since in Eq. (5) the operators $\hat{A}$ and $\hat{B}$ do not commute, one can apply Feynman's theorem[5] to calculate $\mathrm{S_p}\exp(\hat{A} + \hat{B})$:

$$Z'_H = \mathrm{Sp}\exp\left(\hat{A} + \hat{B}\right) = \mathrm{Sp}\exp\hat{A}\exp\left[\int_0^1 ds\exp\left(-s\hat{A}\right)\hat{B}\exp\left(s\hat{A}\right)\right],$$

from which

$$\exp\left(-s\hat{A}\right)\hat{B}\exp s\hat{A} = \hat{B} + \exp\left(-s\hat{A}\right)\left[\hat{B}, \exp s\hat{A}\right].$$

Using this theorem again,

$$\mathrm{Sp}\exp\left(\hat{A} + \hat{B}\right) = \mathrm{Sp}\, e^{\hat{A}}e^{\hat{B}}\exp\left\{\int_0^1 ds e^{-s\hat{B}}\left(\int_0^1 ds' e^{-s'\hat{A}}\left[\hat{B}, e^{s'\hat{A}}\right]\right)e^{s\hat{B}}\right\} = \mathrm{Sp}\exp\hat{A}\exp\hat{B}\exp\left[\int_0^1 dsT(s)\right],$$

since the index s' does not play a regulating role,[5] so that

$$T(s) = e^{-s\hat{B} - s\hat{A}}\left[\hat{B}, e^{s\hat{A}}\right]e^{s\hat{B}}.$$

If T(s) is found to be small, then one can use for Z'_H the expression

$$Z'_H = \mathrm{Sp}\exp\hat{A}\exp\hat{B}\sum_{n=0}^{\infty} W_n,\quad W_0 = 1,\quad W_n = \int_0^1 ds_1 \ldots \int_0^{s_{n-1}} ds_n T(s_1)\ldots T(s_n). \tag{6}$$

It should be noted, that as in the case which is under consideration, with $b_0 < 1$, $T(s) \sim b_0^2$, the series in Eq. (6) converges sufficiently rapidly, because the principal term in W_n is proportional to b_0^{2n}.

To calculate the commutator in T(s) we make use of the properties of the displacement operator $\exp(\pm iY \mp \alpha\partial/\partial X)$.

Then

$$\left[\hat{B}, \exp s\hat{A}\right] = \frac{b_0}{2}\left\{\left[e^{sb_0\cos(X+\alpha)} - e^{sb_0\cos X}\right]e^{-iY+\alpha\partial/\partial X} + \left[e^{sb_0\cos(X-\alpha)} - e^{sb_0\cos X}\right]e^{iY-\alpha\partial/\partial X}\right\}.$$

Since it is not possible to calculate Z'_H for all values of b_0, we can make use of the fact, that in our case it is possible to substitute $b_0 < 1$ and to calculate the terms in Z'_H, by expanding in powers of b_0, the quantities $\exp sb_0 \cos X$ and $\exp sb_0 \cos(Y + i\alpha\partial/\partial X)$, keeping in Z'_H only terms proportional to b_0^4.

In what follows in place of $T(s)$ for simplicity we calculate the quantity

$$T_\eta(s) = \exp \eta s\hat{A} \exp s\hat{B}\,[\exp(-s\hat{B}), \hat{A}]\exp(-s\hat{A}), \tag{7}$$

where η is a certain parameter.

In order to obtain an expression for Z'_H to an accuracy including terms of b_0^4, it is necessary to calculate in Eq. (6) the quantities W_1 and W_2, since, as is evident from Eqs. (6) and (7), $W_n \sim b_0^{2n}$:

$$Z'_H = \mathrm{Sp}\,\{e^{b_0}\cos X e^{b_0}\cos(Y + i\alpha\partial/\partial X)(1 + W_1 + W_2)\}. \tag{8}$$

Subsequent calculation of $T(s)$ and $T_\eta(s)$ is straightforward although very cumbersome. Therefore, we shall not go through the details, and will give instead the result of the calculation of Z'_H. It is simple to prove that

$$\mathrm{Sp}\, e^{b_0}\cos X e^{b_0}\cos(Y + i\alpha\partial/\partial x) = I_0^2(b_0) \tag{9}$$

for any H and b_0, and it is independent of H; the dependence on H comes about because of the non-commutability of the operators $\hat{A}$ and $\hat{B}^4$. To an accuracy of terms of the order of b_0^4:

$$\mathrm{Sp}\, e^{b_0}\cos X e^{b_0}\cos(Y + i\alpha\partial/\partial X)\, W_1 = \int_0^1 ds\, \mathrm{Sp}\, e^{b_0}\cos X \cdot T_\eta(s), \tag{10}$$

in which $\eta s = s - 1$. Noting that

$$\mathrm{Sp}\, f(X)\cos(Y + i\alpha\partial/\partial X) = \mathrm{Sp}\, f(X)\cos^3(Y + i\alpha\partial/\partial X) = \ldots = 0,$$

$$\mathrm{Sp}\, f(X)\cos^2(Y + i\alpha\partial/\partial X) = \frac{1}{2}\int_{-\pi}^{\pi} f(X)\,dX,$$

we obtain that

$$\int_0^1 ds\,\mathrm{Sp}\left(1 + b_0\cos X + \frac{b_0^2}{2}\cos^2 X\right)T_\eta(s) = \frac{b_0^4}{12}\sin^2\frac{\alpha}{2}\,. \tag{11}$$

To this degree of accuracy,

$$\mathrm{Sp}\, e^{b_0}\cos X e^{b_0}\cos\left(Y + i\alpha\frac{\partial}{\partial X}\right)W_2 = \frac{b_0^4}{8}\left(\sin^2\frac{\alpha}{2}\,\sin^2\alpha - \sin^6\frac{\alpha}{2} - \sin^2\frac{\alpha}{2}\cos^4\frac{\alpha}{2}\right). \tag{12}$$

Substituting Eqs. (11) and (12) into Eq. (8) and Z'_H of Eq. (5), we obtain:*

$$Z_H = \frac{2}{a^3}I_0(b_0)\cosh\frac{\mu_B H}{kT}\left\{I_0^2(b_0) + \frac{b_0^4}{24}\Phi(\alpha)\right\},\quad \Phi(\alpha) = \sin^2\frac{\alpha}{2}\left(2 - 3\cos^4\frac{\alpha}{2}\right) + 3\sin^2\frac{\alpha}{2}\sin^2\alpha - 3\sin^6\frac{\alpha}{2}\,. \tag{13}$$

Using the methods of contour integration[2,6] and the expression for Z_H one can obtain an expression for the density of electrons η in the impurity band:

$$n(\bar{\mu}, H, T) = n_0\left\{1 + \frac{2}{\pi}\int_0^\infty d\beta\,\frac{\sin\bar{\mu}\beta\cos\bar{\mu}_B H\beta}{\sinh\beta}\left[I_0^3(b\beta) + \frac{b^4\beta^4}{12}\Phi(\alpha)\right]\right\},\quad \Delta n \equiv n(\bar{\mu}, H, T) - n(\bar{\mu}_0, 0, T)$$
$$= \frac{2}{\pi}\int_0^\infty d\beta\left[\frac{I_0^3(b\beta)}{\sinh\beta}(\sin\bar{\mu}\beta\cos\bar{\mu}_B H\beta - \sin\bar{\mu}_0\beta) + \Phi(\alpha)\frac{b^4\beta^4\sin\bar{\mu}\beta\cos\bar{\mu}_B H\beta}{12\sinh\beta}\right], \tag{14}$$

where $\bar{\mu} = \mu/\pi kT$, $b = \Delta/\pi kT$, $\bar{\mu}_0 = \bar{\mu}\,(H = 0)$. Here μ is the chemical potential, $\bar{\mu}_B = \mu/\pi kT$, and I_0 is Bessel's Function of zero order with a real argument.

Using Z_H and the methods of contour integration, one can also obtain an expression for the magnetic susceptibility of the electrons in the impurity band:[6]

$$\chi_1 = \chi_0 + \chi_{sp} + \chi_{0sp},\ \chi_0 = -2s_0\frac{\Phi'(\alpha)}{\alpha}\sum_{i=1,2}\left(1 - \tanh^2\frac{\pi\bar{\mu}_i}{2}\right)\left(1 - 2\tanh^2\frac{\pi\bar{\mu}_i}{2}\right),\quad \chi_{sp} = \frac{n_0\mu_B}{H}\sum_{i=1,2}\tanh\frac{\pi\bar{\mu}_i}{2};$$

*The expression for $Z'_H(T)$ differs from the expression for Z_H for $H = 0$ only by terms of the order of $b_0^4 \ll 1$.

$$\chi_{0sf} = 2s_0\pi\bar{\mu}_B H_0 \sum_{i=1,2} \tanh\frac{\pi\bar{\mu}_i}{2}\left(1 - \tanh^2\frac{\pi\bar{\mu}_i}{2}\right)\left(3 - 4\tanh^2\frac{\pi\bar{\mu}_i}{2}\right),$$
$$s_0 = e^2\Delta b^3 a/768\,\hbar^2 c^2,\quad \Phi'(\alpha) \equiv d\Phi(\alpha)/d\alpha;\quad \mu_{1,2} = \mu_B H \pm \mu,\quad H_0 \equiv \hbar c/ea^2; \tag{15}$$

where the bar over $\mu_{1,2}$ means division of that quantity by πkT.

3. MAGNETIC SUSCEPTIBILITY OF A SEMICONDUCTOR WITH AN IMPURITY BAND

In what follows we shall not take into account the magnetism of atomic impurities. Let us assume that the complete magnetic susceptibility is approximately equal to the sum of the susceptibility of the impurity band χ_1 and that of the electrons in the conductivity band χ_2. An expression for χ_2 can be found for any field, provided that the electron gas in the conductivity band of the semiconductor with an impurity band is non-degenerate.

Making use of the well-known expression for the statistical sum of electrons in a field H, we obtain

$$\chi_2 = \frac{kT}{H^2} N(H, T)\left\{1 + \pi\bar{\mu}_B H \tanh\pi\bar{\mu}_B H - \frac{\hbar\omega_0}{2kT}\coth\frac{\hbar\omega_0}{2kT}\right\},$$
$$\omega_0 = \frac{eH}{m^* c},\quad N(H, T) = Z_0 \frac{\hbar\omega_0 \cosh\pi\bar{\mu}_B H}{2kT\sinh(\hbar\omega_0/2kT)}\exp\left(\pi\bar{\mu} - \frac{D_1}{kT}\right),\quad Z_0 = 2(2\pi m^* kT/h^2)^{3/2}, \tag{16}$$

where $N(H, T)$ is the equilibrium concentration of electrons in the conductivity band, μ is the chemical potential, $D_1 = 3\Delta + \Delta\epsilon$, m^* is the mean value of the effective mass of the electrons, and $\Delta\epsilon$ is the energy gap between the conductivity band and the fundamental impurity band; it is assumed that for $T \lesssim 10°$ K, $\Delta\epsilon > kT$. For larger fields, H, i.e. for $\pi\bar{\mu}_B H \gg 1$,

$$\chi_2 \approx kTH^{-2}N(H, T)\{1 + \pi\bar{\mu}_B H(1 - m/m^*)\}.$$

It is easily shown, that, in the conductivity band, electron gas is non-degenerate as was assumed in deriving the expression for χ_2. To prove this, we must calculate $\mu(H, T)$. To do this, let us make use of the electrical neutrality equation:

$$n_0\left\{1 + \frac{2}{\pi}\int_0^\infty d\beta\, \frac{\sin\bar{\mu}\beta\cos\bar{\mu}_B H\beta}{\sinh\beta}\left[J_0^3(b\beta) + \frac{b^4\beta^4}{2}\Phi(\alpha)\right]\right\} + N(H, T) = n_0. \tag{17}$$

To calculate χ_1 to an accuracy of b_0^4, it is sufficient to solve for μ from Eq. (17), neglecting corrections of the order of b_0^2. With reference to $\chi_0 + \chi_{0sp}$, further explanation is not required since $\chi_0 + \chi_{0sp} \sim b_0^4$. As regards χ_{sp}, this quantity will be of interest in what follows only when it is of the same order of magnitude as $\chi_0 + \chi_{0sp}$. This occurs, for example, for sufficiently large values of a, since $\chi_{sp} \sim 1/a^3$ and $\chi_0 + \chi_{0sp} \sim a$. Such a situation occurs in a semiconductor with an impurity band if $\eta_0 \sim 10^{15} - 10^{18}$ so that $a \sim 10^{-5} - 10^{-6}$cm.

In this case Eq. (17) becomes (for large H)

$$n_0\left(\tanh\frac{\pi\bar{\mu}_2}{2} - \tanh\frac{\pi\bar{\mu}_1}{2}\right) = N(H, T),$$

or

$$-\sinh\pi\bar{\mu}\,[\cosh\pi\bar{\mu}_B H]^{-1} = Z_0\,\frac{\hbar\omega_0\cosh\pi\bar{\mu}_B H}{2kT\sinh(\hbar\omega_0/2kT)}\,\exp\left(\pi\bar{\mu} - \frac{D_1}{kT}\right). \tag{18}$$

In this equation we have taken into account the fact that $\mu < 0$, which follows from Eq. (17). Consequently $\mu - D_1 < 0$ and therefore in the conductivity band there is no degeneracy. Because of this, one may use Eq. (16) for χ_2.

It has been shown[4] that the approximation (4) is satisfied in all cases for values of H, such that $\mu_B H < \Delta\epsilon$. Since we are considering very low temperatures $T \lesssim 10°$ K, for which $\Delta\epsilon > kT$, and large values of H, for which $\Delta\epsilon/\mu_B > H$, and $\hbar\omega_0/2 > kT$ (usually for a semiconductor of the Ge type $m^* < m$, and $\hbar\omega_0/2 > \mu_B H$) therefore, we can put $N/n_0 \ll 1$; $n_0 \gtrsim 10^{15} - 10^{16}$. Smaller values of n_0 in the presence of an impurity band are hardly worth considering. But then from Eq. (18)

$$\exp\pi\bar{\mu} \approx 1 - S\cosh\pi\bar{\mu}_B H,\quad S = \frac{1}{n_0}\left\{\frac{\hbar\omega_0}{2kT}\cosh\pi\bar{\mu}_B H\left(\sinh\frac{\hbar\omega_0}{2kT}\right)^{-1}\right\}e^{-D_1/kT} \ll 1, \tag{19}$$

i.e.,

$$\bar{\mu} \approx -(S/\pi)\cosh \pi\bar{\mu}_B H, \quad |\bar{\mu}| \ll 1, \quad N \approx n_0 S(1 - S\cosh \pi\bar{\mu}_B H).$$

Consequently, for small values of b, $\bar{\mu}$ is negative and has a small absolute value, which decreases with increasing H/T. As for the susceptibility χ_2, for large values of H, when $N \ll n_0$ and consequently Eq. (19) holds, one finds

$$\chi_2(H, T) \approx \frac{N}{\pi\bar{\mu}_B H}\left[1 + \pi\bar{\mu}_B H \tanh \pi\bar{\mu}_B H - \frac{\hbar\omega_0}{2}\coth\frac{\hbar\omega_0}{2kT}\right]\frac{\mu_B}{H},$$

i.e., $|\chi_2| \ll n_0\mu_B/H \equiv \chi_{sp}$. Thus, in deriving an expression for χ_2, one may neglect corrections to $\bar{\mu}$ of the order of b_0^2. On the basis of what has been established thus far, taking into account the monotonic decrease of $|\bar{\mu}|$ with increasing H, one can readily determine the behavior of the magnitude of χ in a changing field H. The magnetic susceptibility χ_1, consists of three parts. The first term of Eq. (15) can be interpreted basically as the orbital susceptibility of the electrons in the impurity band, χ_0. As is evident from Eq. (15), with increasing H, χ_0 decreases in magnitude and oscillates like $\Phi'(\alpha)/\alpha$ for $\alpha \gtrsim 1$, i.e., for $H \gtrsim H_0$. Incidentally, since $n_0 \sim 10^{15} - 10^{18}$ and $a \sim 10^{-5} - 10^{-6}$, it follows that $H_0 = \hbar c/ea^2$ is not very large, i.e., $H_0 \sim 10^3 - 10^5$. The second term in Eq. (15) which arises from the magnetic susceptibility of the spin of the electrons in the impurity band, monotonically decreases with increasing H. The third term χ_{0sp} arises as a consequence of the coupling between the orbital and spin magnetism. For large H the separation of χ into an orbital and spin susceptibility is ambiguous. This third term also oscillates with increasing H and its magnitude decreases, provided that $H \gtrsim H_0$. For sufficiently large H, but for $D_1 \gg \mu_B H$, χ_2 plays a smaller role than χ_1. Since χ_0 and χ_{sp} are of the same order of magnitude, it can be shown, that the complete expression for χ decreases and changes sign, i.e., it oscillates.

Note that if the impurity has an average valence ξ, such that $1 \leq \xi \leq 2$ (for T = 0 in a narrow band ξ take the place of n_0), for calculating N (H, T) and χ_2 one ought to use the Fermi distribution function for the electrons in the conduction band. This is readily done, using the methods of contour integration.[6] However, by means of such calculations one can verify that the basic conclusion concerning the oscillatory behavior of $\chi(H)$ is still valid. In fact, in case that $1 \leq \xi \leq 2$ the magnitude of $\bar{\mu}$ has an oscillatory dependence on H only in the approximation, in which χ is proportional to b_0^4 and $|\chi_2| \ll |\chi_1|$, for sufficiently large D, large $\hbar\omega_0$ and low temperatures.*

Note also, that since H_0 is not exceedingly large, then for sufficiently low T, it is also possible that $H \gtrsim H_0$, but $kT \gg \mu_B H$ and $kT \gg \hbar\omega_0$.

It should be emphasized that the principal result, i.e., the oscillation of Δn and $\chi(H)$ with increasing H, is obtained for $H_0 \lesssim H \ll H_1 \equiv \Delta\epsilon/\mu_B$. This is the basic criterion which leads to the oscillation of χ. Since $\Delta\epsilon$ and H_0 can be practically independent of T, so also the oscillation of χ, generally speaking, can be realized for $\mu_B H \gtrsim kT$ as well as for $\mu_B H \lesssim kT$, so long as $H_0 \lesssim H \ll H_1$.

For example, if $\mu_B H_0 < kT$, it is possible to have $\mu_B H > kT$, as well as $\mu_B H_0 \lesssim \mu_B H < kT$, provided that, $\mu_B H_1 > kT$. If on the other hand $\mu_B H_0 > kT$, then it must be also that $\mu_B H > kT$: for such low temperatures usually $\mu_B H_1 = \Delta\epsilon > kT$, where $\Delta\epsilon$ is the energy gap between the fundamental impurity band and the conduction band[5] (it is possible to convince oneself, that for normal conditions, in the presence of a fundamental impurity band, rather than impurity levels, excitation of the impurity band does not play a very important role).

It is equally possible for $\hbar\omega_0 > kT$ or $\hbar\omega_0 < kT$, while still being consistent with the condition $H_0 \lesssim H \ll H_1$. Note, however, that for elementary semiconductors like Ge $m^* < m$ and $\hbar\omega_0/2 > \mu_B H$. Since the change in the resistivity in a magnetic field, $\Delta\rho = \rho_H - \rho_0$ is proportional to Δn, and since Δn for $H_0 \lesssim H \ll H_1$ oscillates with increasing H, one can expect that in this case $\Delta\rho$ will also oscillate with increasing H (if $H_0 \lesssim H \ll H_1$). Therefore, if $\mu_B H \ll kT$, then

$$\Delta\rho \sim \Delta n \approx \frac{b^4}{6\pi}\int_0^\infty d\beta \frac{\beta^4}{\sinh\beta} \sin \bar{\mu}_0 \beta \Phi(\alpha).$$

*Note that for $H \ll H_0$ and $\mu_B H \ll kT$, $\chi_0 = S_0(1 - \tanh^2 \pi\bar{\mu}/2)(1 - 2\tanh^2 \pi\bar{\mu}/2)$, the magnitude of χ_0 can be[8] positive for $\bar{\mu} = 0$ (i.e., an approximately half-filled band), and it can be negative for $\bar{\mu} \to \pm\infty$ (i.e., for an almost empty, or an almost filled band: $n/n_0 \ll 1$ or $n/n_0 \approx 2$).

Thus the oscillation of χ with increasing H occurs only if $H_0 \lesssim H \ll H_1$, where H_1 is a particular value of the magnetic field, whose value is limited by the requirement that it should satisfy the approximation in Eq. (4), i.e., for $H_0 \ll H_1$.

The oscillations of $\chi(H)$ are completely distinguishable from the de Haas-van Alphen oscillations for the following reasons: (1) they do not depend on the degree of degeneracy of the electron gas, (2) the period $2H_0$ of these oscillations does not depend on H or T and their amplitude decreases with increasing T, or, more exactly, with decreasing b_0, and (3) from a calculation of the susceptibility for the special case (1) one can prove that the oscillations, by nature, occur as a consequence of the periodic dependence of $\epsilon(\mathbf{k})$ and of the non-vanishing character of the commutator $[\hat{A}, \hat{B}]$. The latter provides the basis of the assertion that in the case of narrow energy bands for any periodic dependence of $\epsilon(\mathbf{k})$, the susceptibility $\chi(H)$ will have an oscillatory part if $H_0 < H < H_1$. In effect, for any periodic behavior of $\epsilon(\mathbf{k})$, the approximate Hamiltonian for $H < H_1$, can be written as

$$\hat{\mathscr{H}} = \sum_i A_i \cos\left(\mathbf{k}\mathbf{a}_i + \frac{e}{2\hbar c}[\mathbf{H}, \mathbf{r}]\,\mathbf{a}_i\right) + s\mu_B H,$$

Here $\mathbf{r} = i\nabla_{\mathbf{k}}$ (The summation occurs for the nearest neighbor approximation, for example, in the model of tightly bound electrons; it is sufficiently good for an investigation of narrow bands).

It can be shown that

$$[(\mathbf{k}\mathbf{a}_i), ([\mathbf{H}\mathbf{r}]\,\mathbf{a}_i)] = 0;$$

where it is understood, that

$$\left[\cos\left(\mathbf{k}\mathbf{a}_i + \frac{e}{2\hbar c}\,[\mathbf{H}\mathbf{r}]\mathbf{a}_i\right),\ \cos\left(\mathbf{k}\mathbf{a}_j + \frac{e}{2\hbar c}\,[\mathbf{H}\mathbf{r}]\,\mathbf{a}_j\right)\right] \neq 0$$

for $i \neq j$. But, in agreement with item (3) above, the oscillatory terms in $\chi(H)$ are obtained by an evaluation of such commutators. Therefore in the case of a narrow band, i.e., for $|A_i/kT| < 1$, consider the case in which Z_H and χ can be evaluated by expanding them in powers of $|A_i/kT|$, and their solution can be obtained using the method of Feynman.[5] In this case, one can expect that $\bar{\mu}$ depends on H, just as before [see Eqs. (17) and (19)], and that the susceptibility χ has an oscillatory part, so that these oscillations give rise to the characteristics referred to above [items (1) and (2)]. To this same degree of precision, Eq. (18) still determines $\bar{\mu}$, even for the general case.

It must be understood, that these assertions require a more detailed investigation.

Let us return now to an examination of the oscillation of $\chi(H)$ in a semiconductor with an impurity band.

It follows from Refs. 9 and 10 that for moderate concentrations of impurities, n_0, when it is still possible to distinguish the impurity band from the conduction band, the random distribution of the impurities leads to a smearing of the edges of the impurity band. This leaves the level-distribution density $g(\epsilon)$ in most of the band nearly equal to the level-distribution density for an ordered distribution of the impurities. The impurities in semiconductors of the Ge-type are of the lattice-substitution type. Thus, while $g(\epsilon)$ has a substantial maximum near the middle of the impurity band, it becomes very small in the smeared edges. If kT is smaller than the width of the principal part of the smeared band, which forms the boundary of the impurity band, it is indeed dangerous to make use of the periodic approximation for $\epsilon(\mathbf{k})$. If, on the other hand, kT is greater than the width of the whole band, which is true for a narrow band, then the smearing of the band probably plays a small role, as a consequence of the small value of $g(\epsilon)$ in the smeared edges of the band.

To calculate the summation of states for $b_0 \ll 1$, it is therefore possibly advisable, to make use of the quantity $g(\epsilon)$, calculated for a particular periodic function, $\epsilon(\mathbf{k})$, which approximates the dispersion law in the impurity band. It is here that it seems to make sense to use the periodicity of $\epsilon(\mathbf{k})$ in the impurity band.

Conversely, if under suitable conditions (low T, $H_0 < H < H_1$, large D and small m^*) the predicted oscillations of $\chi(H)$ are observed experimentally in a semiconductor with an impurity band, this will probably allow one to conclude that $\epsilon(\mathbf{k})$ is nearly periodic. It is possible that oscillations, like those described here, will also occur in other solids, whose energy spectrum contains narrow energy bands. For this to occur, it must also be assumed that the parameter similar to a should have a value, such that $H_0 \ll H_1$. The effect will then occur if $H_0 < H < H_1$. One can anticipate that for semiconductors with impurity bands similar oscillations will also occur in some of the galvanomagnetic phenomena.

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[1] F. Seitz, and G. Castellan, Semiconductor Materials (Russ. transl.), GTTI (1954), p. 7.
[2] A. G. Samoilovich and M. I. Klinger, J. Tech. Phys. (U.S.S.R.) **25**, 1225 (1955).
[3] H. Fritzsche, Phys. Rev. **99**, 406 (1955).
[4] P. G. Harper, Proc. Phys. Soc. **A68**, 879 (1955); R. Peierls, Z. Physik. **80**, 763 (1933).
[5] R. Feynman, Phys. Rev. **84**, 108 (1951), Eqs. (16) – (18).
[6] Iu. B. Rumer, J. Exptl. Theoret. Phys. (U.S.S.R.) **18**, 832 (1948).
[7] R. H. Parmenter, Phys. Rev. **97**, 587 (1955).
[8] A. Wilson, Quantum Theory of Metals, (Russ. transl.), GTTI (1941), p. 149.
[9] E. Conwell, Phys. Rev. **103**, 51 (1956).
[10] H. M. James and A. S. Ginzburg, J. Phys. Chem. **57**, 840 (1953).

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