measurement of the curvature and does not distort the track. On the other hand, kinematic analysis excludes the possibility of interpreting the given event as the decay of a Λ^0, θ^0, or V_3^0-particle.

All this gives reason to suppose that we have observed the decay of a particle heavier than a K-meson. A detailed analysis of this case will be published later.

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APPLICATION OF THE EINSTEIN-FOKKER EQUATION TO FINDING PARTICLE LOSSES DUE TO GAS SCATTERING IN ACCELERATORS

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THE influence of multiple scattering on betatron oscillations of particles was first investigated by Blachman and Courant,[1] who used the Einstein-Fokker equation to describe the scattering process. Somewhat later, in 1951, one of the present authors[2] used the Einstein-Fokker equation in studying the influence of inelastic scattering of electrons on synchrotron oscillations. This method has also been used by other authors.[3,4]*

Let P (y, t) be the probability that the betraton or synchrotron oscillation amplitude of a particle is y at time t. Without accounting for damping of the oscillations, the Einstein-Fokker equation for P(y, t) is

$$\frac{\partial P(y,t)}{\partial t} = -\frac{\partial}{\partial y}(\overline{\Delta y}P) + \frac{1}{2}\frac{\partial^2}{\partial y^2}(\overline{\Delta y^2}P), \tag{1}$$

where

$$\overline{\Delta y^n} = \lim_{\Delta t\to 0}\frac{1}{\Delta t}\int_{-\infty}^{+\infty} Q(y, \Delta y, \Delta t)(\Delta y)^n d(\Delta y),$$

in which Q(y, Δy, Δt) is the probability that the amplitude y changes by an amount Δy in a time $\Delta t \to 0$. In our case Q is found from the interaction cross section between the accelerated particles and the remaining gas in the chamber. Furthermore, Q(y, Δy, Δt) = Q(y, Δy) Δt. As was shown by Kolmogorov,[5] Eq. (1) is valid if

$$(\overline{\Delta y})^3 \ll (\overline{\Delta y})^2, \ \overline{\Delta y}. \tag{2}$$

Equation (2) means that for large Δy, the probability Q(y, Δy) should rapidly approach zero.

Let us investigate whether this condition is fulfilled by the process of exciting betraton oscillations by elastic scattering. In this case Δy is the increase of the amplitude of free vibrations due to scattering through an angle Θ, and Q is the cross section (up to a proportionality factor) for Rutherford scattering through an angle Θ. Since this cross section is proportional to Θ^{-4}, it is clear that (2) is satisfied and the results of Blachman and Courant[1] are valid

The situation is different if Q(y, Δy) has no sharp maximum at small Δy. As an example, let us consider the excitation of synchrotron oscillations due to inelastic collisions between accelerated particles

*Note made in proof. See also D. G. Koshkareva, Приборы и техника эксперимента, (Instr. and Exptl. Tech.) 2, 15 (1957).

and the gas in the chamber. For high-energy electrons, the fundamental process leading to excitation of synchrotron oscillations is bremsstrahlung,* while for heavy particles this process is ionization loss. The change Δy of the synchrotron oscillation amplitude y is linearly related to the energy loss of a particle. In this case, Q is the cross section for collision with energy loss (to within a factor). For inelastic collisions, a particle may lose a significant fraction of its energy with measurable probability, which corresponds to large values of Δy. The maximum value of Δy which is consistent with stable acceleration is given by the equation (see, for example, Bolotovskii[2] or Rabinovich[6])

$$(\Delta y)_{\max} = \omega_0 (2 eV_0 \sin \varphi_0 K/\pi E)^{1/2}, \quad K = 1 + n/\beta^2 (1 - n), \tag{3}$$

where $eV_0 \cos \varphi_0$ is the increase in energy per revolution, E is the particle energy, ω_0 is the frequency of rotation, n is the magnetic field index, and $\beta = v/c$. A change of y by an amount $(\Delta y)_{\max}$ corresponds to an energy loss of

$$(\Delta E)_{\max} = (2 eV_0 \sin \varphi_0 E/\pi K)^{1/2}. \tag{4}$$

If the energy loss of a particle on collision is greater than this amount, the particle immediatedly stops being accelerated. The probability for such a single loss due to inelastic collision is significantly large. For instance, in the case of bremsstrahlung the energy loss is of the same order of magnitude as the energy of the particle, whereas $(\Delta E)_{\max}$ is several orders of magnitude lower.

If we are interested in the excitation of oscillations, we need account only for small losses no greater than those given by (4). Then in calculating $\overline{\Delta y}$ and $\overline{(\Delta y)^2}$ the integration should be in the interval $0 \leq \Delta y < (\Delta y)_{\max}$. Only in this case will Eq. (1) describe excitation of oscillations due to small multiple losses. Large single losses must be treated separately. Separation of losses into classes according to ΔE (such that $\Delta E < (\Delta E)_{\max}$ corresponds to multiple losses, and $\Delta E > (\Delta E)_{\max}$ corresponds to single losses) is only approximate, since strictly speaking losses for which $\Delta E \ll (\Delta E)_{\max}$ are multiple. However, the separation we are using would seem to give the correct order of magnitude for multiple and single losses.

On the basis of the above concepts, we have calculated the losses due to multiple inelastic interactions for a 250 Mev synchrotron and for a 10 Bev proton synchrotron.

Let the solution of Eq. (1) satisfy the conditions

$$P(y, 0) = P_0 = \text{const}\ (0 \leqslant y < y_{\max}); \quad P(y_{\max}, t) = 0. \tag{5}$$

The first of conditions (5) means that at the initial instant of time the amplitude distribution of particles is uniform on the phase plane, and the second that a particle whose amplitude of vibrations is greater than $y_{\max}$ stops being accelerated. Then†

$$P(y, \tau) = 2P_0 \sum_s \frac{J_0(\lambda_s y / y_{\max})}{\lambda_s J_1(\lambda_s)} \exp\left\{-\frac{\lambda_s^2}{2y_{\max}}\tau\right\}, \quad \tau = \int_0^t \overline{(\Delta y)^2}\, dt. \tag{6}$$

Here J_0 and J_1 are Bessel functions, and λ_s is the s'th root of $J_0(x)$. The quantity $\overline{(\Delta y)^2}$ is related to the mean square energy loss of a particle per unit time by the expression

$$\overline{(\Delta y)^2} = (\pi K / eV_0 \sin \varphi_0)\, \overline{(\Delta E)^2} / E; \quad \overline{(\Delta E)^2} = \int E^2 w(E)\, dE, \tag{7}$$

where $w(E)$ is the probability that the energy loss will be E per unit time. The limits of integration with respect to E are 0 and $(\Delta E)_{\max}$, the latter being given by Eq. (4). If the integral is not cut off at this upper limit, $\overline{(\Delta E)^2}$ will contain losses which lead to cases in which the particle stops being accelerated only momentarily. Therefore the final result would be incorrect.

Knowing τ, Eq. (6) can be used to find the fraction of particles lost. For instance, in a 250 Mev synchrotron (taking $y_{\max} \simeq \pi/2$, $eV_0 \cos \varphi_0 \sim 10^3$ ev, and $K \simeq 3$), $\tau_{\max}$ lies between 10^{-3} and 10^{-4} at a chamber pressure between 10^{-5} and 10^{-6} mm Hg, so that losses are negligibly small.

In the case of ionization losses in a 10 Bev proton synchrotron ($eV_0 \cos \varphi_0 \sim 10^4$ ev, $K \simeq 3$) $\tau_{\max}$ is

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*The necessity for accounting for bremsstrahlung in electron accelerators was pointed out by V. I. Veksler.

†It can be shown[2] that for synchrotron oscillations $\overline{(\Delta y)^2} = 2y\overline{\Delta y}$, which simplifies Eq. (1).

about 10^{-2} at a pressure of about 10^{-6} mm Hg, which corresponds to particle losses of several percent.

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CHARGE-EXCHANGE CROSS SECTION OF NITROGEN IONS IN GASES

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We have determined the cross sections for capture and loss of electrons by N^{+2}, N^{+3}, and N^{+4} ions in nitrogen, argon, and hydrogen. The ions were accelerated in the 72-centimeter cyclotron to energies between 1.3 and 9.7 Mev. The extracted and focused beam of ions with charge i passed through a 0.1 cm by 1 cm channel 3 cm long and entered into a cylindrical chamber about 40 cm long. The exit channel of the chamber had a cross section 0.3 cm by 1 cm and a length of 3 cm. After passing through the chamber the beam was analyzed in a magnetic field and recorded with proportional counters. The pressure was measured by an ionization manometer, which was calibrated for the different gases with an oil compression manometer. When the gas was admitted into the chamber, the pressure was increased from between 1 and 2×10^{-5} mm Hg (pressure remaining after evacuation) to between 4 and 10×10^{-4} mm Hg. When this was done, the relative numbers n_{i+1} and n_{i-1} of particles with charges $i \pm 1$ formed as a result of loss and capture of a single electron were increased from 3 — 5% to 8 — 15%, respectively. In calculating the cross sections $\sigma_{i,\,i\pm1}$ for loss and capture of an electron, we accounted both for interactions with the gas remaining after evacuation and for the small deviation from linearity in the relation between $n_{i\pm1}$ and the gas pressure. The error in the value of $\sigma_{i,\,i\pm1}$ was between 10 and 20%. We also evaluated the cross section for loss and capture of two electrons ($\sigma_{i,\,i\pm2}$). This cross section was found to be one order of magnitude smaller than the corresponding value of $\sigma_{i,\,i\pm1}$. From the values obtained for $\sigma_{i,\,i\pm1}$ and the experimental data on the equilibrium distribution of nitrogen ions in gases[1] we calculated the cross sections $\sigma_{i\pm1,\,i}$.

The results of the determination of the electron capture and loss cross sections for nitrogen ions in nitrogen are shown in the figures. These same figures give the values obtained previously.[2,3] The value of $\sigma_{i,\,i-1}$ (Fig. 1a for doubly, triply, and quadruply ionized ions varies as $v^{-k}i^{m}$, where v is the velocity of the ion. It is seen from the figure that the exponent k is close to 5, and that $m \sim 2.5$. When ions pass through argon and hydrogen, the charge dependence is the same as in nitrogen ($m \approx 2.5$), though k is somwhat larger (about 6) only in hydrogen. The absolute magnitude of $\sigma_{i,\,i-1}$ with $v \sim 6 \times 10^8$ cm/sec in argon is about 2 times greater, and in hydrogen is about 4 times less than in nitrogen. The values obtained for the capture cross section of an electron by nitrogen ions in nitrogen and in argon can be written approximately in the form

$$\sigma_{i,\,i-1} = 2\pi a_0^2 (v_0 / v)^5 i^{5/2} Z^{2/3},$$