To calculate the ratios of the probabilities, we make use of the fact that the total probabilities for transitions to states with different parities do not interfere, since their wave functions correspond to orthogonal Legendre polynomials. We then obtain

$$X = \frac{2(\rho_3^2+\sigma_3^2)+2(\rho_1^2+\sigma_1^2)-4(\rho_3\rho_1\cos(\alpha_3-\alpha_1)+\sigma_3\sigma_1)}{(\rho_3^2+\sigma_3^2)+4(\rho_1^2+\sigma_1^2)+4(\rho_3\rho_1\cos(\alpha_3-\alpha_1)+\sigma_3\sigma_1)}, \qquad Y = 3(\rho_3^2+\sigma_3^2)/[(\rho_3^2+\sigma_3^2)+2(\rho_1^2+\sigma_1^2)].$$

Introducing the notation

$$z = \sqrt{(\rho_1^2+\sigma_1^2)/(\rho_3^2+\sigma_3^2)}, \qquad \varkappa = \frac{\rho_3}{\sqrt{\rho_3^2+\sigma_3^2}}, \qquad \mu = \frac{\rho_1}{\sqrt{\rho_1^2+\sigma_1^2}},$$

we obtain

$$X = \frac{2+2z^2-4z[\varkappa\mu\cos(\alpha_3-\alpha_1)\pm\sqrt{(1-\mu^2)(1-\varkappa^2)}]}{1+4z^2+4z[\varkappa\mu\cos(\alpha_3-\alpha_1)\pm\sqrt{(1-\varkappa^2)(1-\mu^2)}]}, \quad Y = \frac{3}{1+2z^2}.$$

This expression differs from the previous one[1] in that $\cos(\alpha_3-\alpha_1)$ is replaced by

$$Q = \varkappa\mu\cos(\alpha_3-\alpha_1)\pm\sqrt{(1-\varkappa^2)(1-\mu^2)},$$

which can take on arbitrary values with $|Q| \leq 1$. It follows from this that the X, Y point lies in a region bounded by the curve

$$X = (2+2z^2\mp4z)/(1+4z^2\pm4z), \quad Y = 3/(1+2z^2),$$

which corresponds to curve (1) of the previous work.[1] This region is the same as that obtained by Gatto[7] in investigating the restrictions following only from the selection rule $\Delta T = 1/2$ without accounting for invariance under time inversion.

It is easily seen that the data of Alvarez lies in the allowed region, and thus does not contradict the assumption of the validity of the selection rule $\Delta T = 1/2$.

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A "SYMMETRIC" CIRCULAR SYNCHROCYCLOTRON WITH OPPOSITELY DIRECTED BEAMS

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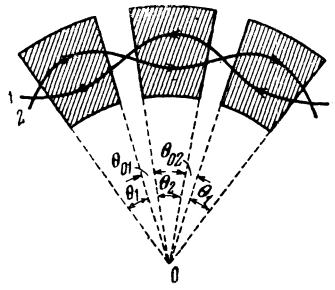
THE possibilities of using the collision of intense beams of particles accelerated to relativistic energies have recently been considered.[1] The energy used directly for a physical experiment (for instance for particle production) is then greater than that available when a beam hits a stationary target by a factor of about $2E/m_0c^2$, where E is the energy of each beam. The suggestions so far made in this regard involve the use of two adjacent or concentric annular accelerators having a common section or sections in which the collision is to occur.[1]

The present note suggests a means for achieving the collision of beams travelling in opposite direction

in a single accelerator, a "symmetric" circular synchrocyclotron. This instrument is a variation on that suggested by the author together with Petukhov and Rabinovich,[2] a strong-focusing accelerator with a magnetic field constant in time and composed of sectors with oppositely directed fields (see also Refs. 3 — 6).

Let θ_1, θ_{01}, θ_2, and θ_{02} be the azimuth angles of sectors with fields $H_1 > 0$, $H_{01} = 0$, $H_2 < 0$, and $H_{02} = 0$, which taken together add up to an element of periodicity θ_{per}. The suggested "symmetric" circular synchrocyclotron should be made of sectors θ_1 and θ_2 having a common point at the center O of the apparatus and having the same geometric and magnetic characteristics (except for the sign of the field), i.e., $\theta_1 = \theta_2$, $H_2(r) = -H_1(r)$, $H \sim r^{-n}$, and $n_0 = \text{const}$. The mean field along a circle of radius r about the point O will obviously be zero. However the closed orbits about which betatron oscillations take place are not circular, being wave-like curves which pass through the sectors with $H_1 > 0$ at higher absolute values of the field than through the sectors with $H_2 < 0$, or vice versa (see figure). Therefore the mean field along the orbit is either positive or negative, depending on the initial direction of motion. It is clear that in a "symmetric" circular synchrocyclotron the two directions of motion (clockwise and counterclockwise) are equally possible for a particle. Therefore with such an accelerator one can simultaneously accelerate particles of the same kind (for instance electrons or protons) in opposite directions. The closed orbits of these two oppositely directed beams are of the same geometric shape, but are rotated with respect to each other by an angle of $\theta_{per}/2$.



Part of a "symmetric" circular synchrocyclotron. 1 and 2 are the orbits of the oppositely directed beams (schematically drawn).

Our calculations,[6] which account for edge effects, show that with appropriate choice of the parameters (in particular when N lies between 6 and 7 times $\sqrt{|n_0|}$, where N is the number of elements of periodicity) one can satisfy the simultaneous conditions for stability of radial and vertical betatron oscillations in the "symmetric" instrument. A disadvantage of this accelerator, which is a general characteristic of circular synchrocyclotrons,[4] is the increase of the radius of the instrument compared with the radius of curvature of the orbit in the sectors (by a factor between about 5 and 6).

For acceleration in resonant (synchrotron) operation, both of the beams will obtain energy from the same rf accelerating system. The equilibrium phases of these beams, obviously, will be out of phase by an angle π, and it follows from the general theory of resonant self-phasing accelerators that such a simultaneous acceleration of oppositely directed beams will be essentially the same as that of a single beam in an ordinary synchrotron or synchrocyclotron. When the particles attain the necessary relativistic energy, the accelerating voltage is turned off, and the free beams continue to rotate in opposite directions undergoing head-on collisions for a time determined by scattering on whatever gas remains in the chamber of the accelerator, and by similar phenomena. The accelerating cycle can then be repeated.

We note that the use of resonant operation makes it possible, in principle, to collect particles in beams of the required orbit during several accelerating cycles. This makes it possible to increase the efficiency of the method of colliding beams.

The case of induction (betatron) acceleration as applied to "symmetric" equipment differs from the resonance method. In each half period of fluctuation of the betatron magnetic flux, the beam moving in one direction accelerates, and that moving in the other direction decelerates. One can, true, obtain in this case head-on collisions of the beams, one of which is accelerated and the other decelerated in a given cycle, the latter having been accelerated in the previous one, but the efficiency would then seem to be much less than in the case of resonance acceleration. Nevertheless, betatron, operation in a "symmetric" circular synchrocyclotron may also, in principal, be of definite interest.

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SECOND ORDER PHASE TRANSITION IN SODIUM NITRATE

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IN the theory of phase transitions[1] a body is usually considered isotropic with respect to its elastic properties, and is characterized by a single modulus of compressibility. But the behavior of the elastic constants of a monocrystal may exhibit various properties in a second order phase transition which are strongly related to specific types of structural changes of the crystal lattice at the transition point. It would hardly be reasonable to investigate this problem in general, in view of the very large number of possibilities that may arise.

We here consider a second order phase transition in sodium nitrate. Kornfel'd and Chudinov[2] have recently measured the temperature dependence of the elastic constants of this substance in the neighborhood of the transition point.

The $NaNO_3$ crystal is rhombohedral. Below the transition point its elementary cell contains two molecules, and the NO_3 groups have two different crystallographic orientations (symmetry group D_{3d}^6; see Wyckoff's[3] description). Above the transition point there is no difference between the NO_3 groups, each of which can have one of two possible orientations with equal probability.[4] This reduces the elementary cell by a factor of two (symmetry group D_{3d}^5). Thus the transition is related to ordering the NO_3 groups.

In the present case the density function $\rho(x, y, z)$, which enters into the general theory of second order phase transitions, can be thought of as the density distribution of oxygen atoms. Using the general methods[1,5] one can show that the change $\delta\rho(x, y, z)$ of the density function corresponding to the given transition has the same symmetry as the function $\sin \pi(x + y + z)$, where x, y, and z are the coordinates relative to the axes of the rhombohedral cell. Therefore the transition being considered in sodium nitrate is described by a single parameter η which transforms as the functions $\sin \pi(x + y + z)$ under all transformations of the symmetry group D_{3d}^5 of the high-temperature phase (including translations).*

From this it follows immediatedly that in the series expansion for the thermodynamic potential there will be no term proportional to η^3, so that the transition may indeed take place as a second order transition.

To determine the change of the elastic constants at the transition point, we write the thermodynamic potential in the neighborhood of this point in the form

$$\Phi = \Phi_0(T) + A(T - T_c)\eta^2 + {}^1/_2 B\eta^4 + a\eta^2(\sigma_{xx} + \sigma_{yy}) + b\eta^2\sigma_{zz} - \alpha_1(T - T_c)(\sigma_{xx} + \sigma_{yy}) - \alpha_3(T - T_c)\sigma_{zz}$$
$$- {}^1/_2 s_{11}(\sigma_{xx}^2 + \sigma_{yy}^2 + 2\sigma_{xy}^2) - {}^1/_2 s_{33}\sigma_{zz} - {}^1/_2 s_{44}(\sigma_{xz}^2 + \sigma_{yz}^2) - s_{12}(\sigma_{xx}\sigma_{yy} - \sigma_{xy}^2) - s_{13}(\sigma_{xx} + \sigma_{yy})\sigma_{zz}$$
$$- s_{14}[(\sigma_{xx} - \sigma_{yy})\sigma_{zz} - 2\sigma_{zx}\sigma_{xy}],$$

where σ_{lk} is the elastic strain tensor, and T_c is the transition temperature in the absence of strain; we have here accounted for the rhombohedral symmetry of the elastic properties of the crystal, and the specific symmetry of the parameter η. The phase transition occurs at the point at which the coefficient of η^2 vanishes, i.e., at the temperature

$$T_c' = T_c - (a/A)(\sigma_{xx} + \sigma_{yy}) - b\sigma_{zz}/A.$$

*The parameter η may be defined as $(w_1 - w_2)/(w_1 + w_2)$, where w_1 and w_2 are the probabilities for the two orientations of the NO_3 group at some lattice site.