$$\Delta\sigma^{\alpha\beta} = -\frac{H}{(1+\mu)\zeta_0}\left\{\left[\mu\left(\overline{q_1^{\alpha\beta}} - \overline{q_2^{\alpha\beta}}\right) + \left(\mu + \frac{1}{3}\right)\zeta_0\overline{\frac{\partial q_1^{\alpha\beta}}{\partial\zeta_0}} + \frac{2}{3}\zeta_0\overline{\frac{\partial q_2^{\beta\alpha}}{\partial\zeta_0}}\right]\Delta M_1^z\right.$$
$$\left. + \left[\mu\left(\overline{q_1^{\alpha\beta}} - \overline{q_2^{\alpha\beta}}\right) + \frac{2}{3}\mu\zeta_0\overline{\frac{\partial q_1^{\alpha\beta}}{\partial\zeta_0}} - \left(\frac{5}{3}\mu + 1\right)\zeta_0\overline{\frac{\partial q_2^{\alpha\beta}}{\partial\zeta_0}}\right]\Delta M_2^z\right\}, \qquad (50)$$

for $\alpha, \beta \neq z,z$ and

$$\Delta\sigma^{zz} = -\frac{H}{(1+\mu)\zeta_0}\left\{\left[\mu\left(\overline{q_1^{zz}} - \overline{q_2^{zz}}\right) - \frac{2}{3}\zeta_0\left(\overline{\frac{\partial q_1^{zz}}{\partial\zeta_0}} - \overline{\frac{\partial q_2^{zz}}{\partial\zeta_0}}\right)\right]\Delta M_1^z + \left[\mu\left(\overline{q_1^{zz}} - \overline{q_2^{zz}}\right) + \frac{2}{3}\mu\zeta_0\left(\overline{\frac{\partial q_1^{zz}}{\partial\zeta_0}} - \overline{\frac{\partial q_2^{zz}}{\partial\zeta_0}}\right)\right]\Delta M_2^z\right\}, \qquad (51)$$

where $\mu = m_2/m_1$, and the mobilities are determined from Eqs. (44) in which one substitutes in the case of the electrons $m^* = m_1$, and for the holes, $m^* = -m_2$.

For the extreme cases $\mu \gg 1$ or $\mu \ll 1$, one can leave out some of the terms in Eqs. (50) and (51); however, the general character of the expressions does not change.

Eqs. (50) and (51) show that the assumption $N^+ = N^-$ does not introduce any fundamental simplification of the general expressions for $\Delta\sigma^{\alpha\beta}$.

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ON THE BRIGHTNESS OF STRONG SHOCK WAVES IN AIR

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The emission and absorption of light at high temperatures in a shock wave front in air are considered. The dependence of the brightness of the shock wave front on its amplitude is derived.

IN our preceding article[1]* we considered in a general form the problem of the internal structure of the front of strong shock waves in gases, taking account of radiation. We operated throughout with integral characteristics of the radiation — the total energy flux and density. Also, passing from the geometrical coordinate to the optical thickness, we excluded from consideration the actual distribution of quantities in space, which is determined by the coefficient of absorption of light in the gas. This approach is in-

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*Hereafter referred to as I.

adequate when we are interested in the flux of visible radiation passing from the wave front to infinity (brightness of the wave front), the measurement of which is one of the important experimental methods for the study of shock waves and the determination of their amplitudes.

The brightness depends in an essential way on the coefficient of absorption of visible light at various points of the wave front, and on the distribution of the light sources, i.e., the temperature distribution in the wave.

In I, temperature profiles were obtained as a function of the mean over the spectrum of the optical thickness. Therefore, to calculate the brightness it is necessary to find the relation between the coefficient of absorption of visible light and the mean over the spectrum. The latter is determined by the character of the radiation spectrum involved in the transfer of energy in the wave, which in turn depends on the coefficients of absorption of the various frequencies at different temperatures.

These questions are considered in the present article as applied to air. The results will allow us to draw conclusions regarding the general character of the relation between the brightness or effective wave temperature and the temperature behind the shock wave front.

Let us recall that by effective (visible) temperature we mean the temperature that a black body would have to have in order to give exactly the same flux of visible light as passes from the shock wave front to infinity.

1. COEFFICIENT OF ABSORPTION OF LIGHT BY AIR

We begin with low temperatures. Cold air, as is well known, is transparent to visible light. Strong molecular absorption begins in the ultraviolet region; the absorption coefficient reaches the value $\kappa_\nu \approx 100$ cm^{-1} for h$\nu \approx$ 8ev * at normal density.[2]

Quanta with energy greater than the ionization potentials of oxygen and nitrogen molecules, I_{O_2} = 12.5 ev, I_{N_2} = 15.5 ev (for atoms, I_O = 13.5 ev, I_N = 14.5 ev), undergo strong photoelectric absorption. The effective absorption crossections from ground levels depend only slightly on frequency in the interval I $\approx$ hν to hν = 25 ev, and are $\sigma_O = 3 \times 10^{-18}$ cm^2, $\sigma_N = 5 + 10^{-18}$ cm^2 per atom,[3] which gives κ_ν = 120 cm. As the frequency increases, the absorption coefficient goes through jumps corresponding to the successive inclusion in the absorption of various electrons filling the L-shells of oxygen and nitrogen atoms. The L-shell levels do not differ too strongly from one another, so that the jumps apparently lie in the frequency region from hν = 13 up to h$\nu \lessapprox$ 30 ev. At higher frequencies the absorption decreases monotonically up to quanta with energy hν = 410 ev, equal to the binding energy of K-electrons in nitrogen (530 ev in oxygen). Experimental data on the absorption of intermediate frequencies between the far ultraviolet and soft x-rays is extremely scanty. Combining the various data, we may set up Table I as an approximate representation of the absorption coefficient in this region.

TABLE I

hν (ev)	κ_ν (cm^{-1})	Ref.
8	100	2
13-25	120	3
182	12	4
280	5.3	5
410	1.6	6
415	35	7

At not too high temperatures, lower than $\approx$ 30,000°, when the degree of ionization is not great, quanta with energies less than the ionization potentials are absorbed principally by photoeffect with excited levels of neutral atoms (and molecules) of oxygen and nitrogen. For an estimate of this absorption we may, as is commonly done in astrophysics, use Kramer's formula, derived for hydrogenlike atoms and ions. The formula for κ_ν, integrated over the levels taking account of their degree of excitation, has the form:[8]

$$\varkappa_\nu = \frac{16\pi^2}{3\sqrt{3}} \frac{e^6}{hc} \frac{kT}{(h\nu)^3} e^{h\nu/kT} \sum_i Z_i^2 n_i e^{-I_i/kT}. \tag{1}$$

The summation is carried out over all the types of atoms and molecules (n_i is the number of these particles per cubic centimeter). Z_i is the effective charge of the atomic residue, which according to Ref. 8 should be set equal to $Z_i = 2.5$.†

The temperature dependence is apparently represented correctly by the Boltzmann exponential factor. Taking account of the equilibrium composition of air at the various temperatures, we may use this formula to calculate the absorption coefficient for red light, hν = 1.91 ev (λ = 6500 A), which is com-

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*It will be convenient in what follows to express the frequency in electron volts.

†Model''s data[9] are in satisfactory agreement with this value.

monly used to photograph shock waves (see Table II; concerning the last two values in this table, see below). κ_ν depends extremely sharply on temperature because of the high ionization potentials.

TABLE II

$10^{-3} \cdot T$	11	12	15	17	20	30	50	100
$\varkappa_\nu$	0.09	0.31	4.1	13.5	60	290	350	2000

The absorption coefficient (1) considered as a function of frequency at a given temperature has a minimum at $h\nu = 3kT$. Eq. (1) may be used only for values of $h\nu$ sufficiently small with respect to unity, when the absorption occurs in highly excited levels which may to some extent be considered "hydrogenlike." Both Kramer's formula and the replacement of summation over levels by integration become inapplicable at $h\nu \approx I$.

A rough estimate of the absorption at $h\nu = I$ may be obtained from Kramer's fromula for the absorption in the ground level of a hydrogenlike atom,[8] which for air gives

$$\varkappa_\nu = 430 Z^{-2} (I/h\nu)^3 \text{ cm}^{-1}. \tag{2}$$

At $h\nu = I$, Eq. (2) gives the correct order of magnitude ($\kappa_\nu \approx 100 \text{ cm}^{-1}$) with $Z = 2$. The absorption coefficient has a maximum at $h\nu = I$.

Absorption at high frequencies, $h\nu \gg I$, and not too high temperatures, when air consists principally of neutral atoms, takes place in the same way as in cold air, i.e., according to Table I.

At high temperatures the atoms are strongly ionized, and as a rule, at every temperature air consists principally of ions of two multiplicities: at T = 50,000° singly and doubly ionized, at T = 100,000° doubly and triply ionized, etc. The absorption of quanta with energies less than the lowest ionization potential of the ions present in sufficient quantity* may be estimated by a formula of type (1). The last three numbers in Table II were obtained in this way.

The maximum absorption, corresponding to $h\nu = I_{\min}$, may be estimated by Eq. (2), setting Z equal to the charge of the ion. The magnitude of the maximum decreases with Z, i.e., with increased temperature, and is of the order $\kappa_\nu \sim 100$ to 10 cm^{-1}. Then, with further increase of frequency, there are successive maxima corresponding to the inclusion in the absorption of different ions (more exactly, different levels of different ions), and at sufficiently high frequencies, κ_ν decreases monotonically with increasing frequency up to the K-jump.

This is the general picture of the absorption of light in air. The state of theory in connection with this question is such that it is very difficult to go further than crude estimates of order of magnitude and the establishment of qualitative relations, which naturally shows in the character of the conclusions regarding the spectrun and effective temperature. The qualitative dependence of the absorption coefficient on frequency at a given temperature is illustrated by Fig. 1.

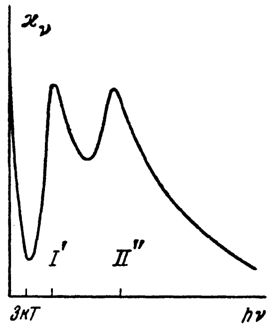

FIG. 1

2. THE CHARACTER OF THE RADIATION SPECTRUM IN THE HEATING ZONE AND THE ABSORPTION COEFFICIENT AVERAGED OVER THE SPECTRUM

The radiation emanating from the surface of the discontinuity of a strong shock wave is absorbed in the gas layers lying in front of it and heats them.

In I, temperature profiles were found in the heating zone, assuming constant heat capacity. In air as well as in other gases, the heat capacity at high temperatures depends on temperature because of ionization processes. In this regard, all the relations in I concerning the heating zone must be generalized somewhat. Thus, the condition that the flux of radiant energy and the hydrodynamic flux are equal (Eq. 12 in I) is now written in the form:

$$S = D\rho_0 \varepsilon(T). \tag{3}$$

Keeping in mind that the compression in the heating zone is small, and that the specific internal energy depends only weakly on density, we may take $\epsilon(T)$ at normal air density in this equation.

The temperature profile in the diffusion approximation is determined by the formula

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*At T < 30,000°: $I_{\min} = I_0 \sim 13$ ev, At T ~ 50,000°: $I_{\min} = I_1 \sim 30$ ev,
At T ~ 100,000°: $I_{\min} = I_2 \sim 50$ ev, At T ~ 150,000°: $I_{\min} = I_3 \sim 75$ ev.

$$\varepsilon(T) = \varepsilon(T_{-}) \exp(-\sqrt{3}\tau) \quad \tau > 0, \tag{4}$$

similar to Eq. (15) in I.

The greatest heating temperature T_{-} in front of the shock discontinuity is found from the equation

$$\sigma T_1^4 = D\rho_0 \varepsilon(T_{-}), \tag{5}$$

in which it is necessary to substitute the actual function $D(T_1)$ for air. Values of T_{-} as a function of T_1, the temperature behind the front, were calculated with this formula and are given in Table III.

TABLE III

$10^{-3}T_1^{\circ}$	$D\left(\frac{\text{km}}{\text{sec}}\right)$	$\varepsilon(T_{-})\left(\frac{\text{ev}}{\text{molec.}}\right)$	$10^{-3}T_{-}^{\circ}$
285	88.1	1020	285
275	86.2	910	240
250	81.6	635	175
150	56.5	122	60
100	40.6	32.7	25
75	32.1	13.1	12
65	28.5	8.4	9
50	23.3	3.7	4

We used Selivanov's[10] data with regard to the calculation of thermodynamic functions and the shock adiabatic of air, taking account of ionization.* It is clear from the table that the temperature behind the front at which the quantities T_{-} and T_1 are comparable, and which is the characteristic temperature for the problem of the structure of a shock wave front, is of the order $T_k \approx 285{,}000°$ in air.

We will now estimate the mean over the spectrum of κ, the absorption coefficient of light in the heating zone.

Our problem is the calculation of the effective temperature of the wave front. It will be shown in the next section that at a temperature $T_1 < 90{,}000°$ behind the front, the effective temperature is practically equal to the temperature behind the front; therefore, we will consider only stronger waves.

It follows from Sec. 1 that the absorption coefficient for the various frequencies at the various temperatures lies, as a rule, between 100 and 10 cm^{-1}. The exceptions consist only of sufficiently low frequencies having insignificant energy.

Consequently, the mean coefficient also has the same order of magnitude. For example, at $T_1 = 100{,}000°$ the maximum of the Planck spectrum (as a function of frequency) occurs at $h\nu = 24$ ev. Here, the greatest heating temperature is $T_{-} = 25{,}000°$. At temperatures less than 25,000°, quanta greater than 13 ev are absorbed in the same way as in cold air. Quanta lower than 13 ev carry little energy. Thus, 85% of the total spectral energy is included in the interval from 13 to 150 ev.

Quanta with lower energy are absorbed more strongly. Therefore, as we go away from the surface of the discontinuity, the spectrum becomes harder and harder and the mean absorption coefficient decreases, varying within the limits from 100 to 10 cm^{-1}.

As the temperature behind the front is raised, the picture does not change in general; it only becomes more complex because the temperature in the heating zone becomes high, and the "leading" frequencies, i.e., the frequencies which carry the principal energy (so to say, "lead" the heating) and to which the mean coefficient κ corresponds, lie in the region of absorption jumps. The order of κ remains as previously: $\kappa \sim 100$ to 10 cm^{-1}.

The picture again becomes clear at sufficiently high temperatures behind the front $T_1 > T_k$ ("isothermal jump") or in the case of a thermal wave.

In both cases there is a region in the wave in which radiation is in local equilibrium with matter. The temperature separating the equilibrium and nonequilibrium regions is $T_k \approx 285{,}000°$.

We shall consider in detail the case of very strong waves, because it is a limiting case with respect to the effective temperature.

At temperatures above $\approx 285{,}000°$ the radiation is in equilibrium with matter; therefore, we may approximately consider that a Planck spectrum of temperature T_k leaves the surface where $T = T_k$, no matter how high the temperature is behind this surface. As we go away from the point at which $T = T_k$ in the direction of lower temperatures, the radiation density exceeds equilibrium at the given temperature to a greater and greater extent. Using Eqs. (12), (16), and (32) of I, and the proportionality $\epsilon \sim T^{1.4}$, we obtain

$$U(T) / U_{\text{r}}(T) = U_{\text{r}}(T_k)\, \varepsilon(T) / U_{\text{r}}(T)\, \varepsilon(T_k) = (T_k/T)^{2.6}. \tag{6}$$

The general tendency of absorption is such that κ decreases with increasing frequency. At $T = 285{,}000°$ the Planck maximum corresponds to $h\nu = 70$ ev, and the principal energy of the primary spec-

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*Let us note that in the temperature interval from 10,000° to 300,000°, $\epsilon(T)$ may be approximated by the power law $\epsilon \sim T^{1.4}$.

trum lies in a region in which κ decreases more or less monotonically with ν. In order to estimate the "leading" frequency ν_m at each point of the wave, we may schematize the spectrum as follows: we will assume that frequencies $\nu < \nu_m$ which are absorbed more strongly than ν_m are in local equilibrium, and that frequencies $\nu > \nu_m$ are distributed in the same way as in the initial spectrum, i.e., arrive at the point x without having undergone absorption along the way from the point at which $T = T_k$. This becomes all the more valid as with increasing x the leading role moves to higher frequencies, so that for a given frequency ν, the ratio $\nu/\nu_m(x)$ increases with decreasing x.

At not too high temperatures, according to Eq. (6), the equilibrium part of the spectrum carries negligible energy. Therefore, we may set

$$U_\nu(x) = \begin{cases} 0 & \nu < \nu_m \\ U_{\nu r}(T_k) & \nu > \nu_m. \end{cases} \tag{7}$$

The "leading" frequency is determined by the formula

$$U(x) = \int_{\nu_m}^{\infty} U_{\nu r}(T_k)\,d\nu = U_r(T_k)\frac{\varepsilon(T)}{\varepsilon(T_k)}. \tag{8}$$

Some calculated values of $\nu_m(T)$ are given in the second column of Table IV. They lie in the sufficiently far Wien region where the spectrum drops rapidly; therefore, the average absorption coefficient corresponds simply to ν_m.

Keeping in mind that at temperatures lower than ≈30,000° energetic quanta are absorbed in the same way as in cold air, we may use the data of Table I to associate a mean absorption coefficient with each temperature at the lower boundary of the wave (third column of Table IV).

3. THE EFFECTIVE TEMPERATURE OF A STRONG SHOCK WAVE AND OF THERMAL WAVES

According to the general formula (26) in I, the flux of visible light passing from the shock wave front to infinity is

$$S_\nu = \frac{c}{2}\int_{-\infty}^{\tau_\nu} U_{\nu r}\,E_2(\tau_\nu - \tau_\nu')\,d\tau_\nu', \tag{9}$$

where τ_ν is the total optical thickness of the heated layer for the frequency ν, corresponding to $x = +\infty$. In distinction from the mean optical thickness τ, equal to $+\infty$ at $x = +\infty$, τ_ν for visible light is finite, thanks to the condition $S = 0$, because the absorption coefficient for visible light drops extremely rapidly with decreasing temperature. Since the mean coefficient κ depends only weakly on temperature, the temperature T^* at which

$$\varkappa_\nu(T^*) = \varkappa(T^*), \tag{10}$$

may be called the boundary of transparency of air in the shock wave front in the sense that for $T < T^*$, air is practically transparent to visible light. A simple calculation shows that the optical thickness of the heated layer from $x = +\infty$ to the point $x = x^*$ at which $T = T^*$ (when $T_- \geq T^*$) is very small; $\Delta\tau_{\nu\max} \approx 0.1$. Therefore, the total optical thickness τ_ν is practically equal to the optical thickness of the layer $0 < x < x^*$. When $T_- < T^*$, the optical thickness of the entire heated layer $\Delta\tau_\nu \ll \Delta\tau_{\nu\max}$.

It is clear from Table II that $T^* \approx 20{,}000$–$17{,}000°$ when $\kappa \sim 100$ to $10\ \mathrm{cm}^{-1}$. When the temperature behind the shock wave front is lower than $T_1^* \approx 90{,}000°$, the maximum heating temperature $T_- < T^*$ as follows from Table III, and the heated layer is transparent to visible radiation. Since under these conditions black body radiation near the temperature T_1 leaves the surface of the discontinuity, the effective temperature T_{eff} of the visible radiation will almost coincide with T_1, the temperature behind the front.

As the shock wave amplitude is raised, for $T_1 > T_1^*$, the greatest temperature T_- in front of the discontinuity becomes higher than the transparency temperature, and an absorbing layer $0 < x < x^*$ with temperatures $T_- > T > T^*$ appears in the heating zone, which strongly screens the visible radiation leaving the surface of the discontinuity.

Let $B_\nu(T)$ denote the flux of Planck radiation of frequency ν leaving the surface of a black body at the temperature T:

$$B_\nu(T) = (2\pi h/c^2)(e^{h\nu/kT} - 1)^{-1}. \tag{11}$$

Than the effective temperature is determined by the expression*

$$B_\nu(T_{\text{eff}}) = 2\int_{-\infty}^{\tau_\nu} B_\nu(T)\,E_2(\tau_\nu - \tau_\nu')\,d\tau_\nu'. \tag{12}$$

We will separate the integral into two parts. The first of them, from $-\infty$ to 0, corresponds to the part of the flux going toward infinity which came from the surface of the discontinuity and was partially screened by the heated layer. The second part, from 0 to τ_ν, corresponds to the visible light generated in the heated layer itself. Using the well known properties of integral exponents, and also the fact that the temperature behind the discontinuity is very close to T_1, we find that the first part is approximately

$$2\int_{-\infty}^{0} B_\nu(T)\,E_2(\tau_\nu - \tau_\nu')\,d\tau_\nu' \approx B_\nu(T_1)\cdot 2E_3(\tau_\nu). \tag{13}$$

To calculate the second integral over the heating zone we note that the radiating capacity which is proportional to the Planck function $B_\nu(T)$, does not depend very strongly on the coordinate, while the screening factor E_2 decreases rapidly as its argument increases. As we advance through the heating zone in the direction of increasing temperature, beginning at the point x^*, the absorption coefficient of visible light increases rapidly, but the mean over the spectrum changes only slightly. Therefore, the radiating layer, i.e., the layer which is the principal contributor to the integral and has a thickness with respect to τ_ν of the order of several units, has a comparatively small optical thickness averaged over the spectrum.

Consequently, the temperature therein varies only slightly [see Eq. (4)], and we may simply set $T \approx \text{const} = T^*$ in the integral. Then

$$2\int_{0}^{\tau_\nu} B_\nu(T)\,E_2(\tau_\nu - \tau_\nu')\,d\tau_\nu' \approx B_\nu(T^*)\,[1 - 2E_3(\tau_\nu)]; \tag{14}$$

$$B_\nu(T_{\text{eff}}) = B_\nu(T_1)\cdot 2E_3(\tau_\nu) + B_\nu(T^*)\,[1 - 2E_3(\tau_\nu)]. \tag{15}$$

A rough estimate of the total optical thickness of the heating zone for visible radiation may be obtained by assuming that the absorption coefficient κ_ν depends on temperature through Boltzmann's law $\kappa_\nu \sim \exp(-I/kT)$, but that κ does not generally depend on temeperature. Taking Eq. (10) into account and changing from x to a new independent variable, the temperature in the integral for τ_ν, we find using Eq. (4) and the proportionality $\epsilon \sim T^{1.4}$,

$$\tau_\nu = \int_0^\infty \varkappa_\nu\,dx \approx \int_0^{x^*} \varkappa_\nu\,dx = \int_{T^*}^{T_-} \exp\left(-\frac{1}{kT} + \frac{1}{kT^*}\right)\frac{1{,}4}{\sqrt{3}}\frac{dT}{T} \approx \frac{1{,}4}{\sqrt{3}}\frac{kT_-}{I}\left[\exp\left(\frac{I}{kT^*} - \frac{I}{kT_-}\right) - 1\right]. \tag{16}$$

Now, using Eq. (15), it is easy to see how the effective temperature varies as the amplitude of the wave increases. For $T_1 < T_1^* \approx 90{,}000°$, $\tau_\nu \approx 0$ and $2E_3(\tau_\nu) \approx 1$, i.e., $T_{\text{eff}} = T_1$. When $T_1 > T_1^*$, τ_ν increases very rapidly with increasing T_1 and the first term in Eq. (15) tends rapidly toward zero because of the screening, in spite of the increase of the flux $B_\nu(T_1)$. Even at $T_1 \approx 110{,}000°$, $T_- = 30{,}000°$: $\tau_\nu = 2.2$ and the first term is less than the second. Since under these circumstances the coefficient is practically equal to unity for $B_\nu(T^*)$, the effective temperature simply coincides with the transparancy temperature T^*. Continuing, $T_{\text{eff}} \approx T^*$ as the shock wave amplitude increases without limit, as well as in the case of a thermal wave. Since the mean absorption coefficient decreases somewhat as the wave amplitude increases, T^* drops slightly from $\approx 20{,}000°$ to $\approx 17{,}000°$.

In the limit of a sufficiently strong wave, $\kappa \approx 10\ \text{cm}^{-1}$ according to Table IV, and the effective tem-

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*Its meaning becomes clear when we remember that

$$2\int_0^\infty E_2(z)\,dz = 1.$$

TABLE IV

$10^{-3}\cdot T^\circ$	$h\nu_m$ (ev)	$\varkappa$ (cm^{-1})
50	140	
20	200	10.5
15	212	9.8
10	225	8.6

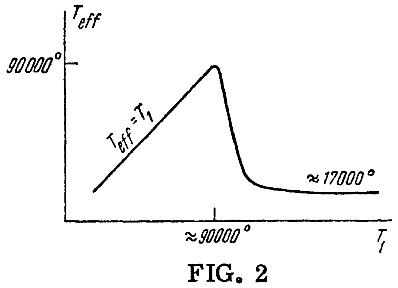

FIG. 2

perature, equal to the transparency temperature, is ≈17,000°. It should be noted that in spite of our extremely poor knowledge regarding the absolute magnitudes of $\kappa_\nu(T)$ and κ, the above value of the effective temperature is fairly accurate because of the sharp dependence of κ_ν on temperature.

A curve of $T_{eff}(T_1)$ is given in Fig. (2).

I wish to express profound thanks to Ia. B. Zel'dovich for his interest in this work, and for valuable discussions and remarks.

APPENDIX

THE ROLE OF ELECTRONIC HEAT CONDUCTIVITY IN THE ESTABLISHMENT OF THE TEMPERATURE PROFILE IN THE HEATING ZONE

At high temperatures, air is strongly ionized, and we should estimate whether the flux of electronic thermal conductivity is comparable to the energy flux transferred by radiation, which generally speaking may lead to some spreading of the temperature profile in the shock wave as compared with the profile accounting for radiant heat exchange alone.

Electrons in a plasma undergo elastic collisions with electrons, ions and neutral atoms. Sufficiently energetic electrons may excite and ionize atoms (and ions). The mean free path for elastic Coulomb scattering of electrons in a plasma may be estimated by Landau's formula,[11] which for air may be written in the form

$$l_e \approx \frac{2\cdot 10^{-7}}{Z(T)}\left(\frac{T^\circ}{10^4}\right)^2 \text{cm}, \qquad (17)$$

where Z(T) is the mean number of free electrons per atom. The coefficient of electronic thermal conduction is then

$$\chi_e = \frac{4{,}5}{Z(T)}\left(\frac{T^\circ}{10^4}\right)^{5/2} \frac{\text{cm}^2}{\text{sec}}, \qquad (18)$$

where ℓ_e is essentially the distance required to establish a Maxwellian distribution in the electron gas. It is very small: at $T = 10^{5\circ}$, $\ell_e \approx 10^{-5}$ cm.

The elastic mean free path of very fast electrons, which are formed as a result of photoelectric absorption of energetic quanta, may be estimated by Eq. (17), if we replace the temperature therein by the electron energy. Thus, for E ~ 100 ev, we obtain $\ell \sim 2 \times 10^{-3}$ cm.

However, such energetic electrons quickly lose energy by ionization and excitation of atoms (the cross section is ~ 10^{-16} to 10^{-17} cm^2, the free path is $\ell_{\text{inelas.}} \sim 2 \times 10^{-4}$ to 2×10^{-3} cm). The inelastic free path is comparable with the elastic free path at E ~ 70 ev.

At low temperatures, where ionization is small, the electrons do not have the ability to ionize (E > 13 ev), and lose energy through elastic collisions with atoms; $\sigma \sim 10^{-16}$ cm^2, $\ell \sim 2 \times 10^{-4}$ cm. In order to become thermalized, they must undergo $N \sim 3 \times 10^4$ collisions, i.e., the thermalization length is $\sim\sqrt{\ell^2 N} \sim 4 \times 10^{-2}$ cm. All of these free paths are less than the free path for the absorption of radiation, which, as was shown in Secs. 1 and 2, is of the order of 10^{-2} to 10^{-1} cm. Therefore, we may assume that the energy of absorbed quanta is transformed into heat at the point of absorption. This energy is transmitted directly to electrons, which then exchange it with ions considerably more slowly than the time required to establish Maxwellian distributions in the ion and electron gases. The exchange time may be estimated by Landau's formula.[11] The characteristic exchange length (the time multiplied by the velocity of the front) is

$$l_{\text{exch}} \sim 10^{-3}\,(10^{-4}\,T_e^\circ)^{3/2}\ \text{cm}.$$

In the heating zone, the difference between the electronic and the ionic temperatures is not great.

It is easy to estimate the flux of electronic thermal conductivity assuming approximately that the electronic and ionic temperatures coincide,

$$S_e = -\chi_e \rho_0 d\varepsilon/dx.$$

Let us compare this with the flux of radiant energy, Eq. (3), using Eq. (4) to obtain the gradient of the internal energy. We obtain

$$S_e / S_{\mathrm{rad}} = (l_e / l_{\mathrm{rad}})\, v_e / D,$$

where v_e is the thermal velocity of the electrons.

Calculation shows that the fluxes are comparable only at very high temperatures ~300,000°. At lower temperatures: $S_e \ll S_{\mathrm{rad}}$.

The difference between the electronic and the ionic temperature has an essential effect only on the structure of the temperature peak behind the discontinuity, but this in no way influences the behavior of the effective temperature of the wave.

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DISPERSION OF SOUND IN A FERMI LIQUID

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On the basis of the theory proposed by Landau for a Fermi liquid the attenuation and dispersion of sound oscillations in such a liquid are investigated. Specific calculations are performed for the case of liquid He^3.

THE characteristics of sound in a Fermi liquid are determined by the kinetic equation for the excitations, which, according to Landau,[1] has the form

$$\frac{\partial n}{\partial t} + \frac{\partial n}{\partial \mathbf{r}} \frac{\partial \varepsilon}{\partial \mathbf{p}} - \frac{\partial n}{\partial \mathbf{p}} \frac{\partial \varepsilon}{\partial \mathbf{r}} = I(n). \tag{1}$$