Investigations of the Equation of State by Mechanical Measurements

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RECENT YEARS have seen successful investigations of the properties of matter at high pressures obtained by explosive means. By this procedure pressures of the order of a million atmospheres are readily attained. However, a distinctive feature of the explosive method is the short duration of the high pressure. Because of this the measurements are limited to determining the velocity of the shock wave (D), the rate of displacement (u) of the material compressed by the shock wave, or the density attained in compression ($\rho = 1/v$, where v is the specific volume). These quantities are related by the expression $D\rho_0 = (D - u)\rho$. In addition to the measurements pertaining to the material compressed by the shock wave, it is also feasible to make certain measurements in the course of the subsequent adiabatic expansion of the material.

A common characteristic of investigations carried out by the explosive procedure is determination of mechanical quantities — the velocities, density, pressure (in the shock wave $p = p_0 + \rho_0 Du$), and energy of the material. In the shock wave $E = E_0 + (v_0 - v)(p + p_0)/2$. The thermal processes and equalizations of temperature do not have time to attain equilibrium, hence conventional temperature measurements prove impossible. Optical measurement of the temperature of the shock wave is possible only in transparent materials, primarily in gases and then only in a temperature range bounded both from below (absence of radiation and transparency of the compressed gas) and from above (formation of a heated layer in front of the shock wave[1]).

Thus in the typical case of investigation of the equation of state by the explosive method, experiments give only information on the energy, pressure and density. By varying the initial state of the material, in particular, by compressing the material in the form of a finely dispersed powder with a reduced initial density by means of the shock wave, one can obtain the different adiabatic curves for shock compression and fill in an entire region in the p, v plane.

In the case of adiabatic (isenotropic) expansion the following relationships hold: $dp = \rho c\, du$; $dp = c^2\, d\rho$ and $dE = p\rho^{-2}\, d\rho$, where c is the velocity of sound. By measuring the velocity and pressure attained during expansion one can also find $E(p, v)$ along an isentrope. Finally in detonation waves and in the expansion of the products of the explosion itself measurements of the rates of detonation and displacement makes it possible to find $E(p, v)$. Thus in principle one can find the function of two variables $p(E, v)$ or $E(p, v)$ at high pressures.

In the present note we examine the problem of how, knowing $E(p, v)$, one can obtain the full thermodynamic characteristic of the material, *i.e.*, find the temperature T and entropy S at the high pressures that can be attained only by the explosive method.

Knowing $E(p, v)$ one can plot the lines of constant entropy. From

$$dE = TdS - pdv = E_p dp + E_v dv \tag{1}$$

we obtain for the isentrope ($dS = 0$):

$$(dp / dv)|_S = -(E_v + p) / E_p \tag{2}$$

(the subscripts at E denote partial derivatives).

However, the absolute value of the entropy cannot be determined from mechanical measurements alone, *i.e.*, from the function $E(p, v)$. The entropy can be determined only in the case when the measurements of the mechanical quantities in the shock wave and in the process of expansion can be linked with the region of low density where calculation or calorimetric measurement of the entropy is possible. Obviously if a constant-entropy line in the p, v plane is known and the value of the entropy at the lower end of this line is known, we will have S for the entire line. It can readily be shown that this also pertains to temperature: if the temperature at one end of an adiabatic curve is known, then when $E(p, v)$ is known, we can calculate the temperature over the entire adiabatic curve.

Let us imagine that we have two neighboring adiabatic lines $p_1(v)$ and $p_2(v)$ drawn in the p, v plane and that the energy E is known over the entire length of the adiabatic lines. If in the region of low pressures and low densities the temperature $T = T_0$ is known at some point of $v \doteq v_0$ [or more accurately, let $T_0 = (T_1 + T_2)/2$], then the entropy difference on the two adiabatic curves is given by

$$S_1 - S_2 = \{E[p_1(v_0), v_0] - E[p_2(v_0), v_0]\} / T_0. \tag{3}$$

In this case the temperature between the two neighboring adiabatic curves at a high pressure and a corresponding density (low specific volume v) is given by

$$T = \frac{E[p_1(v), v] - E[p_2(v), v]}{S_1 - S_2} = T_0 \frac{E[p_1(v), v] - E[p_2(v), v]}{E[p_1(v_0), v_0] - E[p_2(v_0), v_0]}. \tag{4}$$

Consequently, if mechanical measurements giving $E(p, v)$ have been made, all the thermodynamic quantities can be determined along the adiabatic curves extended to where they join the region in which these quantities (T and S) can be determined by calculation or experimentally.

Let us carry out the formal operations substantiating this conclusion and leading to more convenient expressions for the temperature. Determining dS from (1), we then obtain

$$\frac{\partial^2 S}{\partial p \partial v} = \frac{\partial}{\partial v}\left(\frac{E_p}{T}\right) = \frac{\partial}{\partial p}\left(\frac{E_v + p}{T}\right) \tag{5}$$

Equation (5) is a partial differential equation for the absolute temperature; after elementary operations we obtain

$$(E_v + p)\,\partial T / \partial p - E_p \partial T / \partial v = T. \tag{6}$$

The characteristics of this equation are lines, whose differential equation is

$$dp / dv = -(E_v + p) / E_p \tag{7}$$

i.e., adiabatic lines [compare with Eq. (2)]. Along these lines Eq. (6) gives

$$dT / dv \,|_S = -T / E_p, \tag{8}$$

whence

$$T = T_0 \exp\left(-\int_{v_0}^{v} \frac{dv}{E_p}\right) = T_0 \exp\left(\int_{p_0}^{p} \frac{dp}{E_v + p}\right), \tag{9}$$

where the integrals are taken along the adiabatic line. Just as in Eq. (4), the temperature in Eq. (9) is everywhere proportional to the given value T_0 at the end of the adiabatic line.

In the present note we do not examine the question of the accuracy with which it is necessary to make the initial measurements of D and u, as well as of the velocities attained in adiabatic expansion, in order to be able to carry out the calculations for $E(p, v)$, the adiabatic lines, and finally the temperatures according to formulas (4) or (9).

In conclusion I wish to express my gratitude to L. V. Al'tshuler, S. M. Volosov, and N. A. Dmitriev for valuable discussions.

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[1] Ia. B. Zel'dovich, J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 1126 (1957); Soviet Phys. JETP 5, 919 (1957).

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Beta-Emission From Polarized Nuclei

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1. THE ANGULAR DISTRIBUTION of β-radiation from oriented nuclei has been investigated theoretically by Cox and De-Groot[1] and the writer.[2] However, it was assumed in these investigations that parity is conserved in β-decay. Recently it has been found that conservation of parity is violated in β-decay.[3,4] In view of this, the results obtained in the aforementioned investigations[1,2] are valid only for aligned nuclei, but are not valid for polarized nuclei.

In the general case the distribution of the β-particles in energy and in angle should be

$$W(E, \vartheta) = a_0 + a_1 f_1 P_1(\cos\vartheta) + \cdots + a_n f_n P_n(\cos\vartheta), \tag{1}$$