tigations of the interaction between the π-meson in the mesonic atom with the nucleons of the nucleus.[15]

In conclusion, the author expresses his gratitude to Professor D. D. Ivanenko for reviewing the manuscript.

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On the Structure of the Front of Strong Shock Waves in Gases

IU. P. RAIZER

Academy of Sciences, U.S.S.R.

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The internal structure of the front of strong shock waves is investigated, taking account of radiation. Approximate solutions of the equations of the mode are found. Profiles of the hydrodynamic quantities, density and radiation flux, are constructed.

ONE OF THE METHODS of study of shock waves in gases (in particular, in air) is photometric measurement of the brightness of the wave front. In a certain amplitude interval, the shock wave front radiates like a black body. Consequently, it is possible to determine the temperature behind the wave front directly, by photometry. Combined with the measurement of another parameter of the wave, for example, its velocity, this allows us to make some suppositions concerning the thermodynamic functions of the gas being studied. The question arises, up to what amplitudes does the visible temperature coincide with the temperature behind the shock wave, and what is its dependence on the actual temperature behind the front when the latter reaches tens and hundreds of thousands of degrees, since at the present time such powerful shock waves are becoming the subject of experimental investigation.[1] This question leads, first of all, to the problem of the internal structure of a shock wave front, taking account of radiation.

This problem was investigated by Prokof'ev,[2] who obtained correct integrals of the approximate equations in the separate regions in which the variables are continuous. However, as a result of an erroneous analysis of the equations, he joined these solutions in an incorrect way, which led to the continuity of the hydrodynamic variables in the wave. Prokof'ev's error was pointed out by Zel'dovich,[3] who gave a correct qualitative analysis of the approximate equations of the mode, and proved that there is a discontinuity of the hydrodynamic variables in the shock wave.

In the present article, approximate solutions are found of the equations of the mode, encompassing a broad interval of shock wave amplitudes, as well as the case of thermal waves, in which the propagation of energy in the gas takes place not hydrodynamically, but by means of radiant thermal conduction, as proposed by Zel'dovich and Kompaneets.[4]

The physical meaning of the regularities of the behavior of the hydrodynamic quantities and the quantities characterizing the radiation is made quite evident by the extreme simplicity of the approximate solutions, which conserve the fundamental qualitative characteristics of the phenomena and are sufficiently accurate.

In the following article, the theory presented here will be applied to the propagation of waves in air.

1. THE EQUATIONS OF HYDRODYNAMICS AND RADIATION TRANSFER, DESCRIBING THE INTERNAL STRUCTURE OF THE SHOCK WAVE FRONT

We consider a stationary mode in a system of coordinates moving with the wave front. The x-axis is in the direction of propagation of the wave. The undisturbed gas flows into the wave with velocity $-D$ $(D > 0)$, equal in modulus to the velocity of the front. The hydrodynamic equations can be integrated and yield the conditions of conservation of the fluxes of mass, momentum, and energy,

$$\begin{gathered}\rho v = -\rho_0 D, \quad p + \rho v^2 = \rho_0 D^2, \\ \rho v(\varepsilon + p/\rho + v^2/2) + S = -\rho_0 D^3/2.\end{gathered} \tag{1}$$

where p, ρ, v, ε and S are the pressure, density, velocity, specific internal energy and flux of radiant energy* at the point x. ρ_0 is the density of the initial gas. The wave is assumed to be strong, so that the pressure and internal energy of the undisturbed gas may be neglected.

The pressure and density of the radiant energy is negligibly small in comparison with the pressure and energy of matter for the wave amplitudes under consideration. Only the flux of radiation in the visible and near ultraviolet regions of the spectrum reaches "infinity", since air, like other gases, is opaque at higher frequencies. This flux is much smaller than the flux in the wave region that will be of interest to us, and we will omit it.

For the explanation of the general character of the behavior of the variables inside the wave, the heat capacity will for simplicity be considered constant. Then,

$$\varepsilon = p/(\gamma - 1)\rho = RT/(\gamma - 1), \tag{2}$$

where T is the temperature, R is the gas constant, γ is the adiabatic exponent, (at high temperatures the effective magnitude of the adiabatic exponent, taking account of the expenditure of energy for ionization, is approximately 1.25).

Behind the wave front, at $x = -\infty$, the flux S becomes zero. All quantities at $x = -\infty$ will be denoted by the subscript 1.

By a simple calculation, we can obtain from Eqs. (1) and (2) expressions for the temperature and flux at a flowing point in terms of the reciprocal of the compression $\eta = \rho_0/\rho$ at that point*

$$T = T_1\eta(1-\eta)/\eta_1(1-\eta_1), \tag{3}$$

$$\begin{gathered}S = \rho_0 DRT_1(1-\eta)(\eta-\eta_1)/2\eta_1^2(1-\eta_1), \\ \eta_1 = \rho_0/\rho_1 = (\gamma-1)/(\gamma+1),\end{gathered} \tag{4}$$

$\eta_1 = 0.111$ when $\gamma = 1.25$.

The curve $T(S)$ which is obtained from Eqs. (3) and (4) by eliminating η, has two branches: one $(\eta \to 1,\ T \to 0,\ S \to 0)$ corresponds to the region in front of the shock discountinuity, the other $(\eta \to \eta_1,\ T \to T_1,\ S \to 0)$ corresponds to the region behind the discontinuity.[3]

As an approximation, we will consider the angular distribution of radiation in the diffusion approximation, replacing the rigorous kinetic equation for the intensity by a pair of equations for the density and flux of radiation. As a further simplification, we introduce a mean (over the spectrum) absorption coefficient $\varkappa = 1/l$ (l is the free path), having integrated the diffusion equations over the whole spectrum.

Eliminating the explicit dependence of the absorption coefficient on the point by means of a transformation from the geometrical coordinate x to the optical thickness τ by the well known formula

* In general, S should include the flux of electron thermal conduction, which is considerably greater than atomic (ionic) thermal conduction, acting only inside the shock discontinuity. However, calculations presented in the following article indicate that in the first approximation, this flux may be neglected in comparison with the flux of radiant energy (in gases with normal initial density).

* These formulas were derived by Prokof'ev in Ref. 2.

$$d\tau = \varkappa dx, \ \tau = \int_0^x \varkappa dx, \tag{5}$$

(the origin of coordinates $x = 0$ is located at the shock discontinuity), we may write the diffusion equations in the form

$$dS / d\tau = c\,(U_p - U), \tag{6}$$

$$S = -(c/3)\,dU / d\tau. \tag{7}$$

where U is the density of radiant energy, c is the velocity of light,

$$U_{eq} = 4\sigma T^4/c \tag{8}$$

is the equilbrium radiation density, and σ is the Stefan-Boltzmann constant.

In the general case the procedure of averaging over the spectrum is not rigorous, since the coefficient of absorption is averaged differently in Eqs. (6) and (7). As will be evident in what follows, averaging has a well defined meaning only in two limiting cases. Nevertheless, as an approximation, we will consider Eqs. (6) and (7) to be always valid.

Eqs. (3), (4), (6), and (7), together with the natural boundary conditions*

$$\tau = +\infty: \ S = 0, \ U = 0, \ T = 0, \tag{9}$$

$$\tau = -\infty: \ S = 0, \quad U = U_{eq\,1} = 4\sigma T_1^4 / c, \ T = T_1 \tag{10}$$

are the starting point of the present article, just as in Refs. 2 and 3.

The order of the system may be lowered by dividing Eqs. (6) and (7) by one another,

$$dS / dU = (c^2/3)\,(U - U_{eq}) / S. \tag{11}$$

2. APPROXIMATE SOLUTION OF THE EQUATIONS IN THE CASE OF AN "ORDINARY" SHOCK WAVE, $T_1 < T_k$

In the limit of weak waves, the role of radiation is negligibly small, and none of the hydrodynamic quantities depend on the coordinate, except for the discontinuity at the point $x = 0$. As the amplitude of the wave increases, the radiant flux $S^0 = \sigma T_1^4$, issuing from the surface of the discontinuity, and being absorbed by the layers lying in front of it, heats them more and more.

* Of these six conditions, only two are independent, the remaining ones result from the equations.

As was shown by Zel'dovich,[3] the greatest heating temperature T_- that exists at the point $x = 0$ in front of the discontinuity cannot exceed T_1; the greatest compression in the heating zone cannot exceed in this case $1/(1 - \eta_1)$ (equal to 1.13 at $\gamma = 1.25$). Expressing S in terms of T through Eqs. (3) and (4) to within small quantities of the second order with respect to η_1, we obtain the simple equation

$$S = D\rho_0 RT / (\gamma - 1) = D\rho_0 \varepsilon, \tag{12}$$

signifying that the energy of the absorbed radiation goes only toward raising the internal energy of the gas in front of the discontinuity.* The maximum possible error in Eq. (12) is no more than 1.7%.

If S^0, the value of the flux at the point of the discontinuity, is used in Eq. (12), we find the temperature in front of the discontinuity

$$S^0 = D\rho_0 RT_- / (\gamma - 1). \tag{13}$$

In the case of sufficiently weak waves, $S^0 \approx \sigma T_1^4$ and T_- increases very rapidly with increasing wave amplitude.

When the temperature in the heating zone is small compared with T_1, the temperature behind the front, the radiation density at any point in it, which is determined by the initial flux $S^0 \sim T_1^4$, is considerably greater than the equilibrium density $U_{eq} \sim T^4$ at that point. In other words, the radiation generated in the heating zone itself, contributes a small amount to the total flux and density. Under these conditions, neglecting U_{eq} compared with U in Eqs. (6) and (11), it is easy to find the solution of the system in front of the discontinuity†

$$S = cU / \sqrt{3} = S^0 \exp(-\sqrt{3}\tau), \quad \tau > 0, \tag{14}$$

$$T = T_- \exp(-\sqrt{3}\tau), \quad \tau > 0. \tag{15}$$

Its limit of applicability is evidently the temperature T_k at which $U_{eq}(T_k) = U(T_k)$, or, using Eqs. (8), (12) and (14),

$$4\sigma T_k^4 / \sqrt{3} = D\rho_0 RT_k / (\gamma - 1). \tag{16}$$

*The work of compression and the change of kinetic energy, which are proportional to η_1, cancel each other to within η_1^2.

† Essentially, our approximation consists simply of spreading the limiting form of integral curves, which originate at a singular point of the saddle-point type at $\tau = +\infty$, over the entire heating region.

T_k is evidently the temperature at which the hydrodynamic and radiant fluxes are comparable. Since D does not depend strongly on T_1 (for γ = const, $D \sim T_1^{1/2}$), temperature T_k is practically independent of the strength of the wave, when the latter does not vary over too broad a range (for example, in air, $T_k \approx 300{,}000°$).

At the same time, as follows from Eq. (16), T_k is extremely close to that temperature behind the front T_1, at which T_- is comparable with T_1. When $T_1 \ll T_k$ we get $T_- \ll T_1 \ll T_k$ and the solution (14), (15) is valid over the entire heating zone.

Numerical integration of Eqs. (11) and (12) shows that the approximate solution is quite accurate up to values of T_- extremely close to T_k. This occurs because the accuracy is proportional to the third power of the ratio T_-/T_k. Thus, for example, at $T_- = T_k/2$, the greatest error (in front of the discontinuity) is 1.1%.

Now we will find the solution of the equations behind the discontinuity. The curve $U(S)$ has two branches, corresponding to the two branches of $T(S)$. As was shown in Ref. 3, the point at which these branches intersect also determines the position of the discontinuity (the flux and density of radiation are, of course, continuous in the wave).

In the limit of a weak wave, the compression and the temperature are constant behind the discontinuity. Assuming that the change of compression behind the discontinuity is small, we find, as before, the connection between the flux and the temperature from Eqs. (3) and (4) on the second branch, to within small quantities of the second order $\sim \eta_1^2$:

$$(T - T_1)\, D\rho_0 R\,(\gamma + 1)(3 - \gamma)/(\gamma - 1) = S. \quad (17)$$

We will obtain an approximate solution of the equations behind the discontinuity by assuming that the temperature in this region varies slowly, *i.e.*, by replacing U_{eq} by $U_{eq1} = 4\sigma T_1^4/c$ in Eqs. (6) and (11). To this approximation, the system of equations is easily solved, and yields*

$$S = c\,(U_{eq1} - U)/\sqrt{3} = S^0 \exp(\sqrt{3}\,\tau), \quad \tau < 0, \quad (18)$$

$$T - T_1 = (T_+ - T_1)\exp(\sqrt{3}\,\tau), \quad \tau < 0, \quad (19)$$

where T_+ is the temperature at the discontinuity on the upper side of the jump.

We will obtain the values of U^0, S^0 at the discontinuity by joining both branches of the $U(S)$ curves, Eqs. (14) and (18)*

$$U^0 = {}^1/_2\, U_{eq1} = 2\sigma T_1^4, \quad (20)$$

$$S^0 = 2\sigma T_1^4/\sqrt{3}. \quad (21)$$

We will find the values of the temperature on both sides of the jump from Eqs. (13) and (17). Taking (16) into account, we obtain

$$T_-/T_k = {}^1/_2\,(T_1/T_k)^4, \quad (22)$$

$$(T_+ - T_1)/T_k = \frac{3-\gamma}{2(\gamma+1)}(T_1/T_k)^4. \quad (23)$$

As could be expected, the value of the peak temperature $\Delta T_+ = T_+ - T_1$ falls rapidly as the strength of the wave is decreased.

An estimate of the upper limit of the possible error in the approximate solution (18), (19) shows that just as in the heating zone, the accuracy of the approximation is good up to T_1, sufficiently close to T_k. Thus, for example, at $T_1 = T_k/2$, the error in ΔT_+ and S^0 is less than 10%.

Profiles of the dimensionless temperature Θ, flux Σ, and density j of the radiation are shown in Fig. 1. The units of these quantities are T_k, S_k

Profiles of the dimensionless temperature Θ, flux Σ, and density j of the radiation are shown in Fig. 1. The units of these quantities are T_k, $S_k = 4\sigma T_k^4/\sqrt{3}$, and $U_k = 4\sigma T_k^4/c$, respectively.

When the radiation is far from equilibrium $U \gg U_{eq}$, as occurs in the heating zone, the mean of the absorption coefficient over the spectrum in the diffusion equations has a completely defined character. In this case, we may also easily integrate the spectral diffusion equations for the flux and density of the radiant energy of frequency ν, per unit frequency interval, $S_\nu U_\nu$, which may be written in the same way as intergrals (6) and (7). Letting $U_\nu \gg U_{eq}$, where

$$U_{eq\nu} = (8\pi h/c^3)\,[e^{h\nu/kT} - 1]^{-1}, \quad (24)$$

*Exactly as in the region in front of the discontinuity, this solution is the limiting form of an integral curve, originating at a singular point of the saddle-point type at $\tau = -\infty$.

*Note that the value of U^0 was obtained correctly, corresponding to the limiting case $T(\tau) = T_1$ at $\tau < 0$, and $T = 0$ at $\tau > 0$. S^0 is too large by a factor of $2/\sqrt{3}$. This is a consequence of the diffusion approximation.

$$S_\nu = cU_\nu / \sqrt{3} = S_\nu^0 \exp(-\sqrt{3}\tau_\nu), \quad \tau_\nu = \int_0^x \varkappa_\nu dx, \tag{25}$$

so that the law of averaging is the same in both equations.

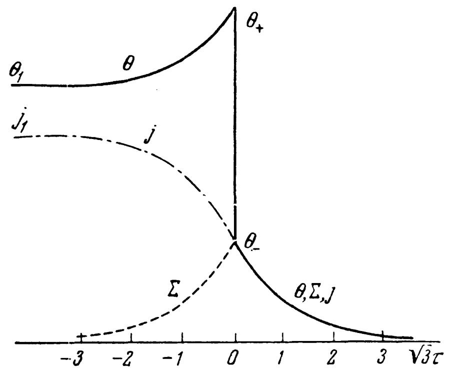

FIG. 1.

Integration over the spectrum of the spectral equation corresponding to Eq. (6) shows that $\varkappa_\nu = 1/l_\nu$ is averaged simply over the spectral density of radiation and the mean coefficient $\varkappa$ corresponds to the frequencies which give the greatest contribution to the flux and energy density of the radiation.

In the nonequilibrium zone we may obtain expressions for the flux and density, which in distinction to the diffusion approximation, take account rigorously of the angular distribution of the radiation. As is well known,[5] in the plane problem, S and U may be written in an integral form

$$S = \frac{c}{2}\int_{-\infty}^{\tau} U_{\mathrm{eq}} E_2(\tau - \tau')\, d\tau' - \frac{c}{2}\int_{\tau}^{\infty} U_{\mathrm{eq}} E_2(\tau' - \tau)\, d\tau', \tag{26}$$

$$U = \frac{1}{2}\int_{-\infty}^{\tau} U_{\mathrm{eq}} E_1(\tau - \tau')\, d\tau' + \frac{1}{2}\int_{\tau}^{\infty} U_{\mathrm{eq}} E_1(\tau' - \tau)\, d\tau', \tag{27}$$

where the functions E_1 and E_2 are special cases of integral exponentials

$$E_n(z) = \int_1^\infty e^{-xz} x^{-n} dx. \tag{28}$$

Since the radiation generated in the heating zone contributes very little to the total flux and density, we may, in Eqs. (26) and (27), neglect integrals over sources located in this zone, *i.e.*, integrals over τ from 0 to ∞. Using the well known properties of integral exponentials, we then obtain

$$S = S^0 2E_3(\tau), \quad U = U^0 E_2(\tau), \quad T = T_{-} 2E_3(\tau). \tag{29}$$

In the limit $T_+ - T_1 \ll T_1$ and $T(\tau) = T_1$ for $\tau < 0$, integrals (26) and (27) give, rigorously

$$S^0 = cU^0 / 2 = \sigma T_1^4. \tag{30}$$

3. APPROXIMATE SOLUTION OF THE EQUATIONS FOR THE CASES OF AN "ISOTHERMAL JUMP" $T_1 > T_k$ AND A THERMAL WAVE

At the lower edge of the heated zone of a strong shock wave with $T_1 > T_k$, in the region where $T < T_k$, the radiation is not in equilibrium, as before, and solutions of the type (14) and (15) are valid. When the temperature becomes of the order of T_k, the density U approaches equilibrium; moreover, as the temperature increases, the relative deviation of U from U_{eq} becomes less and less. This behavior is also indicated by the numerical integration of Eqs. (11) and (12). In this region, so called local equilibrium $U \approx U_{\mathrm{eq}}$ occurs, which is the starting point of the radiant thermal conduction approximation. In the local equilibrium approximation, U_{eq} may be substituted into Eq. (7) in place of U. Combining the equation thus obtained with Eq. (12), we obtain an equation of the first order for the temperature, which is easily integrated.

This solution for the equilibrium part of the heating zone must be joined with the solution in the nonequilibrium part, Eqs. (14) and (15), at the point $\tau = \tau_k$, with temperature T_k, which effectively delimits both regions.

Using Eq. (16), which determines T_k, we find, after an elementary calculation, the solution in the heating zone:
in the nonequilibrium region

$$T / T_k = cU / 4\sigma T_k^4 = \sqrt{3}\, S / 4\sigma T_k^4 = \exp\{-\sqrt{3}(\tau - \tau_k)\}, \quad \tau > \tau_k \tag{32}$$

and in the equilibrium region

$$\frac{T}{T_k} = \frac{\sqrt{3} S}{4\sigma T_k^4} = \left(\frac{cU}{4\sigma T_k^4}\right)^{1/4} = \left[1 - \frac{3\sqrt{3}}{4}(\tau - \tau_k)\right]^{1/8}, \quad 0 < \tau < \tau_k, \tag{33}$$

where τ_k is expressed in terms of the greatest heating temperature

$$\tau_k = 4\cdot 3^{-3/2}[(T_-/T_k)^3 - 1]. \tag{34}$$

Solution (33) is extremely close to the exact solution obtained by a numerical integration of Eqs. (11) and (12). Even at $T = 1.5\ T_k$ the error is less than 13%.

In the nonequilibrium region the more precise solution, Eqs. (29) – (31), is valid if in it we replace τ by $\tau - \tau_k$, and replace S^0, U^0, T_- by S_k, U_k, and T_k, respectively. In exactly the same way, the conclusions at the end of Sec. 2 regarding the law of averaging of the absorption coefficient are valid.

As regards the equilibrium region, since local equilibrium occurs, the free path l_ν is averaged according to Rosseland.[5] Here, the average free path l corresponds to high frequencies, lying in the Wien region ($h\nu \approx 6kT$ corresponds to $l_\nu \sim \nu^3 l$).

It was shown in Sec. 2 that as the amplitude of the wave increases, the temperature T_- in front of the jump becomes comparable with T_1, the temperature behind the front, when the latter reaches T_k. With further increase of T_1, T_- cannot become greater than T because otherwise the radiation density would decrease during the change behind the discontinuity from $U_- \approx 4\sigma T_-^4/c$ to $U_{\text{eq}1} = 4\sigma T_1^4/c$, the flux in this region would be directed toward the other side and density in the wave would not vary monotonically, which clearly does not make sense.*

Therefore, in the case of a sufficiently strong wave, where $T_- > T_k$, the temperature T_- in front of the discontinuity reaches the magnitude of the temperature T_1 behind the front, and with further increase of the amplitude of the wave, remains almost equal to it

$$T_- \approx T_1. \tag{35}$$

Because of the existence of local equilibrium in front of the discontinuity, the radiation density behind the discontinuity is almost independent of the coordinate, *i.e.*, the solution behind the discontinuity is

$$U(S) \approx U_{\text{eq}1} \approx U_- \equiv U^0. \tag{36}$$

The magnitude of the flux at the discontinuity is determined, in accordance with the general rule, from the conditions of intersection of both branches of the $U(S)$ curves. Using solutions (33) and (36), we find

$$S^0 = \frac{4\sigma T_k^4}{\sqrt{3}}\left(\frac{cU^0}{4\sigma T_k^4}\right)^{1/4} = \frac{4\sigma T_k^4}{\sqrt{3}}\left(\frac{T_1}{T_k}\right). \tag{37}$$

The temperature behind the discontinuity may be found from the approximate formula (17), or directly from the exact equations (3) and (4). The latter give *

$$T_+ = T_1(3-\gamma). \tag{38}$$

Had we also used the local equilibrium approximation behind the discontinuity, or, equivalently, the radiant thermal conduction approximation, then since the radiation density is constant, we would have obtained a constant temperature. Then the temperature would not undergo a discontinuity in the wave. This case is called the "isothermal jump".[6] Actually, an "isothermal jump" is never realized, as was noted in Ref. 3, because it contradicts the condition of continuity of flow.†

It is easy to evaluate the accuracy of solution (36). Substituting it into Eq. (11) as a zeroth approximation, and using Eq. (17) and the condition of flow boundedness which is a consequence of the hydrodynamic relations (3) and (4), we find after some straightforward calculations,

$$\frac{U_{\text{eq}1} - U^0}{U_{\text{eq}1}} < \frac{(\gamma+1)^2}{16(\gamma-1)(3-\gamma)}\left(\frac{T_k}{T_1}\right)^6. \tag{39}$$

We see that the relative deviation from $U_{\text{eq}1}$ of the radiation density behind the discontinuity decreases extremely rapidly as the amplitude of the wave increases.

We may estimate the upper limit of the optical thickness of the layer, in which the flux increases from zero to the maximum S^0 and which at the same time is the thickness $\Delta\tau$ of the temperature peak according to Eq. (7), in which as an approximation

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*This simple physical result was obtained by Zel'dovich[3] from a purely mathematical analysis of the integral curves of the equations.

*The approximate formula yields $T_+ = 4T_1/(\gamma+1)$. For $\gamma = 1.25$, the error is 1.7%, which attests to the great accuracy of Eq. (17).

†Indeed, in front of the discontinuity, at the point $x = 0$, S differs from zero and is proportional to T_- according to Eq. (13). But behind the discontinuity, in the local equilibrium approximation at the point $x = 0$,

$$S \sim dU/dx \approx dU_{\text{eq}}/dx \sim dT/dx = 0 \quad \text{for } T = \text{const.}$$

we replace $(dU/d\tau)^0$, the derivative at the discontinuity, by $(U_{eq1} - U^0)/\Delta\tau$. Using Eqs. (37) and (39), we find that the thickness of the peak decreases rapidly with increasing amplitude of the wave,

$$\Delta\tau < \frac{\gamma+1}{4\sqrt{3}(3-\gamma)}\left(\frac{T_k}{T_1}\right)^3. \tag{40}$$

Profiles of the nondimensional temperature, flux and density of radiation are shown in Fig. 2 for the case of an "isothermal jump". The realization of a strong "isothermal jump" requires special and rather artificial conditions, for example, a piston, "pushing" a strong shock wave ahead of it.

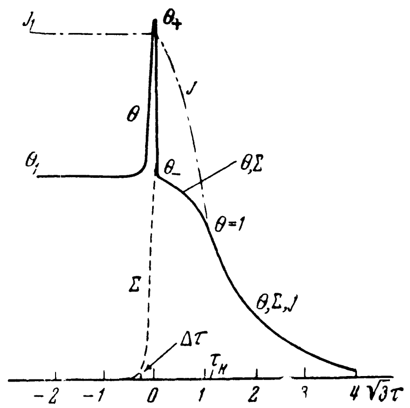

FIG. 2.

If we consider more real sources of such strong waves, such as a local discharge of very great energy, the early stages of the propagation of this energy from the source through the gas takes place not hydrodynamically, but by radiant heat conduction, as was envisaged in Ref. (4).

The radiant thermal conduction or thermal wave mode is essentially nonstationary. In Ref. 4, this mode was self-similar. However, the lowest edge of the wave is stationary in a system of coordinates moving with the velocity of a head wave. The thickness of the stationary layer is determined as follows: during the time that the wave progresses through a distance of the order of this thickness, its velocity D remains practically constant.

In the limiting case of a strong thermal wave, the role of hydrodynamics tends toward zero, and the wave propagates through a nonmoving gas. Under these conditions, the lower edge is described by the same equations, (3), (4), (6), and (7), as the heating zone in a strong shock wave. No conditions are imposed from above on the solution of the equations; it gradually transforms into the nonstationary solution, encompassing the whole space occupied by the wave (for example, in the self-similar solution of Zel'dovich and Kompaneets[4]). The solution of the equations clearly coincides with the solution in the heating zone, Eqs. (32) and (33), in which τ_k simply plays the role of a constant of integration. Since there is now no specially distinguished point in the wave (such as the point of discontinuity in the case of a shock wave), it is convenient to locate the coordinate origin $\tau = 0$ at the point delimiting the equilibrium and nonequilibrium regions, where $T = T_k$. Then the solution at the lower stationary edge of the thermal wave is written in the form

$$\frac{T}{T_k} = \frac{\sqrt{3}S}{4\sigma T_k^4} = \begin{cases} cU/4\sigma T_k^4 = \exp(-\sqrt{3}\tau) & \tau > 0, \\ (cU/4\sigma T_k^4)^{1/4} = (1 - 3\sqrt{3}\,\tau/4)^{1/3} & \tau < 0, \end{cases} \tag{41}$$

where as previously, T_k is determined by Eq. (16), in which D is understood to represent the velocity of the head of the wave.

I take this opportunity to express deep thanks to Ia. B. Zel'dovich for his interest in the work, and valuable discussions and remarks.

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