The Structure of Shock Waves in a Plasma

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The structure of shock waves in a plasma is investigated, taking account of the difference between the electron and the ion temperatures. The following cases are considered: a) nostationary shock wave in which the exchange of energy between electrons and ions may be neglected; b) stationary shock wave; c) stationary shock wave in a strong magnetic field.

THE STRUCTURE OF SHOCK WAVES in a plasma has been investigated in a number of works.[1-3] However, in all of these works it was assumed that the electron and ion temperatures were the same. In fact, however, shock waves in a plasma are characterized just by a difference between the electron and ion temperatures.

As is shown below, shock waves in a plasma without a magnetic field have the following structure. In addition to the usual density jump in a shock wave, there is a region of smooth temperature change. In front of the jump there is a region of raised electron temperature due to the high electron thermal conductivity. Behind the jump there is a comparatively broad region in which equalization of the electron and ion temperatures takes place (Figs. 1, 2).

If the plasma is located in a magnetic field parallel to the plane of the discontinuity, the electron thermal conductivity is strongly lowered and the heated region in front of the density jump is absent. Only the region of temperature equalization behind the jump exists (Fig. 3).

The width of the transition region is considerably greater than the mean free path. Therefore, it can be described by the equations of hydrodynamics, for any shock wave intensity. The structure of the jump itself will not interest us.

1. In the present article we consider a two-component plasma consisting of electrons (mass m, charge $-e$) and ions of one type (mass M, charge ze). The system of equations describing such a plasma consists of Maxwell's equations and the hydrodynamic equations:[4]

$$\partial n_\alpha / \partial t = -\operatorname{div} n_\alpha \mathbf{v}_\alpha, \tag{1}$$

$$m_\alpha n_\alpha \frac{d\mathbf{v}_\alpha}{dt} = -\nabla p_\alpha + (\operatorname{div} \pi)_\alpha + e_\alpha n_\alpha \left(\mathbf{E} + \frac{1}{c}\mathbf{v}_\alpha \times \mathbf{H}\right) + \mathbf{R}_\alpha, \tag{2}$$

$$\frac{n_\alpha}{\gamma-1}\frac{dT_\alpha}{dt} + p_\alpha \operatorname{div}\mathbf{v}_\alpha + \operatorname{div}\mathbf{q}_\alpha - \pi_{\alpha lk}\frac{\partial v_{\alpha l}}{\partial x_k} = Q_\alpha \tag{3}$$

where $\alpha = e, i$ refers to electrons and ions respectively, and the subscript i will be suppressed henceforth; $p_\alpha = n_\alpha T_\alpha$; m_α is the mass of the charge; $(\operatorname{div}\pi)_k = \partial \pi_{lk}/\partial x_l$; $\gamma = 5/3$ is the ratio of heat capacities.

Using the equation of continuity and Maxwell's equations, the last two equations may be written in the form

ions:

$$\frac{\partial (Mn\mathbf{v})}{\partial t} = -\operatorname{div}\Pi + zen\left(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{H}\right) - \mathbf{R}, \tag{4}$$

$$\frac{\partial}{\partial t}\left(\frac{Mnv^2}{2} + \frac{nT}{\gamma-1}\right) = -\operatorname{div}\left\{\mathbf{v}\left(\frac{Mnv^2}{2} + \frac{\gamma}{\gamma-1}nT - \pi\right) + \mathbf{q}\right\} + zen\mathbf{v}\mathbf{E} - \mathbf{v}\mathbf{R} + Q, \tag{5}$$

electrons:

$$\frac{\partial (m, n_e\mathbf{v}_e)}{\partial t} = -\operatorname{div}\Pi_e - en_e\left(\mathbf{E} + \frac{1}{c}\mathbf{v}_e\times\mathbf{H}\right) + \mathbf{R}, \quad (6) \tag{6}$$

$$\frac{\partial}{\partial t}\left(\frac{mn_ev_e^2}{2} + \frac{n_eT_e}{\gamma-1}\right) = -\operatorname{div}\left\{\mathbf{v}_e\left(\frac{mn_ev_e^2}{2} + \frac{\gamma}{\gamma-1}n_eT_e - \pi_e\right) + \mathbf{q}_e\right\} - en_e\mathbf{v}_e\mathbf{E} + \mathbf{v}_e\mathbf{R} + Q_e. \tag{7}$$

where

$$\Pi_{kl} = p\delta_{kl} + Mnv_k v_l - \pi_{kl}, \quad (\mathbf{v}\pi)_k \equiv v_l \pi_{lk},$$

π_{kl} is the viscosity tensor, and $\mathbf{R}$ is the frictional force of electrons and ions. In the absence of a magnetic field (and for $z = 1$)*

$$\mathbf{R} = 0{,}5\,\frac{mn_e}{\tau_e}(\mathbf{v} - \mathbf{v}_e) - 0{,}7\,n_e \nabla T_e,$$

In a strong magnetic field (to within $mc/eH\tau_e$)

$$\mathbf{R} = (mn_e/\tau_e)(\mathbf{v} - \mathbf{v}_e);$$

Q_e and Q represent liberation of heat in the electron and ion gases,[6]

$$Q = \frac{3m}{M}\frac{n_e}{\tau_e}(T_e - T)\,[^6], \quad Q_e = -\mathbf{R}(\mathbf{v} - \mathbf{v}_e) - Q;$$

$\mathbf{q}_e$ and $\mathbf{q}$ are the thermally conducting electron and ion fluxes. In the absence of a magnetic field

$$\mathbf{q}_e = -\varkappa_e \nabla T_e, \qquad \mathbf{q} = -\varkappa \nabla T,$$

$$\varkappa_e = \xi_e(z)\, n_e T_e \tau_e / m, \ \varkappa = \xi(z)\, n T \tau / M,$$

$$\tau_e = 3\sqrt{m} T_e^{3/2} / 4\sqrt{2\pi}\lambda z^2 e^4 n,$$

$$\tau = 3\sqrt{M} T^{3/2} / 4\sqrt{\pi}\lambda z^4 e^4 n,$$

where ξ_e and ξ are coefficients of order unity, $\xi_e(1) = 3.16$, $\xi(1) = 3.92$, and

$$\lambda = \frac{1}{2}\ln\left(\frac{T_e T}{T_e + T}\right)^3 \frac{1}{e^6 n}$$

is the well known Coulomb logarithm, which can be considered constant.

By adding (4) to (6) and (5) to (7) we obtain the laws of conservation of momentum and energy of the entire system

$$\frac{\partial}{\partial t}\left(Mn\mathbf{v} + mn_e \mathbf{v}_e + \frac{1}{4\pi c}\mathbf{E}\times\mathbf{H}\right) = -\operatorname{div}(\Pi + \Pi_e - T), \tag{8}$$

$$\frac{\partial}{\partial t}\left(\frac{Mnv^2}{2} + \frac{mn_e v_e^2}{2} + \frac{nT}{\gamma-1} + \frac{n_e T_c}{\gamma-1} + \frac{H^2+E^2}{8\pi}\right) = -\operatorname{div}\left\{\mathbf{v}\left(\frac{Mnv^2}{2} + \frac{\gamma}{\gamma-1}nT - \pi\right) + \mathbf{q}\right. \tag{9}$$

$$\left. + \mathbf{v}_e\left(\frac{mn_e v_e^2}{2} + \frac{\gamma}{\gamma-1}n_e T_e - \pi_e\right) + \mathbf{q}_e + \frac{c}{4\pi}\mathbf{E}\times\mathbf{H}\right\},$$

where

$$T_{kl} = \frac{1}{4\pi}(H_k H_l + E_k E_l) - \frac{H^2+E^2}{8\pi}\delta_{kl}.$$

2. For our problem, the most significant properties of the plasma are: a) quasi-neutrality, b) large difference between the masses of the ions and electrons. The criterion of quasi-neutrality is, as is well known, the smallness of the Debye length d with respect to the characteristic distance of the given problem. In a shock wave, this characteristic distance is the width of the discontinuity, determined by the mean free path of the particles l. The relative density of charges in the discontinuity $(n_e - zn)/n_e \approx d^2/l^2 \approx 10^{-7}n/T^3$ is small at temperatures $T \gtrsim 10^5$ degrees for all reasonable densities. Therefore, inside the discontinuity as well as outside, the electron and ion charge densities, and consequently also their velocities, can be considered equal: $n_e = zn$, $v_e = v$. The mass difference leads to a difference in the mechanisms through which the particles are heated. In a shock wave, the basic mechanisms for ions are dissipation processes in the discontinuity, transferring part of the kinetic energy of motion with respect to the discontinuity $Mv^2/2$ into energy of thermal motion. Since the velocities of the electrons and ions are equal, the kinetic energy of the electrons $mv^2/2$ is negligible with respect to the ion energy as well as the energy of thermal motion T which is of the same order of magnitude. Electrons in the jump may be heated by adiabatic compression, by acceleration in the electric field of the ions, and directly by elastic collisions with ions. Clearly, the electron temperature will generally differ from the ion temperature.

Furthermore, because of the large mass difference, energy transfer during collisions between one type of particle and the other is small (approximately the mass ratio times the energy of the particles). Therefore, the width x of the region in which temperature equalization takes place is a factor of M/m greater than the distance through which the electrons are

*The second terms in R arises from thermal diffusion. The numerical value of the coefficient of thermal diffusion for a fully ionized plasma is given, for example, in Ref. 5.

displaced between collisions: $x \approx v\tau_e M/m$, or is a factor of $\sqrt{M/m}$ greater than the mean free path: $x \approx l\sqrt{M/m}$ (since $v \approx \sqrt{T/M}$). For a shock wave formed in a system with small dimensions, for example, in a gaseous discharge, this region may be of fundamental interest.

3. In the examination of the structure of a shock wave in the absence of a magnetic field, an essential factor is the high thermal conductivity of the electronic component of the plasma, associated with the mass difference. Actually, the mean free paths of electrons and ions for small z and comparable temperatures are of the same order of magnitude, but the thermal velocity of the electrons is greater by a factor of $\sqrt{M/m}$. Consequently, the electron coefficient of thermal conductivity $\varkappa_e \approx l\sqrt{T/m}$ is also a factor of $\sqrt{M/m}$ greater than the ion coefficient $\varkappa \approx l\sqrt{T/M}$. As a result, the electron temperature in the shock wave does not have a discontinuity, in spite of the existence of a density jump (one may speak of an "isoelectrothermal" jump). The hydrodynamic equations describing the structure of the transition layer outside the density jump differ from the corresponding equations describing the transition layer in the case of an isothermal jump (Ref. 7 p. 421) in that the density of energy flux due to electron thermal conductivity enters into the equation expressing the constancy of the energy flux. Furthermore, since the electron and ion temperatures differ, one more equation is added [it is convenient to take the heat transfer equation for ions (3)]

$$\frac{\varkappa}{j}\frac{dT_e}{dx} = \frac{\gamma}{\gamma-1}(\Theta - \Theta_1) + {}^1\!/_2 M(v^2 - v_1^2),$$

$$n\Theta - n_1\Theta_1 + Mj(v - v_1) = 0, \tag{10}$$

$$\frac{nv}{\gamma-1}\frac{dT}{dx} + nT\frac{dv}{dx} = \frac{3m}{M}\frac{zn(T_e - T)}{\tau_e}.$$

where $\Theta = T + zT_e$, $j = nv = \text{const}$. The electric field due to the separation of charges may be found from the equation of motion of the electrons (2),

$$eE = -\frac{1}{n}\frac{d(nT_e)}{dx} - \frac{1}{n}R_x. \tag{11}$$

Small terms of the order of $\sqrt{m/M}$ and of higher order have been omitted. These equations are rigorous for a stationary shock wave and are applicable to nonstationary waves if the characteristic time of the variable quantities exceeds the time required by particles to pass through the transition layer: $t > x/v \sim \tau_e\sqrt{M/m}$.

To integrate Eq. (10) we must know the conditions in the density jump. These conditions may be obtained from the hydrodynamic equations in the usual way, taking account of the constancy of the electron temperature. They reduce to the following. In the density jump, an electric double layer is formed, which corresponds to a jump of the electric potential. Therefore, the condition that the energy flux density be constant has the form for ions (see Eq. 5),

$$\frac{\gamma}{\gamma-1}(T_{20} - T_{10}) + \frac{1}{2}M(v_{20}^2 - v_{10}^2) + ze(\varphi_{20} - \varphi_{10}) = 0. \tag{12}$$

The subscripts 10 and 20 refer to the values of quantities in the jump on different sides of the discontinuity. The magnitude of the potential jump may be determined from the following considerations. Since the directed motion of the electrons is negligible in comparison with the thermal motion, the electrons in the jump have a Maxwell-Boltzmann distribution, and consequently, their density is $n = \text{const}\cdot\exp(e\varphi/T_{e0})$. Hence,

$$e(\varphi_{20} - \varphi_{10}) = T_{e0}\ln(n_{20}/n_{10}), \tag{13}$$

where T_{e0} is the temperature of the electrons in the jump. The same condition may be obtained directly from Eq. (11). Equations (10)–(13) are the complete solution of the problem of the structure of a shock wave in a plasma.

We will consider two cases. In the first case there is no exchange of energy between the electrons and ions behind the wave front. Such conditions are realized approximately when the wave front has moved from the point of its formation through a distance less than that required for temperature equalization. The shock wave has the following structure. Between region 1 of the undisturbed gas and the density jump, there is a transition region in which the density varies smoothly from n_1 to n_{10}, the ion temperature from T_1 to T_{10}, and the electron temperature from T_{el} to $T_{e0} = T_{e2}$. Furthermore, the density and the ion temperature are changed by the jump from n_{10} to $n_{20} = n_2$, and from T_{10} to $T_{20} = T_2$, respectively. The values of p_2, n_2, v_2, Θ_2, are connected with p_1, n_1, v_1, Θ_1, and the ratio of the velocity of the front to the velocity of sound $M_1 = \sqrt{Mv_1^2/\gamma\Theta_1}$ through the usual equations for an ideal gas (Ref. 5, p. 408). The results of a numerical integration of system (10) are shown in Fig. (1) for $M_1 = 6$, $z = 1$. The wave is moving from the right to the left. The density scale is based on the density n_1 in the undisturbed plasma, the temperature scale is based on $\Theta_2/2$, the length scale is based on $3.16\, v_2 \tau_2 \sqrt{2M/m}\, \Theta_2^2/4M^2 v_1^4$, where τ_2 is the time between collisions of ions for $n = n_2$, $T = \Theta_2/2$. Conditions in the isoelectrothermal jump

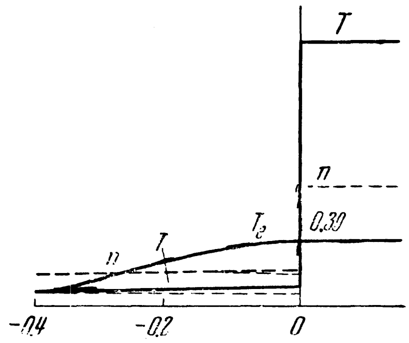

FIG. 1.

for $z = 1$ and various M_1, obtained by numerical integration of Eq. (10), are given in Table 1. For $M_1 < 1.12$, there is no discontinuity. As $M_1 \to \infty$, the ion temperature in the wave increases proportionately to M_1^2, while the electron temperature approaches a constant limit. In this case, our equations lose their meaning since our assertion that the width of the transition region considerably exceeds the width of the jump is violated. Actually, the width of the transition region may be estimated by comparing the electron heat fluxes $nvdT_e/dx$, and $-\varkappa_e d^2T_e/dx^2$ with the transfer of energy by ions $nmT/M\tau_e$, whence $x \approx v\tau_e T_e M/Tm$. The width of the jump is of the order of the mean free path of the ions $l \approx \sqrt{T/M}\tau$. We require $x \gg l$, or $T_e > T(m/M)^{1/5} z^{-4/5}$. This condition is fulfilled at $z = 1$, for example, only for $M \lesssim 7$. In stronger shock waves, the electrons in the discontinuity will be able to attain temperatures of the order of $T_e \approx T_2(m/M)^{1/5} z^{-4/5}$ by collisions with ions.

In the second case, we consider a stationary shock wave. The temperatures of the electrons and ions on both sides of the front and far from it are

TABLE I. Conditions in the jump in the nonstationary mode.

M_1	1.12	1.5	2	3	4	6	8	$M_1 \gg 1$
$n_2/n_1 = 4M_1^2/(3+M_1^2)$	1.18	1.71	2.29	3.00	3.37	3.69	3.82	4
n_{10}/n_1	1.18	1.14	1.11	1.08	1.06	1.04	1.03	$1+3.66M_1^{-2}$
$\Theta_2/2T_1 = (5M_1^2-1)\times(M_1^2+3)/16M_1^2$	1.12	1.49	2.08	3.67	5.86	12.12	20.9	$0.31\ M_1^2$
Te_0/T_1	1.10	1.36	1.73	2.51	3.29	4.78	5.97	13.2
T_{10}/T_1	1.14	1.21	1.28	1.35	1.37	1.33	1.26	1
T_{20}/T_1	1.14	1.62	2.43	4.83	8.43	19.5	35.8	$0.625\ M_1^2$

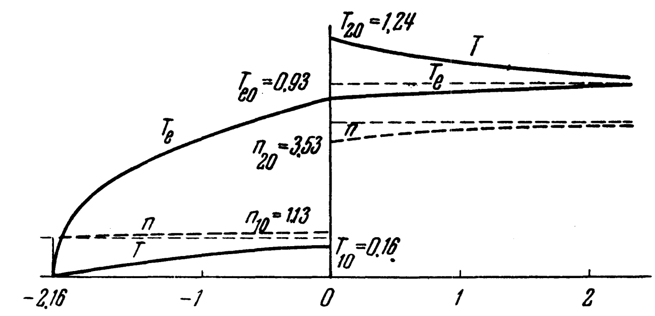

FIG. 2.

equal. In this case, the density jump is divided into two regions in each of which there is a gradual variation of density and ion temperature from the values n_1, n_2 and T_1, T_2 respectively, to the values n_{10}, n_{20}, and T_{10}, T_{20} in the jump, connected by relations (10) – (13). The electron temperature varies continuously from $T_{e1} = \Theta_1/2$ to $T_{e2} = \Theta_2/2$, with the value $T_e = T_{e0}$ in the jump. The derivative dT_e/dx is discontinuous in the jump. The results of a numerical integration of equations (10) – (13) for $z = 1$, $M_1 = \infty$, are shown in Fig. (2). Because of the strong dependence of $\varkappa_e$ on T_e, the electron temperature front is

TABLE II. Conditions in the jump in a stationary shock wave.

M_1	1.20	1.3	1.5	1.8	3	4	6	7	$M_1 \gg 1$
n_{10}/n_1	1.23	1.22	1.21	1.18	1.15	1.143	1.136	1.135	1.131
n_{20}/n_1	1.23	1.36	1.62	1.93	2.72	3.03	3.29	3.35	3.526
T_{e0}/T_1	1.17	1.25	1.43	1.74	3.44	5.48	11.3	15.1	$0.29\,M_1^2$
T_{10}/T_1	1.19	1.23	1.29	1.37	1.73	2.71	3.16	3.83	$0.05\,M_1^2+1.2$
T_{20}/T_1	1.19	1.32	1.58	1.99	4.26	6.97	14.7	19.7	$0.387\,M_1^2$

steep and extends far ahead. The conditions in the isoelectrothermal jump are given in Table 2 for finite M_1. For $M_1 < 1.20$, all quantities in the shock wave vary slowly, without discontinuities.

4. Let us assume that a magnetic field H exists in the plasma, directed along the z-axis, parallel to the plane of the discontinuity. The shock wave moves in the negative direction along the x-axis. The structure of the shock wave is quite dependent on the magnitude of the magnetic field since the latter has a strong influence on the thermal conductivity. For the examination of effects associated with a magnetic field, we will consider the limiting case of a strong magnetic field for which $\Omega^2\tau^2 > 1$, $\Omega_e^2\tau_e^2 \gg 1$ ($\Omega = zeH/Mc$, $\Omega_e = eH/mc$). These conditions signify that the radius of the Larmor circle along which a charge moves in a plane perpendicular to the magnetic field does not exceed the mean free path for ions, and is considerably less than the mean free path for electrons. The coefficient of electron thermal conductivity is strongly lowered in this case (by factor of $\Omega_e^2\tau_e^2$), and is less than the ion thermal conductivity approximately by a factor of $z^{-2}\sqrt{M/m}$. As regards the viscosity, the component of the viscosity tensor of interest to us π_{xx}, equal to $(4\eta/3)\,dv/dx$ at $H = 0$ ($\eta \approx nT\tau$), is changed by the coefficient only slightly. In particular, for $\Omega^2\tau^2 \gg 1$, it is decreased by a factor of four. This result may be explained as follows. The component of the tensor π_{zz} remains the same as for $H = 0$: $\pi_{zz} = 2\eta\,(\partial v_z/\partial z - \tfrac{1}{3}\,\mathrm{div}\,\mathbf{v})$. For a monatomic gas, which the plasma appears to be, $\pi_{xx} + \pi_{yy} + \pi_{zz} = 0$. As a result of axial symmetry, $\pi_{xx} = \pi_{yy}$, and consequently, $\pi_{xx} = -\eta\,(\partial v_z/\partial z - \tfrac{1}{3}\,\mathrm{div}\,\mathbf{v})$. In the present case, $\partial/\partial y = \partial/\partial z = 0$, and $\pi_{xx} = (\tfrac{1}{3})\,\eta\,dv/dx$.

Hence, the width of the discontinuity, as in the usual non-ionized gas, is of the order of the mean free path. As when $H = 0$, temperature changes of the electrons and ions passing through the discontinuity are not equal, and the exchange of energy between them is retarded. Therefore, except for the discontinuity, there is a comparatively broad transition region in which temperature equalization takes place. In the case of ideal conductivity, the equations describing the structure of the shock wave in the transition region have the form

$$\frac{H^2}{4\pi n}+\frac{Mv^2}{2}+\frac{\gamma}{\gamma-1}\Theta=\frac{H_2^2}{4\pi n_2}+\frac{Mv_2^2}{2}+\frac{\gamma}{\gamma-1}\Theta_2,$$

$$\frac{H^2}{4\pi}+Mnv^2+n\Theta=\frac{H_2^2}{4\pi}+Mn_2v_2^2+n_2\Theta_2;$$

$$nv=n_2v_2;\quad Hv=H_2v_2, \tag{14}$$

$$\frac{nv}{\gamma-1}\frac{dT_e}{dx}+nT_e\frac{dv}{dx}+\frac{3m}{M}\frac{n\,(T_e-T)}{\tau_e}=0.$$

The condition in the discontinuity which permits us to determine the electron and ion temperatures separately is easily obtained by noting that an adiabatic compression is the only significant mechanism for the heating of electrons in the discontinuity. Therefore,

$$T_{e0}=T_{e1}\,(n_{20}/n_1)^{\gamma-1}.$$

Since the change of density is bounded, $n_{20}/n_1 \leqslant (\gamma+1)/(\gamma-1)$, the increase of electron temperature is insignificant ($T_{e0} \leqslant 2.5T_{e1}$ for $\gamma = \tfrac{5}{3}$).

The solution of the first two equations of system (14) in the transition region is tribial

$$v=\mathrm{const}=v_2;\quad H=\mathrm{const}=H_2;$$

$$\Theta=\mathrm{const}=\Theta_2.$$

The variation of the electron and ion temperatures behaves according to the last of equations (14) in which we must set $dv/dx = 0$, and according to the condition $zT_e + T = \Theta_2$. Letting $\Theta_2/(z+1)$ be the unit temperature, and $x_1 = v_2\tau_2\sqrt{M/2m}\,z^2/(z+1)$ the unit length, where τ_2 is taken at $n = n_2$ and $T = \Theta_2/(z+1)$, we obtain the solution of this equation in the form

$$x = \frac{1}{2}\ln\frac{1+\sqrt{T_e}}{1-\sqrt{T_e}}\,\frac{1-\sqrt{T_{e0}}}{1+\sqrt{T_{e0}}} - \sqrt{T_e}\left(1+\frac{T_e}{3}\right) + \sqrt{T_{e0}}\left(1+\frac{T_{e0}}{3}\right). \qquad (15)$$

The functions $T(x)$, $T_e(x)$, $\Theta(x)$ for $z=1$, $M_1=\infty$ are shown in Fig. (3). These curves are not assumed to be highly accurate, since as a result of the low electron temperature in the discontinuity, the exchange of energy in the discontinuity becomes substantial. The behavior of all the quantities in the discontinuity may be obtained for weak waves by including the terms in the viscosity $(\eta/3n)\,dv/dx$ and $-(\eta/3)\,dv/dx$ in the left hand sides of the first and second equations respectively of system (14).

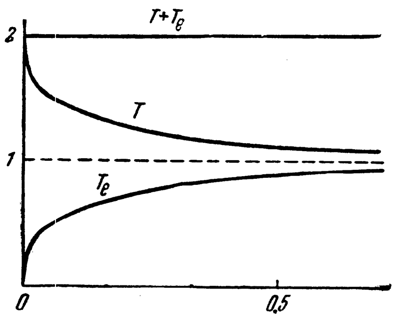

FIG. 3.

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286

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On the Angular Distribution of Deuterons from the $Be_4^9(pd)Be_4^8$ Reaction

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It is shown that even at proton energies ≥ 8 Mev the main contribution is from the region within the Be_4^9 nucleus. This significantly modifies the deuteron angular distribution, good agreement with experiment being obtained for proton energies of ≈ 22 Mev with a Be_4^8 radius $r_0 = 5\times 10^{-13}$ cm.

WHEN ANALYZING the angular distribution of deuterons from the $Be_4^9(pd)Be_4^8$ reaction on the basis of the theoretical angular distribution from the $Be_4^8(pd)Be_4^9$ stripping reaction and the principle of detailed balance, difficulties arise related to the choice of the nuclear radius r_0. This is determined by agreement between the theoretical and experimental distribution curves at some single point. For nuclei that are not too light, the radius r_0 for the stripping reaction (using Butler's formula for the angular distribution) is given by $r_0 = (1.2A^{1/3} + 1.7)\times 10^{-13}$ cm, where A is the atomic weight of the target nucleus. This value of r_0 is in good agreement with that obtained by scattering of neutrons with energy $E \geq 1$ Mev by nuclei. For light nuclei the value of r_0 is found to be larger than that given by the above formula. Thus, for instance, for the direct and inverse reactions on Li_3^7, B_5^{10}, B_5^{11}, the nuclear radii lie in the interval between 4.5×10^{-13} and 5×10^{-13} cm,[1,2] and depend extremely weakly on the incident particle energies. For the $Be_4^9(dp)Be_4^{10}$ reaction at a deuteron energy of 3.6 Mev, the radius r_0 is found to be 6.1×10^{-13} cm.

At a proton energy $E_p \simeq 16-22$ Mev, however, in order to obtain agreement between the theoretical and experimental deuteron distributions from the