Optical Model of the Interaction of Nucleons with Nuclei in the Resonance Region of the Compound Nucleus

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A study is made of the energy dependence of the real and imaginary parts of the effective potential of the optical model, with all states of the compound nucleus taken into account. An interpretation is given of the broad resonances in the interaction of neutrons with nuclei.

INTRODUCTION

IN PAPERS of Brueckner, Watson, and their collaborators[1] a basis has been given for the optical model of the interaction of low-energy nucleons with nuclei, which had been introduced by Feshbach, Porter, and Weisskopf[2] to explain the scattering by nuclei of neutrons of energies up to 3 Mev. In the optical model the interaction of a nucleon with a nucleus, which involves the many-body problem, is reduced to a consideration of the motion of the nucleon in a certain complex effective potential $V(1 + i\zeta)$. Here the results of the interaction of the nucleon with the nucleus are separated into the elastic scattering (caused by the real part of the potential) and all other processes, which go through the stage of the compound nucleus (caused by the imaginary part of the potential). The imaginary part of the optical potential is determined by the coupling between the one-nucleon excited states and the many-particle excitations of the compound nucleus.

A study of the energy dependence of the real and imaginary parts of the effective potential is of great interest, especially in the range of energies corresponding to those at which isolated resonances are observed in the cross-sections. This problem has already been studied in a paper by Brueckner, Eden, and Francis.[3] In that paper, however, only the two-particle states of the compound system are considered; it is shown that in the energy range corresponding to two-particle excitations of the compound system the real and imaginary parts of the effective potential undergo considerable changes.

In the present paper an attempt is made to study the energy dependences of the real and imaginary parts of the effective potential with the inclusion of all states of the compound nucleus. Unlike what was found in Ref. 3, we do not get a rapid variation of the real part of the potential in the energy range corresponding to the isolated resonances in the reaction cross-sections. Even the imaginary part of the potential increases relatively smoothly with the energy outside the isolated resonances and in the region of overlapping levels. In the region of isolated resonances the imaginary part of the effective potential rises sharply.

In order not to complicate the problem by inclusion of the Coulomb interaction, we consider only the interaction of a neutron with the nucleus.

1. ENERGY DEPENDENCE OF THE REAL PART OF THE OPTICAL POTENTIAL

According to the optical model[1] the real part of the effective interaction potential of a neutron and a nucleus is given by

$$V(r) = \mathrm{Re} \sum_{\alpha=1}^{A} \langle t_\alpha \rangle, \tag{1.1}$$

where $\langle t_\alpha \rangle$ denotes the average, over the wave function φ_0 of the ground state of the target nucleus A, of the operator of the interaction of pairs, which is determined by the integral equation

$$t_\alpha = v_\alpha + v_\alpha D^{-1} t_\alpha, \tag{1.2}$$

where v_α is the interaction of the incident nucleon with the αth nucleon of the target nucleus; the operator D is given by

$$D = E - H_A - K_n + i\eta, \tag{1.3}$$

where E is the energy of the system, H_A is the Hamiltonian operator of the nucleus A, K_n is the operator for the kinetic energy of the relative motion of neutron and nucleus, and η is a small positive number, which assures the presence of outgoing waves only in the scattering. After the inte-

grations have been carried out one must let η go to zero.

By direct substitution one can verify by the use of the operator identity

$$D^{-1} + D^{-1}v_\alpha (D - v_\alpha)^{-1} \equiv (D - v_\alpha)^{-1},$$

that the operator equation (1.2) has the solution

$$t_\alpha = v_\alpha + v_\alpha (D - v_\alpha)^{-1} v_\alpha. \qquad (1.4)$$

In the approximation of the optical model the wave function χ_ϵ of the incident neutron must satisfy the Schrödinger equation

$$[\varepsilon - K_n + V(r)]\chi_\varepsilon(r) = 0,$$

where ε is the energy of the relative motion of neutron and nucleus.

Let us determine the dependence of the mean value of the real part of the optical potential

$$\bar{V} = (\chi_\varepsilon(r), \quad V(r)\chi_\varepsilon(r)). \qquad (1.5)$$

on the energy ε of the relative motion. Substituting Eq. (1.1) into Eq. (1.5) and taking into account Eq. (1.4), we get

$$\bar{V} = \mathrm{Re}\sum_{\alpha=1}^{A}\{(\Phi_{0\varepsilon}, v_\alpha\Phi_{0\varepsilon}) + (\Phi_{0\varepsilon}, v_\alpha (D - v_\alpha)^{-1} v_\alpha\Phi_{0\varepsilon})\}, \qquad (1.6)$$

where $\Phi_{0\epsilon} = \varphi_0\chi_\epsilon(r)$.

We are interested in the behavior of the expression (1.6) in the energy range corresponding to the resonances in the cross-sections for nuclear reactions. We introduce the wave functions X_λ and energies E_λ of the compound nucleus $A + 1$ by means of the Schrödinger equation

$$HX_\lambda = E_\lambda X_\lambda, \qquad (1.7)$$

where

$$H = H_A + K_n + \sum_{\alpha=1}^{A} v_\alpha \qquad (1.8)$$

is the Hamiltonian operator of the total system.

With natural boundary conditions the solutions (1.7) for $E_\lambda > 0$ correspond to disintegrating states. Because of the large lifetimes of the states of the compound nucleus in comparison with the lifetimes of single particle excitations, we shall make the approximation of assuming that the E_λ are real and that the X_λ form a complete normal orthogonal system of functions for the compound nucleus. Then we can write

$$v_\alpha\Phi_{0\varepsilon} = \sum_\lambda R_\lambda^\alpha(\varepsilon) X_\lambda,$$
$$R_\lambda^\alpha(\varepsilon) = (X_\lambda, v_\alpha\Phi_{0\varepsilon}). \qquad (1.9)$$

Now we can write

$$(D - v_\alpha)^{-1} v_\alpha\Phi_{0\varepsilon} = \sum_\lambda R_\lambda^\alpha(\varepsilon)(D - v_\alpha)^{-1}X_\lambda. \qquad (1.10)$$

Introducing the matrix $f_{\lambda\mu}^\alpha(\varepsilon)$ by the equation

$$(D - v_\alpha)^{-1}X_\lambda = \sum_\mu f_{\lambda\mu}^\alpha(\varepsilon) X_\mu, \qquad (1.11)$$

we bring Eq. (1.10) into the form

$$(D - v_\alpha)^{-1} v_\alpha\Phi_{0\varepsilon} = \sum_{\lambda,\mu} R_\lambda^\alpha(\varepsilon) f_{\lambda\mu}^\alpha(\varepsilon) X_\mu. \qquad (1.12)$$

To determine the matrix $f_{\lambda\mu}^\alpha(\varepsilon)$ we apply to both sides of Eq. (1.11) the operator

$$D - v_\alpha \equiv E - H + \sum_{\beta\neq\alpha} v_\beta + i\eta;$$

then, taking account of Eq. (1.7), we get

$$X_\lambda = \sum_\mu f_{\lambda\mu}^\alpha(\varepsilon)\left\{E - E_\mu + i\eta + \sum_{\beta\neq\alpha} v_\beta\right\} X_\mu.$$

Multiplying this equation by $X_{\lambda_1}^*$ and integrating over all variables of the compound nucleus, we get the equation

$$\sum_\mu f_{\lambda\mu}^\alpha(\varepsilon)\{(E - E_\lambda + i\eta)\delta_{\mu\lambda_1} + Q_{\mu\lambda_1}^\alpha\} = \delta_{\lambda\lambda_1}, \qquad (1.13)$$

where

$$Q_{\mu\lambda}^\alpha \equiv \left(X_\lambda, \sum_{\beta\neq\alpha} v_\beta X_\mu\right). \qquad (1.14)$$

Assuming that the nondiagonal matrix elements $Q_{\lambda\mu}$ are smaller in absolute value than the diagonal elements, we can solve Eq. (1.13) for $f_{\lambda\mu}$. Then, substituting Eq. (1.12) into Eq. (1.6) and taking into account Eq. (1.9), we get

$$\bar{V} = \mathrm{Re}\sum_{\alpha}\Big\{(\Phi_{0\varepsilon}, v_\alpha\Phi_{0\varepsilon}) + \sum_{\lambda}\frac{|R_\lambda^\alpha(\varepsilon)|^2}{E-E_\lambda+Q_{\lambda\lambda}^\alpha+i\eta} - \sum_{\lambda\neq\mu}\frac{R_\lambda^\alpha(\varepsilon)}{E-E_\lambda+Q_{\lambda\lambda}^\alpha+i\eta}Q_{\lambda\mu}^\alpha\frac{R_n^{*\alpha}(\varepsilon)}{E-E_\mu+Q_{\mu\mu}^\alpha+i\eta}\Big\}. \tag{1.15}$$

From Eq. (1.14) we see that $Q_{\lambda\lambda}^\alpha$ is the average energy of interaction of one nucleon with $A-1$ nucleons in the compound nucleus in the state X_λ. Consequently, $Q_{\lambda\lambda}^\alpha < 0$ and its absolute value is some tens of millions of electron volts. Thus it follows from Eq. (1.15) that for energies of relative motion $\varepsilon < 30$ Mev the real part of the potential (1.15) varies smoothly with the energy. At energies of the relative motion greater than 30 Mev the excitation energy of the compound nucleus falls in the region of the continuous spectrum, and the imaginary part of the optical potential becomes so appreciable that the concept of single-particle states becomes less justified.

2. THE IMAGINARY PART OF THE EFFECTIVE INTERACTION POTENTIAL BETWEEN NEUTRON AND NUCLEUS

The imaginary part (ζV) of the potential in the optical model of nuclear interactions describes all processes that go through the stage of the compound nucleus. In obtaining Eq. (1.1) we took into account only the coherent scattering of the neutrons inside the nuclear matter. Therefore the imaginary part $\mathrm{Im}\sum_\alpha\langle t_\alpha\rangle$ gives only the decay of the state $\Phi_{0\epsilon}$ owing to the presence of outgoing waves. A special argument is required for the calculation of the imaginary part of the potential caused by transitions into states of the compound nucleus with the same energy.

If the probability per unit time for a transition from a single-particle excitation $\Phi_{0\epsilon}$ into excitations of the compound nucleus is denoted by Γ, then the imaginary part ζV of the potential for the optical model is given in terms of Γ by the relation

$$2(\chi_\varepsilon, \zeta V\chi_\varepsilon) = \hbar\Gamma. \tag{2.1}$$

The system consisting of the nucleus A and the neutron is described in the optical model by the Hamiltonian

$$H_0 = H_A + K_n + V(r), \tag{2.2}$$

where H_A is the Hamiltonian of the target nucleus, K_n is the operator for the kinetic energy of the relative motion of nucleus and neutron, and $V(r)$ is given by Eq. (1.1). Let us denote by $E_{n\epsilon}$ and $\Phi_{n\epsilon}$ respectively the eigenvalues and eigenfunctions of this operator. Here $E_{n\epsilon} = E_n + \varepsilon$, and

$$\Phi_{n\varepsilon} = \varphi_n\chi_\varepsilon, \quad H_A\varphi_n = E_n\varphi_n,$$
$$[K_n + V(r)]\chi_\varepsilon = \varepsilon\chi_\varepsilon.$$

Using Eq. (2.2), we can write the complete Hamiltonian of the system, Eq. (1.8), in the form

$$H = H_0 + H', \tag{2.3}$$
$$H' = \sum_\alpha\{v_\alpha - \mathrm{Re}\langle t_\alpha\rangle\}. \tag{2.4}$$

The operator (2.4), which represents the difference between the complete Hamiltonian and the Hamiltonian of the optical model, is the cause of the formation of the compound nucleus, in which the excitation energy is distributed over a large number of degrees of freedom.

In order to calculate the transition probability Γ we shall look for a solution of the time-dependent Schrödinger equation

$$i\hbar\partial\Psi/\partial t = (H_0 + H')\Psi \tag{2.5}$$

in the form

$$\Psi = a(t)\Phi_{0\varepsilon}\exp\{-iE'_{0\varepsilon}t/\hbar\} + \sum_{n,\xi}{}'b_{n\xi}(t)\Phi_{n\xi}\exp\{-iE'_{n\xi}t/\hbar\} \tag{2.6}$$

with the initial conditions

$$a(0) = 1, \quad b_{n\xi}(0) = 0, \tag{2.7}$$

and where

$$E'_{n\xi} = E_{n\xi} + (\Phi_{n\xi}, H'\Phi_{n\xi}). \tag{2.8}$$

Substituting Eq. (2.6) into Eq. (2.5), we get (for $t < \Gamma^{-1}$) the system of equations

$$i\hbar da/dt = \sum_{n\xi}{}'(\Phi_{0\varepsilon}, H'\Phi_{n\xi})b_{n\xi}\exp\{i(E'_{0\varepsilon} - E'_{n\xi})t/\hbar\},$$
$$i\hbar db_{n\xi}/dt = a(\Phi_{n\xi}, H'\Phi_{0\varepsilon})\exp\{i(E'_{n\xi} - E'_{0\varepsilon})t/\hbar\} + \ldots. \tag{2.9}$$

We solve the system (2.9) with the initial conditions (2.7) by the substitution

$$a(t) = \exp\{-\Gamma t/2\}.$$

Then in the usual way we get

$$b_{n\xi} = (\Phi_{n\xi}, H'\Phi_{0\varepsilon}) \times \frac{\exp\{-i(E'_{0\varepsilon} - E'_{n\xi} - i\Gamma\hbar/2)\, t/\hbar\} - 1}{E'_{0\varepsilon} - E'_{n\xi} - i\Gamma\hbar/2},$$

$$\Gamma = \frac{2i}{\hbar} \sum_{n\xi}{}' |(\Phi_{0\varepsilon}, H'\Phi_{n\xi})|^2 \times \frac{1 - \exp\{i(E'_{0\varepsilon} - E'_{n\xi} - i\Gamma\hbar/2)\, t/\hbar\}}{E'_{0\varepsilon} - E'_{n\xi} - i\Gamma\hbar/2}. \quad (2.10)$$

Introducing the number of states $\omega(E)dE$ in the energy interval dE, we can go in Eq. (2.10) from the sum to an integral; then

$$\Gamma = \frac{2\pi}{\hbar} |(\Phi_{0\varepsilon} H'\Phi_{n_0\xi_0})|^2\, \omega(E_{n_0\xi_0}), \quad (2.11)$$

where $E_{n_0\xi_0} \approx E_{0\epsilon}$.

To find the energy dependence of Γ in the region of the resonances of the compound nucleus, we expand the function $\Phi_{0\epsilon}$ in terms of the complete set of the functions X_λ of the operator (2.3),

$$\Phi_{0\varepsilon} = \sum_\lambda A_{\lambda,0\varepsilon} X_\lambda. \quad (2.12)$$

It is easy to verify that the expansion coefficients are given by the formula

$$A_{\lambda,0\varepsilon} = (\Phi_{0\varepsilon}, H'X_\lambda)/(E_\lambda - E_{0\varepsilon}). \quad (2.13)$$

From the normalization of the functions X_λ and $\Phi_{n\xi}$ we have the equations

$$\sum_{n,\xi} A_{\lambda,n\xi} A^*_{\lambda',n\xi} = \delta_{\lambda\lambda'}; \quad \sum_\lambda A_{\lambda,n\xi} A_{\lambda,n'\xi'} = \delta_{nn'}\delta_{\xi\xi'}. \quad (2.14)$$

Substituting Eq. (2.12) into Eq. (2.11) and using Eq. (2.13), we get

$$\Gamma = \frac{2\pi}{\hbar} \left| \sum_\lambda{}' \frac{(\Phi_{0\varepsilon}, H'X_\lambda)(X_\lambda, H'\Phi_{n_0\xi_0})}{E_\lambda - E_{0\varepsilon}} \right|^2 \omega(E_{n_0\xi_0}). \quad (2.15)$$

In the region of overlapping resonances the sum over λ can be replaced by an integral; then

$$\Gamma = \frac{2\pi}{\hbar} \left| \int \frac{(\Phi_{0\varepsilon}, H'X_\lambda)(X_\lambda, H'\Phi_{n_0\xi_0})}{E_\lambda - E_{0\varepsilon}} \omega(E_\lambda)\, dE_\lambda \right|^2 \times \omega(E_{n_0\xi_0}). \quad (2.16)$$

If, on the other hand, the energy $E_{0\epsilon}$ falls in the region of isolated resonances, for example $E_{0\epsilon} \approx E_\lambda$, then in the sum over λ in Eq. (2.15) only one term will be important; consequently,

$$\Gamma \approx \frac{2\pi}{\hbar} \left| \frac{(\Phi_{0\varepsilon}, H'X_\lambda)(X_\lambda, H'\Phi_{n_0\xi_0})}{E_\lambda - E_{0\varepsilon}} \right|^2 \omega(E_{n_0\xi_0}) + \Gamma_0, \quad (2.17)$$

where Γ_0 is the part of Γ that is weakly energy dependent.

To calculate Γ from Eqs. (2.15) and (2.17) one must know the wave functions X_λ and $\Phi_{n\xi}$ and the energy dependence of the density of states $\omega(E)$.

Under certain simplifying assumptions, an estimate of the quantity Γ is made in the following section. Here we only remark that the expressions (2.16) and (2.17) make it possible to draw qualitative conclusions about the energy dependence of Γ. Namely, in the region of isolated resonances of the compound nucleus the energy dependence of Γ is of a resonance character. Outside the isolated resonances and in the region of overlapping resonances the quantity Γ increases with the energy, mainly owing to the change of the density of states $\omega(E)$. It must be noted, however, that our arguments are valid only for not too high excitation energies ($\varepsilon < 50$ Mev).

3. INTERPRETATION OF THE BROAD RESONANCES IN THE INTERACTION OF NEUTRONS WITH NUCLEI

The quantity $\hbar\Gamma$ defined by Eq. (2.15) characterizes the width of the single-particle levels of the nucleus. Therefore $\hbar\Gamma$ is to be compared with the experimentally observed widths ($\sim$ 2 Mev) of the broad maxima of the scattering cross-sections of low-energy ($\sim$ 5 Mev) neutrons averaged over the resonances.

For an estimate of the quantity Γ in the region of closely spaced resonances, using Eq. (2.13) we put Eq. (2.15) in the following form:

$$\hbar\Gamma = 2\pi\omega(E_{n_0\xi_0}) \Big| \sum_{\lambda} A_{\lambda,0\varepsilon} A_{\lambda,n_0\xi_0} (E_\lambda - E_{n_0\xi_0}) \Big|^2.$$

Now, neglecting off-diagonal terms of alternating sign, we get

$$\hbar\Gamma \approx 2\pi\omega(E_{n_0\xi_0}) \sum_{\lambda} |A_{\lambda,0\varepsilon}|^2 |A_{\lambda,n_0\xi_0}|^2 (E_\lambda - E_{n_0\xi_0})^2.$$

Introducing the density of states $\omega(E_\lambda)$ of the compound nucleus, we can replace the summation over λ by an integral over the energy. Then

$$\hbar\Gamma \approx 2\pi\omega(E_{n_0\xi_0}) \times \int |A_{\lambda,0\varepsilon}|^2 |A_{\lambda,n_0\xi_0}|^2 (E_\lambda - E_{n_0\xi_0})^2 \omega(E_\lambda)\, dE_\lambda. \tag{3.1}$$

In this same approximation the orthogonality condition (2.14) is written in the form

$$\int A_{\lambda,n\xi} A^*_{\lambda,n'\xi'} \omega(E_\lambda)\, dE_\lambda = \delta_{nn'}\delta_{\xi\xi'}. \tag{3.2}$$

To estimate the value of Γ we assume that the energy dependence of $|A_{\lambda,0\epsilon}|^2\omega(E_\lambda)$ can be represented by the expression

$$|A_{\lambda,0\varepsilon}|^2 \omega(E_\lambda) = \frac{B}{\Omega_{0\varepsilon}} \exp\left\{-\left(\frac{E_\lambda - E_{0\varepsilon}}{\Omega_{0\varepsilon}}\right)^2\right\}, \tag{3.3}$$

where B is determined from the condition (3.2). Then

$$B = 2\pi^{-1/2}[1 + \Phi(x)]^{-1}, \tag{3.4}$$

$$\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt, \quad x \equiv E_{0\varepsilon}/\Omega_{0\varepsilon}. \tag{3.5}$$

The quantity $\Omega_{0\epsilon}$ is expressed in terms of the value of the "second moment" W^2 introduced in the paper of Lane, Thomas, and Wigner[4]

$$W^2 \equiv \sum_{\lambda} (E_\lambda - E_{0\varepsilon})^2 |A_{\lambda,0\varepsilon}|^2. \tag{3.6}$$

Going from sum to integral and inserting (3.3), we get

$$W^2 = \frac{\Omega_{0\varepsilon}^2}{2[1 + \Phi(x)]} \left\{1 + \Phi(x) - \frac{2x}{\sqrt{\pi}} e^{-x^2}\right\}. \tag{3.7}$$

From Eq. (3.7) it follows that for $x < 1$ and for $x > 1$

$$\Omega_{0\varepsilon}^2 = 2W^2. \tag{3.8}$$

On the other hand, substituting Eq. (2.13) into Eq. (3.6), we have

$$W^2 = \sum_{\lambda} |(\Phi_{0\varepsilon}, H'X_\lambda)|^2 = (\Phi_{0\varepsilon}, (H')^2 \Phi_{0\varepsilon}), \tag{3.9}$$

where, according to Eqs. (2.4) and (1.4), the operator H' is given by

$$H' = \sum_{\alpha} (v_\alpha - \langle v_\alpha \rangle) - \mathrm{Re} \sum_{\alpha} \langle v_\alpha (D - v_\alpha)^{-1} v_\alpha \rangle. \tag{3.10}$$

Substituting Eq. (3.3) into Eq. (3.1) and using the equations $E_{0\epsilon} \approx E_{n_0\xi_0}$, $\Omega_{0\epsilon} \approx \Omega_{n_0\xi_0}$, we can write

$$\hbar\Gamma = \frac{8\omega(E_{n_0\xi_0})}{\Omega_{0\varepsilon}^2 [1 + \Phi(x)]^2} \times \int_0^\infty \omega^{-1}(E)(E - E_{0\varepsilon})^2 \exp\left\{-\frac{2(E - E_{0\varepsilon})^2}{\Omega_{0\varepsilon}^2}\right\} dE. \tag{3.11}$$

Equation (3.11) gives the width of a "giant" resonance in the neutron scattering cross-section (in the region of overlapping levels of the compound nucleus) and its dependence on the energy. To estimate the value of $\hbar\Gamma$ and its dependence on the energy $E_{0\epsilon}$ it is necessary to know the energy dependence of the density of states $\omega(E)$. According to the statistical model of the nucleus

$$\omega(E) = c \exp\left(\sqrt{bE}\right), \tag{3.12}$$

where the parameters c and b are functions of the mass number A. According to Ref. 5, satisfactory agreement with experimental values of the density of levels is obtained with the choice $b = 0.14(A - 12)$ for nuclei with mass numbers in the range $15 < A < 70$.

Equation (3.12) is inconvenient for analytical calculation, and therefore we set

$$\omega(E) = a \exp\{\beta E\} \tag{3.13}$$

and choose the coefficient β in such a way that in the range $7 < E < 30$ Mev, which is most important in the integral (3.11), Eq. (3.13) gives about the same increase of the levels as does Eq. (3.12).

For nuclei with mass numbers close to 60 we can take $\beta = 0.3$. It must, of course, be kept in mind that in the interaction of neutrons of energy less than 3 Mev with nuclei only those states of the compound nucleus are excited that have spins differing from that of the original nucleus by $\pm \frac{1}{2}$. Equation (3.12) refers to all possible levels of the compound nucleus. Therefore the energy dependence of the density of levels with a definite spin value must be given by a value of β less than 0.3.

Substituting Eq. (3.13) into Eq. (3.11), we find after simple calculations

$$\hbar\Gamma = \frac{2\sqrt{2}\,\Omega_{0\varepsilon}}{[1+\Phi(x)]^2} \int_{-x\sqrt{2}}^{\infty} y^2 \exp\{-y(y+2a)\}\,dy,$$

$$a \equiv \frac{\beta\Omega_{0\varepsilon}}{2\sqrt{2}}. \qquad (3.14)$$

For $\beta = 0$ and, consequently, for $a = 0$, using Eq. (3.8) and $x < 1$, we have

$$\hbar\Gamma = (\sqrt{\pi}\,\Omega_{0\varepsilon}/\sqrt{2})[1 + \Phi(x)]^{-2} \approx W_{0\varepsilon}.$$

This value agrees with that obtained in the paper of Lane, Thomas, and Wigner on the basis of the assumption that $\hbar\Gamma = W_{0\epsilon}$, which leads as follows from their calculation, to the excessive value $\hbar\Gamma = W_{0\epsilon} = 23$ Mev. It must, however, be noted that in the calculation of $W_{0\epsilon}$ in Ref. 4 account was taken only of the terms

$$\sum_{\alpha} (v_\alpha - \langle v_\alpha \rangle).$$

in the operator H' of Eq. (3.10).

If we adopt for $W_{0\epsilon}$ the value 23 Mev obtained in Ref. 4, then according to Eq. (3.8) $\Omega_{0\epsilon} \approx 32$ Mev. When the inequality $x \equiv \frac{E_{0\epsilon}}{32} < 1$ is satisfied, it follows from Eq. (3.14) that

$$\hbar\Gamma \approx 2\sqrt{2}\,\Omega_{0\varepsilon} \int_0^{\infty} y^2 \exp\{-y(y+2a)\}\,dy. \qquad (3.15)$$

It follows from Eq. (3.15) that the value $\hbar\Gamma = 2.5$ Mev is obtained, if we set $\beta \approx 0.13$. But if $\Omega_{0\epsilon} = 20$ Mev, we must take $\beta = 0.2$.

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