charge of the electron, while σ is the conductivity of the metal). Therefore, taking the spin magnetic moment into account leads to an albeit small but slowly-damped addition. In this paper, the coefficient of wave transmission through a sufficiently thick film $[d \gtrsim 2\delta \ln (\delta_{\text{eff}}/4\chi\delta)]$ is calculated taking this addition into account.

In this case, the complete system of equations is [see Eq. (14) of Ref. 1]

$$\operatorname{curl} \mathbf{E} = -i\omega \mathbf{B}/c; \quad \operatorname{curl} \mathbf{H} = 4\pi \mathbf{j}/c;$$

$$\mathbf{H} = \mathbf{B} - 4\pi \mathbf{M}; \quad \mathbf{M} = \chi(\mathbf{B} - \overline{\mathbf{w}});$$

$$\frac{\partial \mathbf{w}}{\partial z} v \cos\theta + \frac{\mathbf{w}}{t_0^*} = \frac{\overline{\mathbf{w}}}{t_0} + i\omega \mathbf{B};$$

$$\frac{1}{t_0^*} = \frac{1}{t_0} + \frac{1}{T_{\text{sp}}} + i\omega; \quad \overline{f} = \frac{1}{2}\int_0^\pi f \sin\theta \, d\theta.$$

The solution of this system is completely analogous to the solution of the system (26) of Ref. 1 and leads to the following formula for the transmission coefficient:

$$K \sim \left| \frac{\chi c^3 Z^2}{2\pi d (\omega + 1/T_{\text{sp}})} \right|^2, \quad 2\delta \ln \frac{\delta_{\text{eff}}}{4\pi\chi\delta} \ll d \ll \delta_{\text{eff}}.$$

In particular, for normal skin effect and $\omega \gg 1/T_c$

$$K \sim \left(\frac{2\chi c}{\sigma d}\right)^2 \gtrsim \frac{2\pi\chi^2\omega}{\sigma} \ln^{-2} \frac{c}{v\sqrt{2\pi\sigma t_0}} \sim 10^{-17}.$$

In conclusion, I would like to express my gratitude to I. M. Lifshitz and M. I. Kaganov for valuable discussions.

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Translated by M. D. Friedman
260

Radiation From a Point Charge, Moving Uniformly Along the Surface of an Isotropic Medium

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(Submitted to JETP editor February 18, 1957)
J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 1260-1261 (May, 1957)

THE MOTION OF CHARGED particles close to the surface of a dielectric material has been treated in a number of papers[1,2]. The case of a straight charge line moving close to a medium with arbitrary ε' and μ' was already treated by the author[3].

We shall now give some results of the corresponding case of a point charge. It is known[4] that in a uniform motion of a particle at a distance l from the surface the radiation into the surface is limited to wavelengths $\lambda\!\!\!^{-} \gtrsim l\sqrt{\mu'\varepsilon'}$ (more accurately $\lambda\!\!\!^{-} \gtrsim l\beta$). This allows to obtain a qualitative picture assuming a nondispersive medium. Then the energy emitted per unit time by Cerenkov radiation by a particle moving at a distance l from the medium with a a velocity $v = \beta c$ is given by

$$P = -\frac{e^2}{2\theta l^2}\frac{c}{\sqrt{\mu'\varepsilon' - 1}}\left\{\theta^2\mu'\left(1 - \frac{\mu'}{\sqrt{\mu'^2 + \Gamma^2}}\right) + \frac{\mu'\varepsilon - 1}{\varepsilon' - \mu'}\left(\frac{\mu'}{\sqrt{\mu'^2 + \Gamma^2}} - \frac{\varepsilon'}{\sqrt{\varepsilon'^2 + \Gamma^2}}\right)\right\}, \tag{1}$$

$$\theta^2 = 1 - \beta^2, \quad \Gamma = \gamma/\theta, \quad \gamma^2 = \mu'\varepsilon'\beta^2 - 1.$$

We note for comparison that the radiation per unit length from a charge filament of linear charge density ρ is[3]

$$P_l = (2v\rho^2/l)\,\varepsilon'\Gamma/(\varepsilon'^2 + \Gamma^2). \tag{2}$$

From these formulae we deduce that:

a) In the relativistic case $\theta \to 0$, independently of the characteristics of the medium, the radiated power approaches the limits

$$P \to e^2 c/2l^2; \quad P_l \to 0. \tag{3}$$

b) In the nonrelativistic case but if $\beta\sqrt{\varepsilon'} \gg 1$

$$P \approx \frac{e^2}{2l^2}\frac{c}{\sqrt{\varepsilon'}}\beta^2; \quad P_l \approx 2\frac{\rho^2}{l}\frac{c}{\sqrt{\varepsilon'}}\beta^2. \tag{4}$$

This means physically that at low velocity the field enters the medium practically perpendicularly to the surface if $\varepsilon' \gg 1$.

c) For a magnetic material ($\varepsilon' = 1$) at $\beta^2\mu' \gg 1$ and nonrelativistic velocity the radiated power is given by

$$P \approx e^2 c/2l^2\sqrt{\mu'}, \quad P_l \approx 2\rho^2 c/l\sqrt{\mu'}. \tag{5}$$

The form of these expressions is due to the fact that at particle velocities above the inversion velocity (see below) the field is being expelled from the magnetic medium.

d) Finally, for a medium of the type of a ferrite ($\mu' \gg \varepsilon' \gg 1$) we have for $\mu'\beta^2 \gg \varepsilon'$ and $\theta \approx 1$

$$P \approx (e^2c/2l^2)\sqrt{\varepsilon'/\mu'}, \quad P_l \approx (2\rho^2 c/l)\sqrt{\varepsilon'/\mu'}. \tag{6}$$

These expressions can be interpreted in an analogous fashion as the previous ones.

The force acting on the particle in a direction normal to the surface of the medium is given by

$$F = -\frac{e^2}{\pi\theta l^2}\left\{\frac{\mu'\varepsilon'-1}{\mu'-\varepsilon'} L(\varepsilon') - \left(\frac{\mu'\varepsilon'-1}{\mu'-\varepsilon'} + \mu'\theta^2\right) L(\mu') + \frac{\pi}{4}\theta^2\right\}, \tag{7}$$

$$L(\lambda) = \frac{\lambda}{\sqrt{(\lambda^2-1)(\lambda^2+\Gamma^2)}} \tan^{-1}\left(\sqrt{\frac{\lambda^2-1}{\lambda^2+\Gamma^2}}\,\Gamma\right) + \int_0^{\tan^{-1}\Gamma} \frac{d\alpha}{\lambda + \sqrt{1-(\Gamma^2+1)\sin^2\alpha}}. \tag{8}$$

The equivalent force per unit length acting on a charged line is

$$F_l = -(\rho^2/l)(\varepsilon'^2 - \Gamma^2)/(\varepsilon'^2+\Gamma^2). \tag{9}$$

One sees from (8) that in the case of a magnetic medium ($\mu' \gg 1$, $\varepsilon' = 1$) the force is attractive up to $\Gamma \sim 1$ and becomes attractive again at $\Gamma \approx \mu'$. For $\mu' \gg \Gamma \gg 1$ the repulsive force equals $e^2/(4l^2)$. On the other hand, in the case of a dieletric, the force is always attractive and has a value close to $-e^2/(4l^2)$.

For comparison we note that for a charged line the force is repulsive beginning at the inversion velocity (*i.e.*, at the velocity where $\Gamma = \varepsilon'$) and remains repulsive from there on for all velocities.

In conclusion, the author wishes to express his gratitude to Prof. A. A. Sokolov for his interest in this work, to I. Kvasnitsa for the discussion of the calculations, and to V. Pafomov for acquainting the author with his work[2].

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Translated by M. Danos
261

Photon Green Function Accurate to e^4

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(Submitted to JETP editor March 1, 1957)
J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 1261-1262
(May, 1957)

THE SUM OF DIAGRAMS for the self energy of the photon, inclusive of fourth-order diagrams, was calculated. After carrying out renormalization in the usual way and evaluating the integrals over the momenta, the following expression was obtained

$$\begin{aligned} iG_{\mu\nu} = \frac{k^2\delta_{\mu\nu} - k_\mu k_\nu}{k^4}\Bigg\{ & 1 + \frac{e^2}{2\pi^2}\int_0^1 dx\, X \ln\left(1 + \frac{k^2}{m^2}X\right) + \left[\frac{e^2}{2\pi^2}\int_0^1 dx\, X\ln\left(1+\frac{k^2}{m^2}X\right)\right]^2 \\ & + \frac{e^4}{16\pi^4}\Bigg[\left[\frac{1}{2}\int_0^1 dx \ln\left(1+\frac{k^2}{m^2}X\right)\right]^2 + \int_0^1 dx\,(2-3x+2x^2)\ln\left(1+\frac{k^2}{m^2}X\right) \\ & + \int_0^1 dx\int_0^1 dy\left(y - 2y^2 - \frac{y}{x}\right)\ln\left(1 + \frac{k^2 Y x}{m^2(1-y+yx)}\right) + \int_0^1 dx\int_0^1 dy\int_0^1 dz\int_0^1 dt \\ & \times \frac{k^2[1-x(1-t)-zt][x(1-t)+zt] + m^2\{1+2[x(1-t)+zt][yx(1-t)+yzt+x(1-y)]\}}{(1-t)(k^2X+m^2) + y[t(k^2Z+m^2)+k^2T(z-x)^2]} \\ & - \int_0^1 dx\int_0^1 dy\int_0^1 dz\,\frac{2m^2(3-x)yz^2(1-yz)}{k^2x(Yz+Zy^2)+m^2(1-z+zx)} + \frac{49}{54} + \frac{5}{12} - \frac{11}{6}\zeta(2)\Bigg] + \cdots\Bigg\}, \end{aligned} \tag{1}$$