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Phenomenological Study of the Effect of Nonconducting Medium in Quantum Electrodynamics

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AS IS KNOWN (see, for instance, the review by Feinberg [1]), the influence of the surrounding medium on the collision of particles with a small longitudinal momentum transfer can become considerable even for high energies. Furthermore, the influence of the surrounding medium must necessarily be important in the higher order perturbation calculations because, in the integration over the 4-momentum of virtual photons, one necessarily has to include the wavelength region for which the presence of the neighboring atoms cannot be ignored. This was pointed out for the first time by Landau and Pomeranchuk [2] who noted that the magnitude of the radiative corrections is strongly influenced by the multiple scattering of the electron in the medium. Ter-Mikaelian [3] noted that the radiative corrections should be especially strongly influenced by the deviation, for soft quanta, of the dielectric susceptibility $\varepsilon(\omega)$ from unity.

In view of this, it would be of some interest to construct a covariant Feynman-Dyson perturbation theory for a phenomenological quantum electrodynamics in a medium.

A non-covariant formulation of quantum electrodynamics was given by Ginzburg [4] and Sokolov [5] and was later expanded by Watson and Jauch [6]. For the construction of a covariant perturbation theory in a medium, it is convenient to use the formulation of phenomenological quantum electrodynamics proposed by Tamm, in which the properties of the medium are described by the dielectric and magnetic susceptibility tensor $\varepsilon_{\nu\lambda\rho\sigma}$ which relates the field tensor $F_{\nu\lambda}$ to the induction tensor $H_{\nu\lambda}$:

$$H_{\nu\lambda} = \varepsilon_{\nu\lambda\rho\sigma} F_{\rho\sigma}. \tag{1}$$

In homogeneous isotropic matter, $\varepsilon_{\nu\lambda\rho\sigma}$ has the form

$$\varepsilon_{\nu\lambda\rho\sigma} = \mu^{-1} (\delta_{\nu\rho} + \varkappa u_\nu u_\rho)(\delta_{\lambda\sigma} + \varkappa u_\lambda u_\sigma). \tag{2}$$

Here u_ρ is the 4-velocity of the medium, μ is the magnetic susceptibility, $\varkappa = \varepsilon\mu - 1$, $\varkappa$ and μ are invariants; the Feynman notation is used. If the potential of the electromagnetic field is introduced in the usual way

$$F_{\rho\sigma} = \partial_\rho A_\sigma - \partial_\sigma A_\rho \tag{3}$$

and the components are constrained to the auxiliary condition

$$\partial_\rho (A_\rho + \varkappa u_\rho u_\sigma A_\sigma) = 0, \tag{4}$$

then, from the field equation

$$\partial_\nu H_{\nu\lambda} = -j_\lambda \tag{5}$$

and from (2) – (4) one obtains the following equation for the potential

$$\mu^{-1} (\partial_\rho^2 + \varkappa (u_\rho \partial_\rho)^2)(\delta_{\lambda\sigma} + \varkappa u_\lambda u_\sigma) A_\sigma = -j_\lambda. \tag{6}$$

In the Heisenberg representation of quantum theory, the field operators satisfy the same equation (5); the commutation relations for the free field operators have been found in Ref. 6. The rules for the computation of the scattering matrix elements can be easily obtained, for instance by the method of Galanin [8]. But, instead of the usual Green function for photon, the formulae will involve the Green function for the free equation (6), determined by

$$\mu^{-1} (\partial_\rho^2 + \varkappa (u_\rho \partial_\rho)^2)(\delta_{\lambda\sigma} + \varkappa u_\lambda u_\sigma) G_{\lambda\nu}(x, x') = -\delta_{\sigma\nu}\delta(x - x'), \tag{7}$$

Changing to the momentum representation, it is easy to obtain the following expression for the Green function:

$$G_{\lambda\nu}(x, x') = (2\pi)^{-2} \int d^4k \mu \left(\delta_{\lambda\nu} - \frac{\varkappa}{1+\varkappa} u_\nu u_\lambda\right) \times \{k_\rho^2 + \varkappa (u_\rho k_\rho)^2\}^{-1} \exp ik(x - x'). \tag{8}$$

The choice of the integration path with respect to the poles is determined in the following way: for positive frequencies, it is required that an infinitesimal absorption occur. For negative frequencies, the contour is chosen in such a way as to make the theory symmetric with respect to past and future.

It is also necessary to note that the denominator of the integrand in (8) vanishes if both $u_\rho k_\rho$ and k_ρ^2 tend to zero. This is due to the fact that perturbation theory is not applicable to soft quanta and leads to singularities in the expressions for the radiative corrections ("infrared catastrophe"). In order to obtain converging expressions, it is necessary to limit, in a covariant way, the integration region from the soft-quanta end. In analogy with perturbation theory in vacuum, let us introduce an additional term in the denominator of the Green function (8) – a constant λ^2 which cuts off the infrared region. The magnitude of λ depends only on the conditions of applicability of the perturbation theory and can be left unspecified if one is interested only in final results independent of λ. Taking this into account, the final expression for the Green function can be obtained in the following form

$$G_{\lambda\nu}(x, x') = (2\pi)^{-2}\int d^4k g_{i\lambda}\{k_\rho^2 + \varkappa (u_\rho k_\rho)^2 - \lambda^2\}^{-1} g_{i\nu}\exp ik(x-x'),$$
$$g_{i\lambda} = e_{i\sigma}(\delta_{\lambda\sigma} - u_\lambda u_\sigma[1-(1+\varkappa)^{-1/2}]);\ e_{i\lambda}e_{i\nu} = \delta_{\lambda\nu};\ e_{i\lambda}e_{k\lambda} = \delta_{ik}. \tag{8'}$$

This way, the matrix elements can be computed using the standard diagram technique, where an internal photon line corresponds in the momentum representation, to the factor

$$\{k_\rho^2 + \varkappa (u_\rho k_\rho)^2 - \lambda^2\}^{-1}, \tag{9}$$

and the vertices at its ends correspond to the operators

$$g_{i\nu}\gamma_\nu \tag{10}$$

To obtain the factor corresponding to a vertex with an exterior photon line, it is convenient to make use of a method mentioned by Feynman in a footnote in the second section of his paper[9]. After integrating (8) over k_4 it is easy to see that the factor corresponding to an internal line can be interpreted as the result of exchange by real photons of all possible momenta and polarizations. Since an external line corresponds to emission (absorption) of a real photon with definite momentum and polarization, it is easy to determine the relationship between the factors. The result can be formulated in the following way: in the momentum representation, a vertex with an external photon line corresponds to the operator

$$g_{i\nu}\gamma_\nu [2k_4 + u_4 2(u_\rho k_\rho)\varkappa + u_4 (u_\rho k_\rho)^2 \partial\varkappa / \partial (u_\rho k_\rho)]^{-1/2}. \tag{11}$$

Here, in contrast with (9) and (10), the components k_ρ are related by

$$k_\rho^2 + \varkappa (u_\rho k_\rho)^2 = \lambda^2, \tag{12}$$

which has to be considered as a definition of the dependence $k(k_4)$. The other factors of the matrix element have the same form as in the case of vacuum. Let us illustrate the application of the theoretical apparatus with an example of Cerenkov radiation. The matrix element of the process is

$$S^{(1)} = -e(2\pi)^4 (\bar{u}_2,\ g_{i\nu}\gamma_\nu u_1)$$
$$\times \left\{2\omega\left(1+\varkappa+\frac{1}{2}\omega\frac{\partial\varkappa}{\partial\omega}\right)\right\}^{-1/2}\delta(p_1 + k_1 - p_2). \tag{13}$$

The emission probability of a non-polarized photon by a non-polarized electron can be obtained in the form (medium at rest):

$$dW = \mu\frac{e^2}{4\pi}d\omega\beta\left\{1-\beta^{-2}(1+\varkappa)^{-1} + \frac{\omega}{p\beta}\frac{\varkappa}{1+\varkappa} - \frac{\omega^2}{4p^2}\left[\varkappa + \frac{\varkappa}{1+\varkappa}\right]\right\}. \tag{14}$$

The energy radiated by the electron per unit time is equal to

$$W = \frac{e^2}{4\pi}\mu\beta\int_{\omega_{\min}}^{\omega_{\max}}\omega d\omega\left\{1-\beta^{-2}(1+\varkappa)^{-1} + \frac{\omega}{p\beta}\frac{\varkappa}{1+\varkappa} - \frac{\omega^2}{4p^2}\left[\varkappa + \frac{\varkappa}{1+\varkappa}\right]\right\}. \tag{15}$$

The conservation laws determine the direction of the radiation and the limits of integration

$$\cos\vartheta = (1/\beta\sqrt{1+\varkappa}) - (\omega/2m\beta)\sqrt{1-\beta^2}\varkappa/\sqrt{1+\varkappa}. \tag{16}$$

The results coincide completely with those of Refs. 4 and 5 for $\mu = 1$ and go into the classical formula as the electron's recoils are neglected.

As a conclusion, I wish to express my gratitude

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Two Possible Schemes of Non-Conservation Of Parity in Weak Interactions

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ONE OF THE possible explanations of the decay of K^+-mesons into two and three π-mesons consists in the supposition that spatial parity is not conserved in weak interactions[1]. If one accepts this hypothesis then the question arises: should charge parity and parity relative to reflection in time be conserved in weak interactions. As is well known[2], the connection between spin and statistics requires that all interactions be invariant under the product of the three transformations: reflection of the three spatial coordinates I, reflection in time T and charge conjugation C, *i.e.*, symbolically $ITC = 1$. Therefore[3] with violation of spatial parity in weak interactions ($I \neq 1$) there are three possibilities: I) weak interactions are invariant under reflection in time ($T = 1$), but are not invariant under charge conjugation, so that $IC = 1$; II) weak interactions are invariant under charge conjugation ($C = 1$) but are not invariant under reflection in time and $IT = 1$; III) weak interactions are not invariant under either charge conjugation or reflection in time, but $ITC = 1$. If one accepts the last possibility, then the fact that a K^0-meson with a long lifetime exists[4] would appear to be a pure coincidence in so far as the argument of Gell-Mann and Pais[5], on the basis of which it was predicted, and would be valid only under conservation of either charge parity or parity relative to reflection in time. This forces us to discard the third possibility and consider only the first two.

In this article we consider what physical phenomena could occur with either of these alternative possibilities.

The first of these possibilities, as remarked by Landau[6], corresponds physically to the assumption that all interactions are invariant under simultaneous interchange of right and left and change from particle to antiparticle. The physical significance of the second assumption is that all interactions remain unchanged only if the motion proceeds backwards in time together with the transition from right to left.

We consider first scheme I, *i.e.*, when, together with violation of spatial parity, invariance relative to reflection in time is conserved. At $t \to -\infty$ let there be a system of particles in state a, with particle momenta $\mathbf{p}_a$ and a mean value of spins $\mathbf{s}_a$. Let, further, as a result of interaction, this system go into a different system of particles (at $t \to \infty$) with momenta $\mathbf{p}_b$ and mean values of the spins $\mathbf{s}_b$. From the invariance under reflection in time it follows[7] that the transition matrix element $S^I_{ab}(\mathbf{p}_a, \mathbf{s}_a; \mathbf{p}_b, \mathbf{s}_b)$ is connected in the following way with the matrix element of the inverse process $S^I_{ba}(\mathbf{p}_b, \mathbf{s}_b; \mathbf{p}_a, \mathbf{s}_a)$

$$S^I_{ab}(\mathbf{p}_a, \mathbf{s}_a; \mathbf{p}_b, \mathbf{s}_b) = S^I_{ba}(-\mathbf{p}_b - \mathbf{s}_b; -\mathbf{p}_a, -\mathbf{s}_a). \quad (1)$$

The matrix element S_{ba}, viewed as a function of its arguments $\mathbf{p}_a$, $\mathbf{p}_b$, *etc.*, does not have, in general, the same functional form as the function S_{ab}. Thus, we cannot extract any help directly from Eq. (1). However, if the transition $a \to b$ is considered to go as a result of a weak interaction, then in the first non-vanishing approximation of this interaction, the relation of detailed reversibility holds:

$$S_{ab}(\mathbf{p}_a, \mathbf{s}_a; \mathbf{p}_b, \mathbf{s}_b) = -S^*_{ba}(\mathbf{p}_b, \mathbf{s}_b; \mathbf{p}_a, \mathbf{s}_a). \quad (2)$$

[For the validity of (2) it is important that the transition proceed as a result of a weak interaction, but it is not necessary that the particle motion as a whole in the initial or final states be describable