The Problem of Saturation of the Hall "Constant" in Semiconductors in Strong Magnetic Fields

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An expression is derived for the Hall "constant" in strong magnetic fields; it is valid for semiconductors that contain narrow bands.

1. A RECENTLY PUBLISHED work[1] explains the asymptotic behavior of the resistance and Hall "constant" of metals in strong magnetic fields. In particular, the following expression is obtained for the Hall "constant":

$$R = -1/ec(n_1 - n_2) \qquad (1)$$

(n_1 and n_2 are the electron and hole densities). The expression is valid in those cases in which only closed equal-energy surfaces play a role. An analogous expression has also been obtained for semiconductors[2]. It reduces to the usual expressions in the special cases of a donor ($n_2 = 0$) and of an acceptor ($n_1 = 0$) semiconductor.

Formula (1) fails for an intrinsic semiconductor, since in this case $n_1 = n_2$. The subject of the present communication is the derivation of an expression for the Hall constant that is valid for intrinsic semiconductors and is especially effective for semiconductors with narrow bands.

For simplicity, a semiconductor is considered that possesses the energy spectrum represented in the figure. It is supposed that at $T = 0$, bands a and b are completely filled, band c empty. Upon increase of temperature, electrons from band b get into band c. The distance between the bottom of band b and the top of band a is so great that excitation of electrons from band a is known to be negligible. This is a representative model of an intrinsic semiconductor. By narrowing band b, we pass over to a donor semiconductor. Generalization to the case of several bands located close together, and likewise passage to an acceptor semiconductor in such a scheme, are trivial.

A considerable simplification results from the assumption that in band b there is *only one open surface*, and that in band c the open surfaces are located so high that excitation of electrons into these states may be neglected. A similar relation must clearly exist if band b is appreciably narrower than band c.

2. The treatment presented in Ref. 1 shows that in our case the conductivity matrix $\sigma_{ik}(H)$ has a form determined by expression (24) of that reference. This means that the Hall "constant" R in large fields is determined by

$$R = \rho_{yx}/H = 1/\sigma_{xy}H. \qquad (2)$$

It should be mentioned that in consequence of the smallness of the numbers of conduction electrons in semiconductors, use of formula (2) in this case is permissible at larger magnetic fields than in the case of metals.

In accordance with Eq. (25) of Ref. 1, the asymptotic expression for σ_{xy} may be written in our case in the following form:

$$\sigma_{xy} = -\frac{2ce}{H}\left\{\int_0^{\varepsilon_0} f_0'(\varepsilon)V(\varepsilon)\,d\varepsilon - \int_{\varepsilon_0}^{\varepsilon_1} f_0'(\varepsilon)V(\varepsilon)\,d\varepsilon + \int_{\varepsilon_2}^{\infty} f_0'(\varepsilon)V(\varepsilon)\,d\varepsilon\right\}. \qquad (3)$$

Here ε_0 is the value of the energy on the open surface; the values ε_1 and ε_2 are evident from the figure (see below); $V(\varepsilon)$ is the volume *inside* the surface $\varepsilon(\mathbf{p}) = \varepsilon$.* Upon integrating (3) by parts, we get

$$\sigma_{xy} = (2ce/Hh^3)\left\{-f_0(\varepsilon_0)[V(\varepsilon_0 - 0) + V(\varepsilon_0 + 0)] + \int_0^{\varepsilon_0} f_0(\varepsilon)V'(\varepsilon)\,d\varepsilon - \int_{\varepsilon_0}^{\varepsilon_1} f_0(\varepsilon)V'(\varepsilon)\,d\varepsilon + \int_{\varepsilon_2}^{\infty} f_0(\varepsilon)V'(\varepsilon)d\varepsilon\right\}.$$

* The exact definition of $V(\varepsilon)$ is evident from (22′) of Ref. 1.

Here

$$V(\varepsilon_0 - 0) = \lim_{\substack{\varepsilon \to \varepsilon_0 \\ (\varepsilon < \varepsilon_0)}} V(\varepsilon); \quad V(\varepsilon_0 + 0) = \lim_{\substack{\varepsilon \to \varepsilon_0 \\ (\varepsilon > \varepsilon_0)}} V(\varepsilon),$$

and we have used the fact that $V(0) = V(\varepsilon_1) = V(\varepsilon_2) = 0$. Obviously $2h^{-3}[V_0(\varepsilon_0 - 0) + V(\varepsilon_0 - 0)] = N$ is the number of states in band b, and

$$2V'(\varepsilon)/h^3 = \begin{cases} n(\varepsilon) & 0 < \varepsilon < \varepsilon_0; \ \varepsilon_2 \leqslant \varepsilon; \\ -n(\varepsilon) & \varepsilon_0 < \varepsilon \leqslant \varepsilon_1, \end{cases}$$

where $n(\varepsilon)$ is the density of states per unit energy interval. We recall that $m > 0$ for $\varepsilon < \varepsilon_0$; but that for $\varepsilon_1 > \varepsilon > \varepsilon_0$, $m < 0$. From the normalization condition we have

$$\int_0^{\varepsilon_1} f_0(\varepsilon)\, n(\varepsilon)\, d\varepsilon + \int_{\varepsilon_2}^{\infty} f_0(\varepsilon)\, n(\varepsilon)\, d\varepsilon = N$$

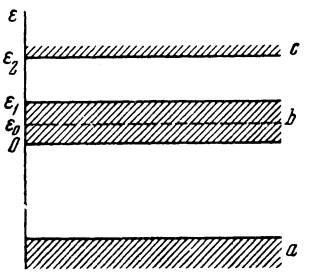
Thus

$$R = 1/ecN\,(1 - f_0(\varepsilon_0)); \qquad f_0(\varepsilon_0) = [\exp\{(\varepsilon_0 - \mu(T))/kT\} + 1]^{-1}. \tag{4}$$

If the band b is broad, the chemical potential $\mu(T)$ is located approximately in the middle of the forbidden band[3]. Then $e^{(\varepsilon_0 - \mu)/kT} \ll 1$, and expansion in the exponential gives

$$R = e^{(\mu - \varepsilon_0)/kT}/ecN. \tag{5}$$

We are assuming that the distance on an energy scale from the open surface in band b to the chemical potential is less than the distance from μ to an open surface in band c. The appropriate generalization to take account of an open surface in band c is carried out in an obvious manner. The formula (5) obtained corresponds to an intrinsic semiconductor.



Qualitatively, the formula is evidently valid not only in the case of a single open surface, but also when a layer of open surfaces is narrow in comparison with the width of the band.

Passage to the limit of a donor level, $\varepsilon_1 = 0$, is accomplished as follows:

$$\int_0^{\varepsilon_1} f_0(\varepsilon)\, n(\varepsilon)\, d\varepsilon = N f_0(0); \quad f_0(\varepsilon_0) = f_0(0).$$

Hence (*cf.* for example, Ref. 4)

$$R = \frac{1}{ecN(T)}; \quad N(T) = \int_{\varepsilon_2}^{\infty} f_0(\varepsilon)\, n(\varepsilon)\, d\varepsilon.$$

Here N is the number of electrons in the conduction band.

If the band b is very narrow ($\varepsilon_1 \ll kT$) but has a finite width, then the Hall "constant" can be written in the following form:

$$R = \frac{1}{ecN(T)}\left\{1 - \frac{\Delta N}{N(T)}\right\}.$$

Here $N(T)$ is the number of electrons in band c, and

$$\Delta N = f_0'(\varepsilon_0) \int_0^{\varepsilon_1} (\varepsilon - \varepsilon_0)\, n(\varepsilon)\, d\varepsilon.$$

It is necessary to set special limits to the applicability of the formulas obtained here. According to Swanson,[2] formula (1) is applicable when $H \gg H_0$, where H_0 is the magnetic field at which the period of revolution of an electron is equal to the relaxation time. However, since the electrons in a narrow band must have large effective masses, *i.e.*, small mobilities (as the width of the band approaches zero, the effective mass becomes infinite*), it may be concluded that H_0 is appreciably larger for semiconductors with narrow bands than for metals or for semiconductors of other types. Furthermore, one must keep in mind that the expression (3) that we have used for σ_{xy} is the first term of an expansion in powers of $1/H$. In the case of *low temperatures*, for an intrinsic semiconductor, the coefficient of $1/H$ will be so small [*cf.* Eq. (5)] that the next term of the expansion in powers of $1/H$ will be significant, and the formulas obtained here will be invalid. A similar situation always exists (practically at all temperatures) for metals with an equal number of electrons and "holes" (*cf.* Ref. 1).

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* For example, Samoilovich and Klinger[5] showed that $m_{eff} \sim 1/\varepsilon_1$. True, according to an estimate made in the the same work $m_{eff} \sim m_0$ (m_0 = mass of a free electron).

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[2] J. A. Swanson, Phys. Rev. **99**, 1799 (1955).

[3] F. F. Vol'kenshtein, *Electrical Conductivity of Semiconductors* (Gostekhizdat, 1947).

[4] A. H. Wilson, *Theory of Metals* (Cambridge University Press, 1953), Chap. VIII.

[5] A. G. Samoilovich and M. I. Klinger, J. Tech. Phys. (U.S.S.R.) **25**, 2050 (1955).

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