[8] A. E. Glauberman and I. P. Iukhnovskii, J. Exptl. Theoret. Phys. (U.S.S.R.) 22, 562 (1952).

[9] I. P. Iukhnovskii, J. Exptl. Theoret. Phys. (U.S.S.R.) 27, 690 (1954).

Translated by R. T. Beyer
236

SOVIET PHYSICS JETP VOLUME 5, NUMBER 5 DECEMBER, 1957

Relation Between Scattering and Multiple Particle Production

S. Z. BELEN'KII

P. N. Lebedev Institute of Physics, Academy of Sciences, U.S.S.R.

(Submitted to JETP editor July 5, 1956)

J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 1171-1175 (May 1957)

According to Fermi's theory, the multiple production of particles in high-energy collisions should be determined by the statistical weight of the respective states. In the present investigation, the statistical weight is calculated while taking into account the mutual interaction of pairs of particles, and it is shown that in this case the phase shift of the interacting particles is involved as a characteristic of the interaction. In resonance scattering, the effect of the scattering reduces to the appearance of an intermediate "isobaric" state which should be included in the statistical theory.

THE STATISTICAL THEORY of multiple production as originally suggested by Fermi[1] is well known to be in disagreement with experiments. In neutron-proton collisions, at energies around 2 Bev, the production of two mesons is considerably more probable than predicted by the theory[2].

Theoretical and experimental results disagree by a factor of 20. It was suggested by Nikishov and the author[3] to modify Fermi's theory according to the following considerations. The cross section of the scattering of π-mesons against nucleons has a strong maximum at a center-of-mass energy of about 160 Mev. This maximum corresponds to a P-state and isotopic spin $^3\!/_2$. These experiments indicate the presence of a strong interaction between π-mesons and nucleons in these circumstances. Such an interaction must also manifest itself in multiple particle production. It is evidently not taken into account in the original form of Fermi's theory.

A strong interaction between mesons and nucleons may be described qualitatively through the presence of an intermediate, rapidly decaying state of the meson-nucleon system with spin $^3\!/_2$ and isotopic spin $^3\!/_2$.

This concept of a so-called "isobaric" state was used in a number of articles[5,6], including also the work of Tamm and his collaborators[7]. We have suggested (see Ref. 3) that this isobaric state should be included in the statistical theory of multiple production, thus effectively allowing for the strong interaction between the particles. It was suggested that in a collision between two nucleons or a nucleon and a meson, there may appear "quasi-particles" (isobaric states of the system) of mass $1.32\, M_0$, where M_0 is the nucleon mass, having isotopic spin $^3\!/_2$ and ordinary spin $^3\!/_2$, and rapidly decaying into a nucleon and a meson. It was also suggested that the probability for the production of such "particles" may be determined from their statistical weight. A comparison was made between the modified Fermi theory and experimental results on the multiple production of mesons in nucleon-meson and meson-nucleon collisions in the energy range of 1.4 to 5 Bev (see Refs. 3 and 8). Comparison shows that the theory agrees with experiments regarding the probability of various multiple production processes, and also regarding the energy distribution of secondary particles.

In the present article we want to bring up certain theoretical considerations regarding the relation between resonance scattering and multiple particle production.

First we shall introduce the following example suggested by Landau. We shall consider the statistical sum of the states of a non-ideal gas. In the first approximation we shall assume an ideal gas, and in the second we shall include only two-particle interactions (see Ref. 9). Uhlenbeck and Beth[10] have shown that the additional term appearing in the sum over the states when two-particle interaction is included, may be represented in the following form:

$$Z = Z_1 + Z_2, \tag{1}$$

where Z_1 is the sum over states of the discrete spectrum of the two particles, and Z_2 is the sum over the continuous spectrum including two-particle interaction:

$$Z_2 = \frac{1}{\pi} \sum_l \int_0^\infty (2l+1) \frac{\partial \eta_l}{\partial p} e^{-p^2/mT} dp, \tag{2}$$

where $\eta_l(p)$ is the phase shift, p the momentum of the particle in the center-of-mass system, l the orbital angular momentum, T the temperature, and m the mass of the particle. If the scattering shows resonance character for some value of l, then

$$\eta_l = \eta_l^0 + \tan^{-1} \frac{\Gamma}{E_0 - E}, \tag{3}$$

where η_l^0 is the phase shift due to resonance. The phase changes by π when going through resonance. If the resonance is sufficiently sharp, then the expression for Z_2 reduces to

$$Z_2 = (2l+1)\, e^{-p_0^2/mT}, \tag{4}$$

where p_0 is the momentum at which maximum scattering takes place. Expression (4) corresponds to an extra "discrete" state of positive energy for the two-particle system.

Now, we shall turn directly to the problem of interest and consider a π-meson-nucleon collision. According to the statistical theory, the production of n mesons as a result of the collision is proportional to the statistical weight of the system consisting of one nucleon and n mesons.

Excluding the interaction of secondary particles, the statistical weight is computed by the following formula

$$S_0(W, n, M_0) = \left[\frac{\Omega}{(2\pi\hbar)^3}\right]^n \frac{1}{n!} \int \prod_{i=1}^{n} d\mathbf{p}_i d\mathbf{p}\, \delta(W - \Sigma W_i - W_M)\, \delta(\Sigma \mathbf{p}_i + \mathbf{p}). \tag{5}$$

W is here the energy of the system, M_0 the nucleon mass (in energy units), Ω the effective volume (a quantity of the order of $(4\pi/3)(\hbar/\mu c)^3$, where μ is the meson mass), $\mathbf{p}_i$ the momentum of the ith meson, $\mathbf{p}$ the momentum of the nucleon, W_i the energy of the ith meson, W_M the energy of the nucleon, and the δ-function insures conservation of energy and momentum. The integration is to be carried out over the momentum space of all the mesons and of the nucleon.

Now let us compute the statistical weight including the interaction of the nucleon with the mesons. We shall assume two-particle interaction. First let us single out in Eq. 5 a pair of particles, say a nucleon and a meson. We transform to the variables $\mathbf{p}_c$ and $\mathbf{p}'$, where $\mathbf{p}_c$ is the momentum of the center of mass of the nucleon-meson system, and $\mathbf{p}'$ is the momentum of the meson (or the nucleon) in the center-of-mass system. The formula for the statistical weight may then be written in the following form:

$$S^{(1)}(W, n, M_0) = \sum_{\mathbf{p}'} \left[\frac{\Omega}{(2\pi\hbar)^3}\right]^{n-1} \frac{1}{n!} \int \prod_{i=1}^{n-1} d\mathbf{p}_i d\mathbf{p}_c \delta\Big(W - \sum_{i=1}^{n-1} W_i - W_c - W'\Big)\, \delta\Big(\sum_{i=1}^{n-1} \mathbf{p}_i + \mathbf{p}_c\Big). \tag{6}$$

W_c is here the kinetic energy of the center of mass of the two particles, and W' is their total energy in the system where their center of mass is at rest.

We shall sum over all possible states of the nucleon-meson system in the same fashion as this was done for a non-ideal gas (see Ref. 9). At large distances, the wave function for the state with orbital angular momentum l and momentum p' has the following well-known asymptotic form

$$\psi = \frac{\text{const}}{r} \sin\Big(\frac{p'}{\hbar} r - \frac{l\pi}{2} + \eta_l\Big). \tag{7}$$

The phase shift η_l depends on the specific form of the particle interaction. Let the range of r be limited to some large but finite radius R_0. The possible values of the momentum are then determined by the condition

$$R_0 p'/\hbar - l\pi/2 + \eta_l = s\pi, \tag{8}$$

where s is an integer. If R_0 is sufficiently large, one may assume that s varies continuously and transform the summation in Eq. 6 to an integral

$$S^{(1)}(W, n, M_0) = \sum_{p'} S^{(1)}_{p'} = \int \sum_{l} (2l+1) S^{(1)}_{p'} \frac{1}{\pi} \left(\frac{R_0}{\hbar} + \frac{\partial \eta_l}{\partial p'}\right) dp' , \tag{9}$$

where $S^{(1)}_{p'}$ is the expression appearing in Eq. 6 after the summation sign; the factor $(2l + 1)$ and the sum over l is related to the angular momentum degeneracy. The first term in Eq. 9 gives the statistical weight exclusive of the interaction, *i.e.*, it leads to Eq. 5; the second term which includes the two-particle interaction reduces to the following expression:

$$S^{(2)} = \sum_{l} \frac{2l+1}{\pi} \left[\frac{\Omega}{(2\pi\hbar)^3}\right]^{n-1} \frac{1}{n!} \int \prod_{i=1}^{n-1} d\mathbf{p}_i d\mathbf{p}_c \delta\left(W - \sum_{i=1}^{n-1} W_i - W_c - W'\right) \times \delta\left(\sum_{i=1}^{n-1} \mathbf{p}_i + \mathbf{p}_c\right) \frac{\partial \eta_l}{\partial W'} dW' . \tag{10}$$

We have transformed here from the variable p' to the variable W' (W' is the total energy in the center-of-mass system), which is uniquely related to p'. Equation 10 should be multiplied by n to allow for the presence of the n mesons. The resulting expression for the statistical weight including two-particle interaction between nucleon and mesons is written as follows:

$$\begin{gathered} S(W, n, M_0) = S_0(W, n, M_0) \\ + \sum_{l} \frac{2l+1}{\pi} \int S_0(W, n-1, W') \frac{\partial \eta_l}{\partial W'} dW' . \end{gathered} \tag{11}$$

The formula for $S_0(W, n, M_0)$ is obtained here from Eq. 5, and $S_0(W, n-1, W')$ is the statistical weight for n non-interacting particles: $n - 1$ mesons and one heavy "particle" of mass W'. Let us assume that the scattering exhibits a resonance character for some value of l. If the resonance is sufficiently sharp, the function $S_0(W, n-1, W')$ may be removed from under the integral sign and its value taken at the point $W' = W_0$ corresponding to the maximum of $\partial\eta_l/\partial W'$, and the integral $\int(\partial\eta_l/\partial W')dW' = \pi$. If we substitute for the second term of Eq. 11 just the term corresponding to resonance scattering, we obtain

$$\begin{gathered} S(W, n, M_0) = S_0(W, n, M_0) \\ + (2l+1) S_0(W, n-1, W_0), \end{gathered} \tag{12}$$

where $W_0 = M_0 + \mu + E_0$, and E_0 is the kinetic energy of the nucleon and the meson for the case of resonance.

Thus the second term of Eq. 12 which includes the meson-nucleon interaction formally corresponds exactly to a system of $n - 1$ mesons and a nucleon in an "isobaric" state. At the same time, Eq. 12 gives the statistical weight for the production of one nucleon and n mesons. It should be noted that it is not clear whether the interaction may generally be limited to a two-particle type of interaction. This is a natural assumption, however, for the case of the specially strong two-particle interaction which corresponds to resonance scattering. Similarly, if we consider the collision between two nucleons, it is necessary to include the simultaneous interaction of the two nucleons with the mesons. Augmenting Eq. 12, we obtain

$$\begin{gathered} S_{NN} = \sum_{l'} \frac{2l'+1}{\pi} \sum_{l} \frac{2l+1}{\pi} \\ \times \iint S_0(W, n-2, W', W'') \frac{\partial \eta_l}{\partial W'} \frac{d\eta'}{\partial W''} dW' dW'' . \end{gathered} \tag{13}$$

$S_0(W, n-2, W', W'')$ is here the statistical weight for n non-interacting particles: $n - 2$ mesons and two "heavy particles" of mass W' and W''. If the scattering exhibits resonance character for some value of l, Eq. 13 reduces to the following expression:

$$S_{NN} = (2l+1)^2 S_0(W, n-2, W_0, W_0). \tag{14}$$

This expression formally corresponds to the statistical weight for a system consisting of $n - 2$ mesons and two nucleons in an "isobaric" state. If it turns out that the scattering of π-mesons against π-mesons exhibits resonance character at certain energy values (there is an indication of this in the literature[12]), it will be necessary to include analogously the π-meson interaction in computing the statistical weight.

Let us consider the following example. We shall compute the statistical weight corresponding to the production of a system of one nucleon and two mesons as a result of a meson-nucleon collision.

This statistical weight is determined by the following expression:

$$S(W, 2, M_0) = S_0(W, 2, M_0) + \sum_l \frac{2l+1}{\pi} \int S_0(W, 1, W') \frac{\partial \eta_l}{\partial W'} dW'. \quad (15)$$

The formula for $S_0(W, 1, W')$ has a simple analytical form (see for example Ref. 3):

$$S_0(W, 1, W_1) = C(W^2 - \mu^2 - W_1^2)(1 - (W_1^2 - \mu^2)^2 / W^4). \quad (16)$$

C is here a coefficient which is independent of $W_1 \equiv W'$. If $(W_1/W)^4 < 1$, the second term in parenthesis is practically equal to 1. We shall assume that this condition is satisfied. Indeed, when $W_1 = 1.23$ Bev and $W = 2$ Bev, the ratio $(W_1/W)^4$ is on the order of 12 percent. Let us assume that the scattering shows resonance character for some value of l, and the phase shift is obtained from Eq. 3. We shall assume the quantity Γ is independent of the energy; we find, then

$$\partial \eta_l / \partial W_1 = \partial \eta_l / \partial E_1 = \Gamma / [\Gamma^2 + (E_0 - E_1)^2], \quad (17)$$

where $E_1 = W_1 - M_0 - \mu$.

The second term of Eq. 15 may now be written as follows:

$$S_2 = \frac{C_1}{\pi} \int_0^{E_{\max}} [W^2 - \mu^2 - (M_1 + E_1)^2] \frac{\Gamma dE_1}{\Gamma^2 + (E_0 - E_1)^2}, \quad (18)$$

where C_1 is a constant, and

$$M_1 = M_0 + \mu, \quad E_{\max} = \sqrt{W^2 - \mu^2} - M_1.$$

Let us transform to the variable $\varepsilon = E_1 - E_0$; the integration is then easily carried out, and we obtain the following result:

$$S_2 = \frac{C_1}{\pi} \Big[(W^2 - \mu^2 - M_2^2) \Big(\tan^{-1} \frac{\sqrt{W^2 - \mu^2} - M_2}{\Gamma} - \tan^{-1} \Big(-\frac{E_0}{\Gamma} \Big) \Big) - \Gamma M_2 \log \frac{\Gamma^2 + (\sqrt{W^2 - \mu^2} - M_2)^2}{\Gamma^2 + E_0^2} - \Gamma (\sqrt{W^2 - \mu^2} - M_2) + \Gamma^2 \Big(\tan^{-1} \frac{\sqrt{W^2 - \mu^2} - M_2}{\Gamma} - \tan^{-1} \Big(-\frac{E_0}{\Gamma} \Big) \Big) \Big]. \quad (19)$$

$M_2 = M_0 + \mu + E_0$ is here the mass of the nucleon in the "isobaric" state. It is easily seen that as Γ tends to zero, $S_2 = C_0(W^2 - \mu^2 - M_2^2)$. This expression corresponds to the statistical weight of a system of two particles having respectively mass μ (meson) and M_2 ("isobaric" state of the nucleon). Let us assume now that $\Gamma = E_0/2$ and $E_0 = 160$ Mev. These values are close to the values obtained from experiments on the scattering of π-mesons against nucleons (see Ref. 13). Γ is given here the value it had at an energy of E_0. Actually it turns out to be a function of E_1. We shall evaluate Eq. 19 for $W = 1.9$ Bev. This corresponds, in the laboratory system, to an incident π-meson energy of 1.37 Bev. We find that the value of S_2 for $\Gamma = E_0/2$ is 0.78 times the value for S_2 for $\Gamma = 0$. In this case the computation of the level width leads to a correction which it is hardly worth considering in view of the approximate nature of the whole theory. This correction is even smaller at higher energies.

An attempt to compute the nucleon-meson interaction in the framework of the statistical theory was also made by Kovacs[14]. That analysis, however, introduces, along with statistical notions, certain matrix elements which are evaluated by applying a specific type of meson theory. The computations are extremely difficult. It seems to us that it is inconsistent to combine a statistical theory of phenomenological character with computations based on a certain variant of meson theory which appears doubtful itself.

In conclusion, the author wishes to express his gratitude to L. D. Landau for a discussion of the subjects considered in this article.

[1] E. Fermi, Progr. Theoret. Phys. 5, 570 (1950); Phys. Rev. 92, 452 (1953); 93, 1434 (1954).

[2] Fowler, Shutt, Thorndike and Whittemore, Phys. Rev. 95, 1026 (1954).

[3] S. Z. Belen'kii and A. I. Nikishov, J. Exptl. Theoret. Phys. (U.S.S.R.) 28, 744 (1955); Soviet Phys. JETP 1, 593 (1955).

[4] H. A. Bethe and F. de Hoffmann, *Mesons and Fields*, Vol. 2, Row, Peterson and Co., New York (1955).

[5] K. A. Brueckner, Phys. Rev. 86, 106 (1952).

[6] F. J. Belinfante, Phys. Rev. 92, 145 (1953); D. C. Peasly, Phys. Rev. 94, 1085 (1954); R. Gatto, Nuovo cimento 1, 159 (1955).

[7] Tamm, Gol'fand and Fainberg, J. Exptl. Theoret. Phys. (U.S.S.R.) 26, 649 (1954).

[8] A. I. Nikishov, J. Exptl. Theoret. Phys. (U.S.S.R.) **29**, 246 (1955); Soviet Phys. JETP **29**, 161 (1956); **30**, 601 (1956); Soviet Phys. JETP **3**, 634 (1956); **30**, 990 (1956); Soviet Phys. JETP **3**, 783 (1956); V. M. Maksimenko and A. I. Nikishov, J. Exptl. Theoret. Phys. (U.S.S.R.) **31**, 727 (1956); Soviet Phys. JETP **4**, 614 (1957).

[9] L. Landau and E. Lifshitz, *Statistical Physics*, GITTL (1951), § 75.

[10] G. E. Uhlenbeck and E. Beth, Physica **3**, 729 (1936); **4**, 915 (1937).

[11] R. H. Milburn, Rev. Mod. Phys. **27**, 1 (1955); I. L. Rozental', J. Exptl. Theoret. Phys. **28**, 118 (1955); Soviet Phys. JETP **1**, 166 (1955).

[12] F. J. Dyson, Phys. Rev. **99**, 1037 (1955); G. Takeda, Phys. Rev. **100**, 440 (1955).

[13] J. Orear, Phys. Rev. **100**, 288 (1955).

[14] J. S. Kovacs, Phys. Rev. **101**, 397 (1956).

Translated by M. A. Melkanoff
237

SOVIET PHYSICS JETP VOLUME 5, NUMBER 5 DECEMBER, 1957

Contribution to the Theory of Transport Processes in a Plasma Located in a Magnetic Field

E. S. FRADKIN

P. N. Lebedev Institute of Physics, Academy of Sciences, U.S.S.R.

(Submitted to JETP editor July 5, 1956)

J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 1176-1187 (May, 1957)

The mean statistical characteristics (velocity, heat current, stress tensor, etc.) have been determined for transport phenomena in a plasma located in a magnetic field.

THE PRESENT ARTICLE develops the theory of a plasma located in electric and magnetic fields (a brief account of the principal results of the author's analysis[1] is presented here). A method of approximations is used to solve the Boltzmann equation. The essence of this method lies in computing the terms of the distribution function, expanded in powers of some small parameter characteristic of the specific problem. One thus finds a so-called "local" distribution function which must be understood in the following sense: the complete distribution function is a function of 6 variables (3 coordinates and 3 velocity components) while the "local" distribution function depends explicitly upon the velocity only, and its dependence upon the coordinates enters only through the externally-acting forces and the mean statistical characteristics (temperature, density, and in general, velocity of the center of mass), which play the role of parameters. The dependence of these characteristics upon the coordinates is obtained by solving a certain system of equations which also arise from the Boltzmann equation. These equations represent a generalization of the well-known hydrodynamical equations.

1. As is well known[2], the Boltzmann equation for particles of type s, mixed with particles of other types, has the following form in the presence of magnetic and electric fields (the notation is that used by Chapman and Cowling):

$$\frac{\partial f_s}{\partial t} + \mathbf{v}_i \frac{\partial f_s}{\partial x_i} + \frac{e_s}{m_s}\left\{\mathbf{E} + \frac{1}{c}[\mathbf{vH}]\right\}_i \frac{\partial f_s}{\partial \mathbf{v}_i} = -\sum_{n=1} J_{sn}(f_s, f_n). \tag{1}$$

It will later be found convenient to rewrite the Boltzmann equation (1) in terms of a new set of independent variables; specifically, we transform the velocity of particles of a given type into their velocity with respect to the center of mass. The latter is given by the following formula:

$$\mathbf{v}_0 = \sum_{s=1}^{n} \int m_s \mathbf{v} f_s d\mathbf{v} \Big/ \sum_s \int m_s f_s d\mathbf{v}. \tag{2}$$

Eq. (1) takes the following form in terms of the new variables (henceforth, we write v for v^{rel}):