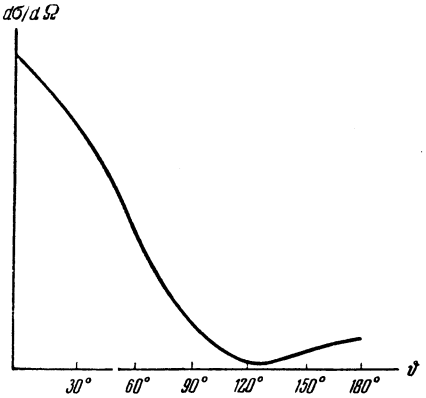

FIG. 7. Angular distribution of optical elastic scattering. Parameters $1/\alpha = 0.71 \times 10^{-13}$ cm, $E = 0.44$ Mev, $A = 100$.

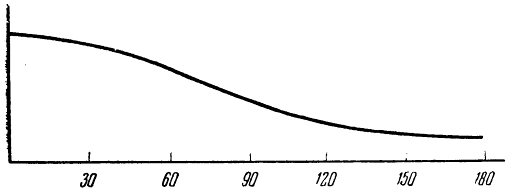

FIG. 8. Angular distribution of optical elastic scattering. Parameters $1/\alpha = 0.71 \times 10^{-13}$ cm, $E = 0.44$ Mev, $A = 84$.

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Contribution to the Theory of the Molecular Generator

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Processes of resonance interaction between an electromagnetic field and a molecular beam and also auto-oscillation processes in a molecular generator are examined in the present paper. Unlike other papers, the case of a beam of monochromatic (with respect to velocity) molecules is considered in detail and the peculiarities of this case are elucidated. The result obtained by taking into account non-monochromatic molecules in the beam is discussed qualitatively.

INCREASED STABILITY requirements have led to the development of a molecular generator in which the electromagnetic oscillations in the resonator are excited by radiation of excited gas molecules passing through the resonator. The possibility of such a generator was indicated in 1952 by Basov and Prokhorov[1, 2]. The molecular generator was almost simultaneously developed by a group of American physicists [3, 4] and by Basov and Prokhorov in the U.S.S.R. Even before the experimental success of the molecular generator, Basov and Prokhorov suggested a theory of its operation[5-7], which in essence consists of the following. Using the theory of dispersion, and taking account of saturation effects, the dielectric constant of the molecular beam passing through the resonator is determined. They then examine oscillatory processes in a circuit with a capacitor whose dielectric has the same constant as the molecular beam. The dielectric constant depends on the square of the electric field. Therefore this oscillatory process is described by a non-linear differential equation whose solution determines the amplitudes and frequencies of the oscillation. The theoretical method used by Basov and Prokhorov is not sufficiently complete and makes difficult an analysis of more complicated processes connected with the operation of a molecular generator. We therefore give in the present work a more rigorous statement of the problem, on the basis of which a thorough examination of molecular generator operation in the stationary state is made. We first examine the case of a single-velocity molecular beam ($v = v_0$) and afterwards give a qualitative evaluation of the effect of molecules with speeds other than v_0.

1. In analysis of the operation of a molecular generator it suffices to examine two energetic states

of the molecules, which we shall call E_1 and E_2. Then for the description of the molecular beam state in the resonator we shall use the density matrix:

$$\left| \begin{matrix} C_{11}(x, t) & C_{12}(x, t) \\ C_{21}(x, t) & C_{22}(x, t) \end{matrix} \right|. \qquad (1)$$

For definiteness we assume that $E_2 > E_1$. The matrix elements C_{11} and C_{22} determine at point x and at time t the density of molecules in states 1 and 2 moving with velocity v_0. The matrix elements C_{12} and C_{21} determine the polarization vector P of the molecular beam, which polarization is due to molecular transitions between states 1 and 2 as a consequence of the action of the field.

$$P(x, t) = p_{12}(t) C_{21} + p_{21}(t) C_{12}, \qquad (2)$$

where $p_{12}(t)$ and $p_{21}(t)$ are the matrix elements of the molecular dipole moment. Let us denote by

$$D(x, t) = C_{22}(x, t) - C_{11}(x, t) \qquad (3)$$

the difference in density of molecules of velocity v in states 2 and 1 at the point x at the instant t. For D, C_{12} and C_{21} we have the following equations:

$$\partial D / \partial t + v \, \partial D / \partial x = -(2 / ih) [p_{21}(t) C_{12} - p_{12}(t) C_{21}] E(t), \qquad (4)$$

$$\partial C_{12} / \partial t + v \, \partial C_{12} / \partial x = -(1 / ih) p_{12}(t) D E(t), \qquad (5)$$

$$\partial C_{21} / \partial t + v \, \partial C_{21} / \partial x = (1 / ih) p_{21}(t) D E(t). \qquad (6)$$

The x axis is directed along the axis of the resonator. Equations (4)–(6) are obtained from the general equations for the density matrix $C_{ik}(x, t)$, in which account is taken of the kinetic energy of the molecules. We assume that the molecular beam does not interact with itself and that the interactions of the separate molecules with the field are determined by the expression $-(\hat{\mathbf{p}}\mathbf{E})$, where $\hat{\mathbf{p}}$ is the dipole moment operator of the separate molecule, and $\mathbf{E}$ is electric field intensity of that normal component, whose frequency ω_p is close to the transition frequency $\omega_{21} = (E_2 - E_1)/n$. This normal oscillation has a homogeneous distribution of the electric field along the resonator. The direction of the E field is along the x axis. If the molecular-generator resonator is replaced by an equivalent circuit with a capacitance shunted by a resistance, then the equation for E has the following form:

$$\frac{d^2 E}{dt^2} + \frac{\omega_p}{Q} \frac{dE}{dt} + \omega_p^2 E = -4\pi \frac{d^2 \bar{P}}{dt^2}. \qquad (7)$$

Here Q is the quality of the resonator and $\bar{P}$ the average polarization along the x axis.

If the generator resonator is replaced by a circuit with a series resistance as was done in Refs. 6 and 7, we must add to the right half of Eq. 7 the term $-4\pi(\omega_p/Q) d\bar{P}/dt$. However, if we limit our examination to an accuracy of Q^{-2}, we can ignore this term. Thus it is first necessary to find an expression for the polarization P of the molecular beam, and then substitute this expression into (7) to analyze the resulting solution.

2. The beam polarization $P(x, t)$, according to (2), is expressed in terms of the solutions of the system (4)–(6). Let us examine the solutions of this system in the case that oscillations are already established, assuming that the electric field strength is $E = E_0 \cos \omega t$. In the steady state the distribution of the molecules does not depend explicitly on time. We therefore set $\partial D/\partial t = 0$. If the expression for E is substituted into Eqs. (4)–(6), terms appear on the right which are rapidly varying in time (with a frequency $\omega + \omega_{21}$), and slowly varying in time (with a frequency $\omega_{21} - \omega = \delta$). Neglecting on the right hand side of the equations for D, C_{12} and C_{21} the terms which are rapidly varying in time and assuming that $\partial D/\partial t = 0$, we obtain the following system of equations:

$$v \, \partial D / \partial x = -(1 / ih) [p_{21} e^{i\delta t} C_{12} - p_{12} e^{-i\delta t} C_{21}] E_0, \qquad (8)$$

$$\partial C_{12} / \partial t + v \, \partial C_{12} / \partial x = -(^1/_2 \, ih) \, p_{12} e^{-i\delta t} D E_0, \qquad (9)$$

$$\partial C_{21} / \partial t + v \partial C_{21} / \partial x = (^1/_2 \, ih) \, p_{21} e^{i\delta t} D E_0. \qquad (10)$$

In equations (8)–(10) we take it into account that $p_{ik}(t) = p_{ik} e^{i\omega_{ik} t}$. These relationships will be solved with the boundary conditions:

$$D = D_0; \quad C_{12} = C_{21} = 0 \quad \text{for} \quad x = 0. \qquad (11)$$

The solutions of equations (9) and (10) satisfying the boundary conditions (11) have the form

$$C_{12}(x, t) = -\frac{p_{21} E_0}{2ihv} \int_0^x \exp\left\{-i\delta\left[t - \frac{1}{v}(x - x')\right]\right\} D(x') \, dx', \qquad (12)$$

$$C_{21}(x, t) = \frac{p_{21}E_0}{2ihv}\int_0^x \exp\left\{i\delta\left[t - \frac{1}{v}(x - x')\right]\right\} D(x')\,dx'. \quad (13)$$

Substituting (12) and (13) into Eq. (8), we obtain an equation for $D(x)$

$$\frac{dD}{dx} = -\frac{p^2E_0^2}{h^2v^2}\int_0^x \cos\frac{\delta}{v}(x - x')\,D(x')\,dx'. \quad (14)$$

The solution of Eq. (14) satisfying the boundary conditions has the form

$$D(x) = D_0\left[\frac{p^2E_0^2}{\Omega^2h^2}\cos kx + \frac{\delta^2}{\Omega^2}\right], \quad (15)$$

$$\Omega^2 = p^2E_0^2/h^2 + \delta^2; \quad k = \Omega/v. \quad (16)$$

Expression (15) describes the level distribution of the constant-velocity beam molecules moving along the resonator, as a function of the detuning δ and of other parameters. It follows from this that for a small detuning δ there is a resonance action on the molecular beam: almost all molecules go from the upper level to the lower and back again as they move along the resonator. For large detuning the indicated transition is made only by a small part of the beam molecules: the function $D(x)$ is almost constant along the resonator.

We now calculate the polarization vector of the molecular beam. Substituting the expressions (12), (13) for C_{12} and C_{21} into expression (2) for the polarization vector P, and using expression (15) for $D(x)$, we find

$$P(x, t) = -D_0(p^2E_0/h\Omega^2)\,[\Omega\sin kx\sin\omega t + \delta(1 - \cos kx)\cos\omega t]. \quad (17)$$

From expression (17) we find the average polarization along the resonator of the beam molecules moving with constant velocity.

$$\bar{P}(t) = -D_0\frac{p^2E_0\tau}{h}\left[\frac{1 - \cos\Omega\tau}{\Omega^2\tau^2}\sin\omega t + \delta\tau\frac{1 - \sin\Omega\tau/\Omega\tau}{\Omega^2\tau^2}\cos\omega t\right], \quad (18)$$

where $\tau = L/v$ is the transit time through a resonator of length L.

3. Let us now proceed to analyze solution of the equation for the electric field in the case of a molecular beam which is monochromatic in velocity. Using expressions (18) for $\bar{P}(t)$, we write Eq. (7) for E in the form

$$\frac{d^2E}{dt^2} + \frac{\omega_p}{Q}\frac{dE}{dt} + \omega_p^2E = -B\omega^2\frac{1 - \cos\Omega\tau}{\Omega^2\tau^2}E_0\sin\omega t - B\omega^2\delta\tau\frac{1 - \sin\Omega\tau/\Omega\tau}{\Omega^2\tau^2}E_0\cos\omega t. \quad (19)$$

Here $B = 4\pi D_0p^2\tau/h$. Putting $E = E_0\cos\omega t$ and equating the sine and cosine coefficients to zero, we obtain a transcendental equation for the amplitude and frequency of the steady-state oscillations

$$1/QB = (1 - \cos\Omega\tau)/\Omega^2\tau^2, \quad (20)$$

$$\omega^2 - \omega_p^2 = B\omega^2\delta\tau\,\frac{1 - \sin\Omega\tau/\Omega\tau}{\Omega^2\tau^2}. \quad (21)$$

Expression (21) can be written in the following manner:

$$\omega_p - \omega_{21} = (\omega - \omega_{21}) + \frac{B}{2}\omega\tau(\omega - \omega_{21})\frac{1 - \sin\Omega\tau/\Omega\tau}{\Omega^2\tau^2}. \quad (22)$$

From (22) it follows that when the resonance frequency ω_p equals the frequency ω_{21} of the transition in the molecule, the detuning is $\delta = \omega_{21} - \omega = 0$ and therefore the generator frequency coincides exactly with the frequency ω_{21}. In expression (22), for very large values of $\Omega\tau$ the second term is larger than the first. We therefore have approximately

$$\omega_p - \omega_{21} = -\frac{B}{2}\tau\delta\,\frac{1 - \sin\Omega\tau/\Omega\tau}{\Omega^2\tau^2}. \quad (23)$$

Let us introduce the notation

$$F(x) = \frac{1 - \cos x}{x^2}, \quad \Phi(x) = \frac{1}{x^2}\left(1 - \frac{\sin x}{x}\right),$$
$$\Delta = \omega_p - \omega_{21}. \quad (24)$$

Then

$$1/QB = F(\Omega\tau), \quad \Delta = -{}^1/_2\,B\omega_{21}\tau\Phi(\Omega\tau)\,\delta. \quad (25)$$

The functions $F(x)$ and $\Phi(x)$ are plotted in Figs. 1 and 2. Figure 1 shows that for a molecular beam which is monochromatic in velocity there exist values of the transit phase $\Omega\tau$ for which oscillation is impossible for any value of the coefficient QB. These transit phases have the values $2\pi n$, where n is an integer. In these cases there is no transfer of energy during the passage of the molecular beam through the resonator and all the active molecules entering the resonator leave it. For all other values of transit phase, self-oscillation is possible for sufficiently large values of the coefficient QB. From Figs. 1 and 2 it is seen that slightly above

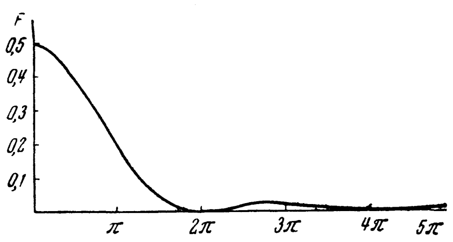

FIG. 1

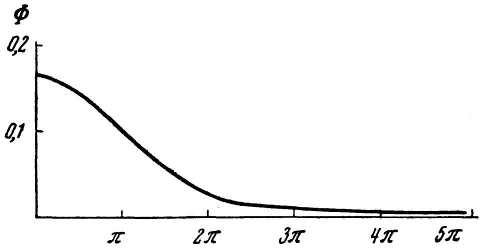

FIG. 2

the self-excitation threshold*, when $1/QB \lesssim 1/2$, self-oscillations are possible corresponding to small values of $\Omega\tau$ that are negligible compared with 2π. The generator frequency which is determined with the aid of Fig. 2 and relationship (23) is close in this case to the value $\omega = \omega_{21} + 6Q\Delta/(\omega_{21}\tau$ and approaches ω_{21} with an increasing margin above the self-excitation threshold. When the coefficient QB becomes sufficiently large that the inequality $1/QB \lesssim 0.024$ is satisfied, we have the possibility of generation for transit phases in the region $2\pi < \Omega\tau < 4\pi$. It is essential to note that values of the transit phase corresponding to increasing branches of $F(x)$ determine unstable regions of self-oscillation, and therefore cannot be realized. For large increase in the intensity of a beam that is monochromatic in velocity, it becomes possible to generate in the transit phase region $4\pi < \Omega\tau < 6\pi$.

So far we have been examining the case of a beam that is monochromatic in velocity. In reality the molecules have some velocity distribution. For a properly selected length of level-sorting quadrupole capacitor and for a small entrance aperture to the resonator, the molecular velocities concentrate more or less closely about the values v_0, $v_0/3$, $v_0/5$ *etc.* If we limit ourselves to examination of the velocities v_0, $v_0/3$, $v_0/5$ in the molecular beam the following changes will take place in the above derivations. Equation (25) takes on the form

$$1/Q = d_1 B(\tau_0) F(\Omega\tau_0) + d_3 B(3\tau_0) F(3\Omega\tau_0) + \ldots, \tag{26}$$

where $d_1, d_3, \ldots$ are the relative molecular concentrations with velocities v_0, $v_0/3$, ... The presence of velocities $v_0/3$, $v_0/5$, *etc.*, which differ from v_0 by 3, 5, *etc.*, times, results in a somewhat different effective value for the expression $\tilde{B}\tilde{F}$. However, the new function $\tilde{F}$ has almost the same characteristics as the function F for a beam monochromatic in velocity. In particular, when the argument of $\tilde{F}$ is equal to $2\pi n$ this function vanishes. Between these values of the argument the function $\tilde{F}$ is different from zero. Therefore there exist various isolated regions of transit phase for which oscillation is possible. The second relationship in (25) can also be rewritten in a form analogous to (26). We obtain in this case instead of (24) a new function $\tilde{\Phi}$ which qualitatively does not differ from the old one. Thus the existence of the several indicated molecular velocities in the beam does not cause a qualitative change in the basic relationships (25) which determine the self-oscillatory process in the generator. Taking account of the small spread in the velocities around v_0, $v_0/3$, $v_0/5$, *etc.*, leads to a smearing of the function F: its minima increase and the maxima decrease; the function $\tilde{F}$ will no longer vanish for values of the argument $2\pi n$. However, for small velocity spreads the function $\tilde{F}$ retains its important characteristic—the existence of various isolated regions of transit phase values for which oscillations are possible. For a considerably larger spread of velocities all regions of transit phase which allow generation merge into one region. It should be noted that for small transit angles even a considerable spread in velocity has little effect on the function $F(x)$ and it is therefore unnecessary in this case to take account of this spread.

4. Let us now evaluate the importance of various accidental factors on the stability of molecular-generator operation. Let us first evaluate the order of magnitude which characterizes the oscillations in the molecular generator. Let $Q = 10^4$, $\Delta = 10^3$ sec^{-1}, $\omega_{21} = 1.5 \times 10^{11}$, $\tau = 10^{-4}$ sec. For these values of the parameters the oscillation frequency for a slight margin above the self-excitation threshold is determined by the equation $\omega = \omega_{21} + 6\,Q\Delta/\omega_{21}\tau$ and is $\omega = \omega_{21} + 5$. With increasing beam intensity, the oscillation frequency approaches ω_{21}. The oscillation amplitude can be evaluated from Eq. (19). We have for a transit phase $\Omega\tau \approx 1$, $E = h\Omega/p = 10^5$ CGS electrostatic units. The order of magni-

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* The conditions for self-excitation differ from those developed in reference (2) by an unimportant constant coefficient, which is the result of a different averaging of the beam polarization vector.

tude of the output power is determined by the relationship $W = E^2\omega_{21}\theta/2Q$ where θ is the volume of the resonator which is in order of magnitude 10 cm^3. Substituting here the given quantities we have $W = 7\times10^{-3}$ erg/sec. To produce self-oscillation it is essential that $B > 2/Q$. Let us take therefore a value $B = 10^{-3}$. Then from Eq. (19) we have $D_0 = Bh/4\pi p^2\tau \approx 10^9\,\text{cm}^{-3}$. The number of molecules falling on an area of 1 cm^2 is $D_0 v = 4\times10^{13}$ cm^{-2} sec^{-1}. It should be noted that in real operating conditions the molecules are concentrated in a narrow beam whose area is significantly smaller than the transverse area of the resonator. The value that has been used above is characteristic of an average molecular density in the resonator. The total number of molecules which enter the resonator in a unit time is $N = D_0 vS \approx 10^{14}\,\text{sec}^{-1}$ where S is the transverse area of the resonator. The order of magnitude of the power emitted by the molecule is the same as the input power of the generator $Nh\omega/2 = 7\times10^{-3}$ erg/sec. This value agrees with the value obtained above.

Let us evaluate the effect of a varying resonator temperature on the generator stability. The order of magnitude of resonator frequency variation due to unconstant temperature is determined by the following relationship:

$$\Delta\omega_p/\omega_p = \Delta R/R = \alpha\Delta T, \tag{27}$$

where α is the linear expansion coefficient, R the radial dimension of the resonator, and ΔT is the accuracy with which the resonator temperature is maintained. For example for invar we have $\alpha = 1.5\times10^{-6}$. For small values of transit phase

$$\Delta\omega = \Delta\omega_p(6Q/\omega_{21}\tau) \approx 4\cdot10^{-3}\,\Delta\omega_p. \tag{28}$$

From (27) and (28) it follows that in order to maintain a stability of 5×10^{-11} near the self-oscillation threshold it is essential to maintain the temperature constant with an accuracy of 10^{-2}. With an increase in beam intensity, the accuracy with which it is necessary to maintain the temperature can be decreased by one order of magnitude.

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On the Nonlinear Generalization of the Meson and Spinor Field Equations

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Exact periodic solutions of the nonlinear generalized Klein-Gordon and Dirac equations are considered. The energy of the nonlinear classical meson field is compared with that derived from quantum theory.

THE NECESSITY for nonlinear generalizations of scalar, spinor, and other field equations, as well as the possible importance of the nonlinearities in specific effects, makes desirable detailed examinations of this problem. Here we consider a purely classical unquantized scalar or pseudo-scalar neutral meson field. We make use of the exact solution of nonlinear wave equations in terms of