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Resonant Pion-Nucleon Interaction and Production of Pions by Nucleons

L. M. SOROKO

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The production of pions by nucleons is studied in an attempt to take approximate account of the strong pion-nucleon interaction. The calculation is based on the assumption that the probability of the process $N + N \to \pi + N + N'$ is determined by the energy of the created pion relative to one of the nucleons. Using experimental values for the matrix elements of the pion-nucleon interaction, we calculate the spectrum of pions and nucleons in the reaction $N + N \to \pi + N + N'$, and also the intensity of pion emission as a function of the angle between pion and nucleon. The results are compared with experiment.

IT IS NOW firmly established that the pion-nucleon interaction is strong in the P-state with total angular momentum $J = 3/2$ and isotopic spin $T = 3/2$. Upon this fact is based the hypothesis of a nucleon isobar state, formulated by Brueckner[1] and by Tamm and his collaborators[2]. To compare the consequences of this hypothesis with experiment, several properties of the production of pions by nucleons have been calculated[3]. Belen'kii and Nikishov[4] evaluated the relative magnitudes of single and multiple pion production, including both direct production and production through an intermediate isobar state. Many authors, for example Aitken *et al.*[5], have calculated pion production by nucleons taking the strong pion-nucleon interaction into account explicitly.

Instead of making such direct calculations, one can establish a phenomenological correspondence between two processes. Assuming that the matrix element for the process

$$N + N \to \pi + N + N', \tag{1}$$

depends only on the relative energy of the pion and one nucleon, the magnitude and the energy-dependence of the production cross-section can be obtained from the experimental values of the total cross-section for pion-nucleon scattering. In order that the strong pion-nucleon interaction shall appear in the process (1), it is only necessary[6] that the isotopic spin of the two nucleons in the initial state should be $T = 1$.

1. THE PION-NUCLEON INTERACTION MATRIX ELEMENT IN STATES WITH $T = 3/2$

The cross-section for a reaction

$$A + b \to C + d \tag{2}$$

is completely determined by a matrix element H according to the formula

$$d\sigma(\theta)/d\Omega$$
$$= (2\pi/\hbar)\,|H(\theta)|^2 p^2/2\pi^2\hbar^3(v_C + v_d)\,v_{Ab}, \qquad (3)$$

where p is the momentum of particles C and d, v_c and v_d are their respective velocities in the center-of-mass system, and v_{Ab} is the relative velocity of collision of particles A and b. The total cross-section, apart from irrelevant constant factors, is given by

$$\sigma = |H_t|^2 p^2/(v_C + v_d)\,v_{Ab}. \qquad (4)$$

The matrix element of the pion-nucleon interaction was deduced[7] from experimental values of the total cross-section for pion-proton scattering in the energy range from 30 to 400 Mev. Figure 1 shows the values we have used for the total cross-section, agreeing satisfactorily with a resonance curve[6]. Figure 2 shows the dependence of the squared matrix element on pion energy; this curve is the basis of the subsequent calculations. In Fig. 3 we show the total cross-section for photo-production of neutral pions in hydrogen, deduced from the matrix element plotted in Fig. 2. From the agreement of the

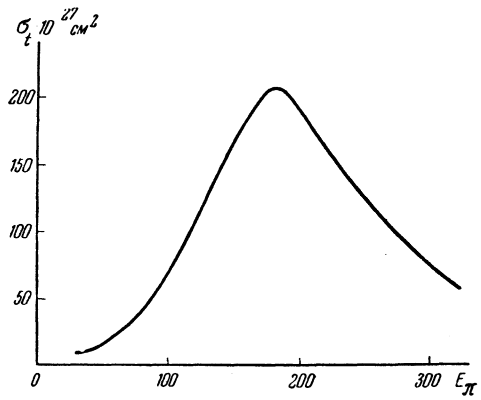

FIG. 1

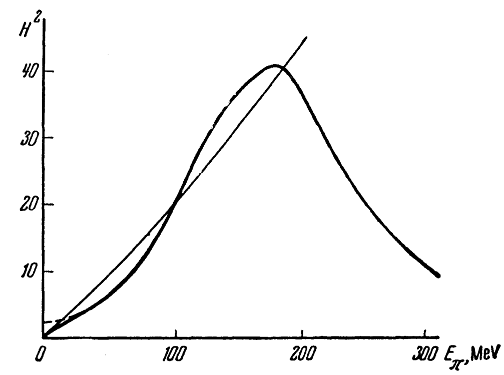

FIG. 2

FIG. 1. Total $\pi^+ - p$ scattering cross-section given by the formula $\sigma_t(\pi^+p) = 2\pi\lambda\!\!\!^-{}^2\Gamma^2/[(E - E_0)^2 + \Gamma^2/4]$; $\Gamma = 2b(a/\lambda\!\!\!^-)^3/[1 + (a^2/\lambda\!\!\!^-)^2]$; $a = 1.4\hbar/\mu c$, $b = 75$ Mev. $E_0 = 154$ Mev, E_π is the pion energy in the center-of-mass system.

FIG. 2. The squared pion-nucleon interaction matrix element in the state with isotopic spin $T = 3/2$, as a function of pion energy. The lighter curve shows the result of a p^2 dependence in the center-of-mass system.

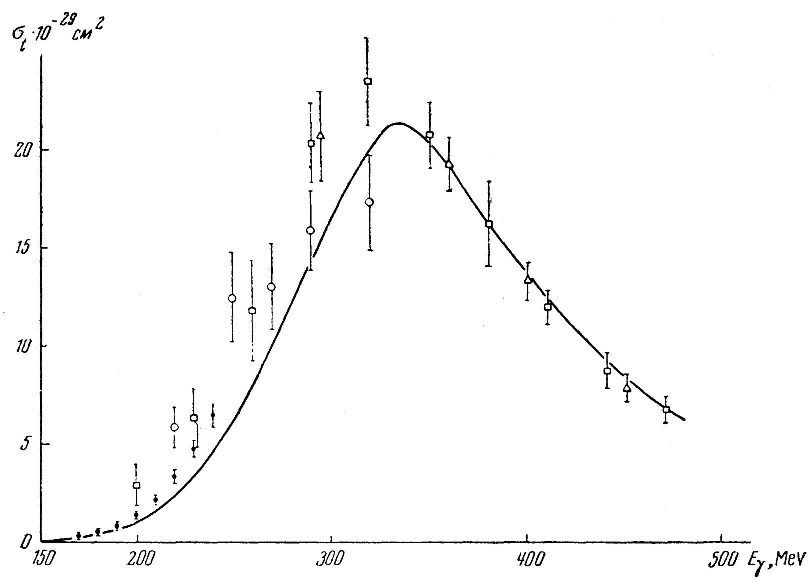

FIG. 3. Total π^0 photoproduction cross-section as a function of photon energy. Experimental points, ●—Ref. 12, □—Ref. 13, ○—Ref. 14, △—Ref. 15.

calculated curve with the experimental points[12–15], one may conclude that the processes $\pi^+ + p \to \pi^+ + p$ and $\gamma + p \to \pi^0 + p$ proceed through the same matrix element.

2. PION SPECTRUM IN THE PROCESS $N + N \to \pi + N' + N$.

The vector diagram of the momenta of three particles (Fig. 4) forms a closed triangle. The position of the vertex A of the triangle is fixed by energy conservation[8], thus

$$(p_1^2 + p_2^2)/2M + \sqrt{p^2 + m^2} = W. \qquad (5)$$

Here W is the combined kinetic energy of the two colliding nucleons in the center-of-mass system, m and M are the pion and nucleon masses, and p, p_1, p_2 are their momenta. In this approximation, the locus of the vertex A of the vector triangle is a sphere in momentum space with radius

$$R = [M(W - \sqrt{p^2 + m^2}) - {}^1/_4\, p^2]^{1/2}. \qquad (6)$$

The probability for emitting a pion of momentum p is

$$1/\tau = |H|^2 R(p;\ W)\, p^2 dp, \qquad (7)$$

and the differential cross-section is

$$d\sigma/dp = v_{NN}^{-1} |H|^2 R(p;\ W)\, p^2. \qquad (8)$$

If H is independent of energy and angle, then

$$d\sigma/dp = (p^2/v_{NN})\, R(p;\ W). \qquad (9)$$

For given pion momentum and auxiliary angle θ (see Fig. 4), the energy of the pion relative to a nucleon at rest is

$$E_{\pi N}(p,\ \theta;\ W)$$
$$= \sqrt{p^2 + m^2}\sqrt{1 + (R^2 + {}^1/_4 p^2 - Rp\cos\theta)/M^2} - pR\cos\theta/M + p^2/2M - m. \qquad (10)$$

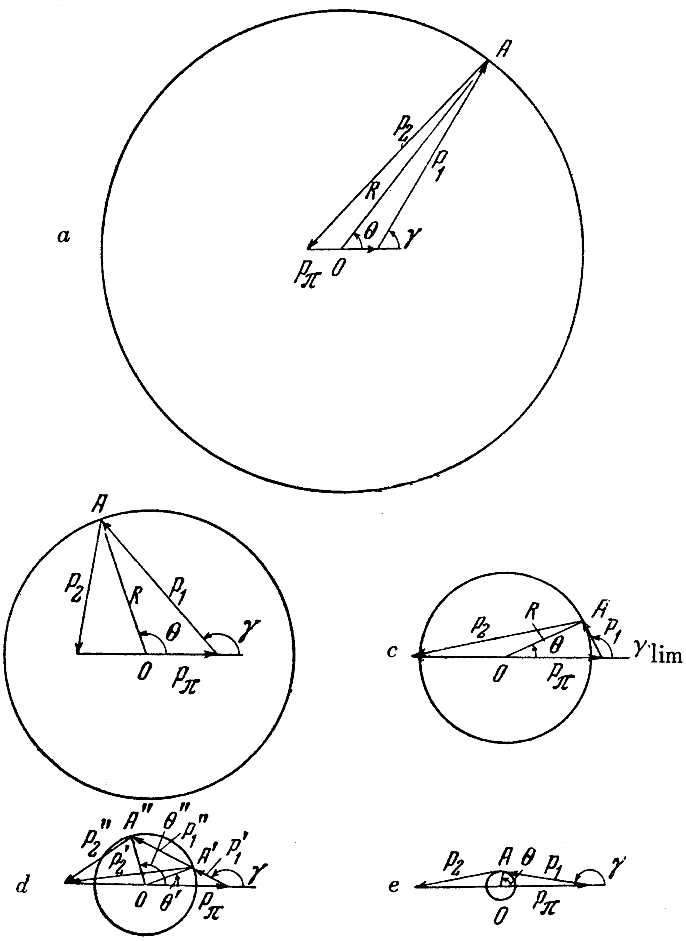

FIG. 4. Vector diagrams for the three particle momenta in the reaction $N + N \to \pi + N + N'$ center-of-mass system. The pion momentum p_π is 100, 200, 232, 240, 248 Mev/c in the five diagrams.

Equation (10) implies that a fixed pion momentum corresponds to a certain range of values for $E_{\pi N}$. The pion spectrum is finally given by

$$\frac{d\sigma}{dp} = \frac{p^2}{v_{NN}(W)} R\,(p; W) \int_{-1}^{+1} d\cos\theta \,\{H\,[E_{\pi N}(p,\ \theta;\ W)]\}^2. \tag{11}$$

Calculations have been made for a total energy $W = 300$ Mev, corresponding to an incident proton energy $E_p = 650$ Mev. Figure 5 compares the averaged matrix element $H(p_\pi)$ defined by the expression

$$H\,(p_\pi) = \left[\int_{-1}^{+1} d\cos\theta\,\{H\,[E_{\pi N}(p,\ \theta;\ W=300)]\}^2\right]^{1/2}, \tag{12}$$

with the empirical matrix element

$$H_e = \left[\frac{d^2\sigma}{dp d\omega}\frac{1}{p^2 R\,(p;\ W)}\right]^{1/2},$$

derived from production experiments[9].

Figure 5 shows that the graph of $H(p_\pi)$ does not start at the origin but has a finite intercept on the vertical axis. This is a consequence of the motion of the nucleons, which have rather large momenta even for $p_\pi = 0$. It is also remarkable that $H(p_\pi)$ has no maximum, although the pion energy in the center-of-mass system can go up to 150 Mev. The cause of this "sluggishness" is the averaging over energy which occurred in the definition of the matrix element. Calculations show that a maximum appears in the graph of $H(p_\pi)$ at an incident energy $E_p \sim 750$ Mev.

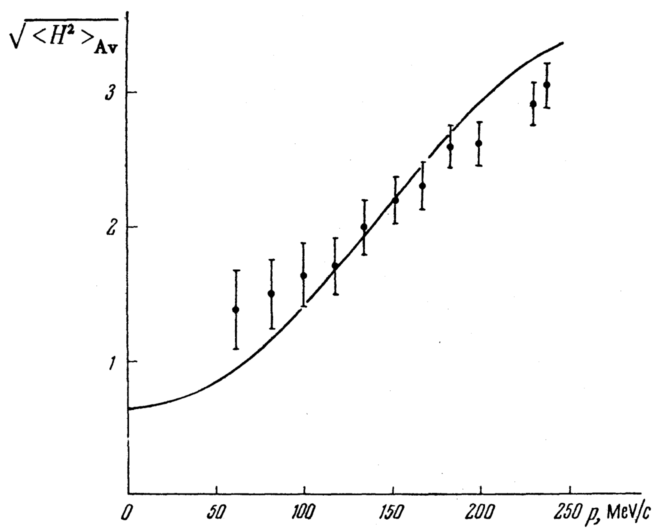

FIG. 5. Dependence of $\sqrt{\langle H^2\rangle_{Av}}$ on pion momentum in center-of-mass system. Experimental points taken from Ref. 9 at $E_p = 657$ Mev.

3. NUCLEON SPECTRUM IN THE PROCESS $N + N \to \pi + N' + N$.

The vector diagram of the momenta which determine the nucleon spectrum has the property that the locus of the vertex A is not a circle with center at O, so that the distance $OA = \rho(\theta_1)$ is a function of the angle θ_1.

If the momentum p_1 of one nucleon and the angle θ_1 are fixed, the relative energy of the pion and nucleon is

$$\begin{aligned} &E_{\pi N}\,(p_1, \theta_1; W) \\ &= [(1 + p_1^2 / M^2)\,(m^2 + \rho^2 + p_1^2/4 - \rho p_1 \cos\theta_1)]^{1/2} \\ &+ p_1^2 / 2M - p_1 \rho \cos\theta_1 / M - m, \end{aligned} \tag{13}$$

and the differential cross-section has the form

$$\begin{aligned} &\frac{d\sigma}{dp_1} = \frac{p_1^2}{v_{NN}(W)} \\ &\times \int_{-1}^{+1} \rho\,(p_1, \theta_1; W)\,\{H\,[E_{\pi N}\,(p_1, \theta_1; W)]\}^2\, d\cos\theta, \end{aligned} \tag{14}$$

with the function $\rho = \rho(p_1, \theta_1; W)$ defined by the parametric representation

$$\rho^2 + {}^1/_4\, p_1^2 - p_1 \rho \cos\theta_1 = p^2,$$
$$p_1^2 + 2p_1 \rho \cos\theta_1 = 2M\,(W - \sqrt{p^2+m^2}) - p^2, \tag{15}$$

where the parameter p is in fact the pion momentum.

Figure 6 shows the nucleon spectra, calculated at energy $E_p = 650$ Mev, for the case of a resonant interaction H and for the case of an interaction whose matrix element is constant. The spectra differ significantly only near the upper limit, where the resonant interaction gives a less steep descent to zero. The results apply only to σ_{10} transitions in which the nucleon isotopic spin changes from 1 to 0. The proton spectra experimentally observed[10] at energy $E_p = 650$ Mev include not only σ_{10} but also σ_{11} transitions, and the latter occur with triple weight because of the existence of the process $p + p \to \pi^0 + p + p$. The spectrum of σ_{11} transitions has not been calculated, and therefore a decisive comparison with the experimental results cannot be made. Still it seems that the σ_{10} and σ_{11} spectra should not differ essentially in the immediate neighborhood of the upper limit.

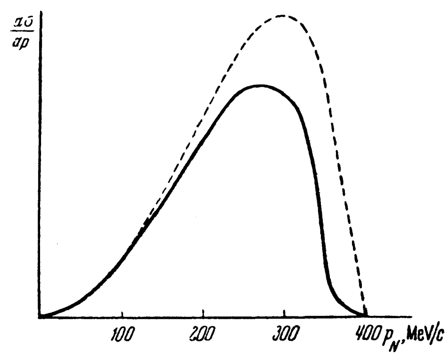

FIG. 6. Nucleon spectra from the reaction $N+N\to\pi+N+N$ at $E_p = 657$ Mev; σ_{10} transitions only.

4. PION-NUCLEON ANGULAR CORRELATIONS

The dependence of the cross-section for reaction (1) upon the angle γ between the pion and one nucleon has some peculiarities arising from energy and momentum conservation alone. Figure 4 shows vector diagrams of the momenta corresponding to various regions of the pion spectrum. When the pion momentum is large, the angle between pion and nucleon becomes concentrated into the region $\gamma > 90°$. For $p_\pi > 230$ Mev/c and $W = 300$ Mev, angles $\gamma < 90°$ cannot occur.

The probability $W(\gamma)$ can be expressed in terms of the pion momentum and the matrix element $H(E_{\pi N})$. We find

$$W(\gamma, p)\, d\gamma\, dp = |H|^2 p^2 R(p; W) \times \frac{d\cos\theta}{d\cos\gamma} \sin\gamma\, d\gamma\, dp. \tag{16}$$

The quantity $(d\cos\theta/d\cos\gamma)$ is obtained from the geometrical relations

$$p_1^2 = R^2 + {}^1/_4\, p^2 - Rp\cos\theta,$$
$$R\cos\theta = {}^1/_2\, p + p_1\cos\gamma, \tag{17}$$

hence the final expression for $W(\gamma)$ is

$$\frac{W(\gamma, p)}{\sin\gamma}\, d\gamma dp = |H|^2 p^2 R(p; W) \left\{\left[1 - \left(\frac{p}{2R}\right)^2 \sin^2\gamma\right]^{1/2} + \left(\frac{p}{2R}\right)^2 \cos^2\gamma \left[1 - \left(\frac{p}{2R}\right)^2 \sin^2\gamma\right]^{-1/2} - \frac{p}{R}\cos\gamma\right\} d\gamma\, dp. \tag{18}$$

Equation (18) shows that $W(\gamma) \approx \sin\gamma$ only for pion momenta close to zero. In the region $p > 2R$, angles $\gamma < 90°$ are forbidden.

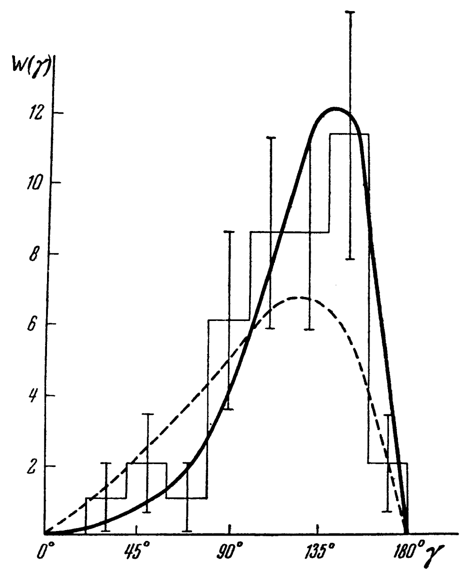

FIG. 7. Pion production probability as a function of angle γ between pion and nucleon in center-of-mass system. Experimental histogram from Ref. 11.

Figure 7 shows the functions $W(y)$ calculated for a resonant H (full curve) and for a constant H (dotted curve). Also shown is the experimental histogram obtained[11] at a proton energy of 650 Mev. Although the statistics are not very good, it is clear from Fig. 7 that the experiment agrees better with a resonant than with a constant interaction.

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Model of a Semi-transparent Nucleus with a Diffuse Boundary, II

P. E. NEMIROVSKII

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A new method is presented for calculating nuclear interaction cross-sections for low energy neutrons. Assuming that no absorption occurs in the surface layer, it is shown that the energy dependence of the cross-section at low energy is the same for a potential with diffuse boundary as for a rectangular well. The capture cross-section is larger for the diffuse than for the sharp boundary. Values of the parameters and of the nuclear potential are found which give satisfactory agreement with experiment over a wide range of nuclear weights and energies.

IN AN EARLIER PAPER[1] we reported results of calculations of cross-sections for a semi-transparent nuclear model with diffuse boundary. We found good agreement of the capture of cross-sections transparent nuclear model with diffuse boundary. We found good agreement on the capture of cross-sections with experiment, up to an energy of a few million volts, with the following values of the parameters:

$$V(r) = 20\,\text{MeV} \quad \text{for} \quad r \leqslant r_0,$$

$$V(r) = 20 \exp\{-(r - r_0)/1.4 \cdot 10^{-13}\} \quad \text{for} \quad r \geqslant r_0,$$

$$r_0 + 1.4 \cdot 10^{-13} = 1.25 \cdot 10^{-13} A^{1/3} \text{ cm}.$$

The imaginary part $\zeta V(r)$ of the potential was variable. However, it has since been reported[2] that the potential $V(r)$ should be 42 Mev. Also it seems appropriate to compare the calculated results with a