makes a transition from the ground state with $\gamma = 0$ to the S state which corresponds to $\gamma = \pi$ the quadrupole moment is changed in sign and magnitude.

In nuclei where the states of the outer nucleons correspond to negative values of β_0 the lowest energy levels occur for $\gamma = \pi$. However it appears from experiment that there are no nuclei with large negative values of β_0.

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[1] A. Bohr, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **26**, 14 (1952).

[2] N. N. Lebedev, *Special Functions and their Applications*, GITTL, 1953.

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Reactions Produced by μ-Mesons in Hydrogen

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AT PRESENT THERE IS EVIDENCE that at Berkeley[1] in a bubble chamber, filled with liquid hydrogen having a varying deuterium content, there could be observed a nuclear reaction catalyzed by μ-mesons. The possibility of such a type of reaction was first pointed out by Frank[2] in connection with the analysis of π-μ disintegrations in emulsions. This process was investigated in liquid deuterium by both of the authors of this article, independently of one another[3,4].

The presence of a μ-meson changes the form of the potential barrier which previously prevented nuclear reactions among slow proton and deuteron nuclei, increasing sharply the penetrability of the barrier and making possible the reactions

$$p + \mathrm{D} = \mathrm{He}^3, \quad \mathrm{D} + \mathrm{D} = \mathrm{He}^3 + n, \quad \mathrm{D} + \mathrm{D} = \mathrm{T} + p.$$

In the presence of tritium there are also possible the reactions

$$\mathrm{D} + \mathrm{T} = \mathrm{He}^4 + n, \quad \mathrm{T} + \mathrm{T} = \mathrm{He}^4 + 2n, \quad p + \mathrm{T} = \mathrm{He}^4.$$

The reaction $p + p = \mathrm{D} + e^+ + \nu$, catalyzed by mesons, is practically impossible, since in addition to the barrier there is also the factor of a low probability for the beta process.

It has been predicted[4] that the probability of the reaction in flight is low, the production of mesomolecules practically always leads to nuclear reactions, the rate of the process is determined by the production of mesomolecules, and the probability of mesomolecule formation during the lifetime of a meson may amount to several hundredths or even tenths, depending on the arrangement of the mesomolecule levels.

The experimental data of Alvarez[1] shows that in natural hydrogen (deuterium content ratio 1 : 7000), the reaction $p + d = \mathrm{He}^3$ occurs on the average once for each 150 mesons. If the deuterium ratio is 1 : 300, the reaction occurs once per 40 mesons, and if the ratio is 1 : 20, the reaction occurs once per 33 mesons. Furthermore, the energy of the resulting He^3 (5.4 Mev) is carried away by the μ-meson, so that monochromatic μ-mesons are observed while the reaction is taking place. The relatively high probability found for the reaction in the natural mixture is explained[1] by the transfer of the meson from the proton to the deuteron (charge exchange): $p\mu + d = p + d\mu$. Due to the difference in reduced mass, the energy of the $\mathrm{D}\mu$ bond (2655 ev) is greater by $\Delta E = 135$ ev than the energy of the $p\mu$ bond. Therefore the charge-exchange process appears to be irreversible under the experimental conditions.

We shall give a rough estimate of the probability of the transition. If ΔE is equal to zero, the cross section should be of the order of πa^2, where a is the radius of the Bohr orbit of the mesoatom, 2.5×10^{-11} cm.

Indeed, if the masses of the two nuclei are equal, with $\Delta E = 0$, the states of the systems $\underset{+}{\Sigma}_g$ and $\underset{-}{\Sigma}_u$ appear to be proper, and the cross section for charge exchange can be expressed by the scattering lengths a_g and a_u of these states in a continuous spectrum: $\sigma = \pi(a_g - a_u)^2$. When $\Delta E \neq 0$, but is still small with respect to the molecular dissociation energy, then

$$\sigma = \pi(a_g - a_u)^2 v_f/v_i,$$

where v_i is the velocity before collision and v_f is the velocity after collision.

In actual fact, ΔE is of the same order as the dissociation energy, so that the formula is corrected at least in order of magnitude. If v_i is small $\sigma \sim 1/v$ $\sigma \sim 1/v_i$. It follows that in order of magnitude,

$$\sigma \approx \pi a^2 v_*/v_i,$$

where v_* is the characteristic velocity corresponding to 135 ev. Using the masses of the proton and deuteron, $v_* = 2 \times 10^7$ cm/sec. Such an estimate yields qualitative agreement with the observed facts. Calculation shows that saturation should be reached at deuterium concentrations of 1 in 300 to 1 in 20. In the natural mixture of hydrogen, the probability of Dμ production and consequently the probability of the reaction should be three-fold less than in enriched mixtures; experimentally it is found to be 4 to 5 times less.

Let us examine the reaction in the pDμ molecule. The observed high probability a of the giving up of the meson energy does not agree with the hypothesis that the reaction proceeds like an electric dipole transition ($E1$), since a becomes 2×10^{-3} for the meson under this hypothesis. Therefore, to estimate the probability we cannot use the experimental cross section for $p + \mathrm{D} = \mathrm{He}^3 + \gamma$ (as was done before[4]), since under the conditions of the measurement, it is precisely the cross section of the $E1$ process which is observed.

In the case of zero orbital momentum, the system $p + \mathrm{D}$ can be found in either of the states $+\frac{3}{2}$ or $+\frac{1}{2}$. The transition to He^3 ($+\frac{1}{2}$ state) is possible in the first case as $M1$ and $E2$, and in the second case as $M1$ and $E0$*.

The conversion coefficients have the following values: for $M1$, $a = 10^{-4}$; for $E2$, $a = 0.1$; for $E0$, only the giving up of meson energy is possible (the probability of pair production e^+ and e^- at the expense of $E0$ is 10^{-3} of the probability of the giving up of meson energy in the case of $p + \mathrm{D}$, but is of the order of unity in the case of $p + \mathrm{H}^3 = \mathrm{He}^4$).

Calculations concerning barrier penetration for the pDμ molecule under adiabatic investigation of the motion of the proton and deuteron yield $\psi^2(0) = 6 \times 10^{-27}$ cm^{-3}.

For the mirror reaction $n + \mathrm{D}$, it is assumed that for the purposes of computation[8] the process proceeds from the state $\frac{3}{2}$ as the result of $M1$. Experimentally for thermal neutrons[9], $\sigma = 5.7 \times 10^{-28}$ with $v = 2200$ m/sec and $\sigma v = 1.3 \times 10^{-22}$ cm^3/sec.

Hence for mesomolecules the probability of the reaction ($\tau = 2.15 \times 10^{-6}$ being the meson lifetime) is

$$w = \frac{\sigma v \psi^2(0)}{(1/\tau) + \sigma v \psi^2(0)} = 0{,}6.$$

During the approach of the proton and deuteron in the spin state $\frac{1}{2}$, the approximate determination of the magnitude of the monopole moment was carried out by examining one charged particle with the wave function $\psi = \frac{1}{\sqrt{2\pi\lambda}} \frac{1}{r} e^{-r/\lambda}$ in the final (combined) state and $\psi = \psi(0)(1 - \lambda/r)$ in the initial state (continuous spectrum, $\psi(0)$ being the previously calculated wave function under the barrier, $\psi^2(0) = 6 \times 10^{-27}$).

The probability of a process with a release of energy, to a meson in a $p + \mathrm{D}$, in spin state $\frac{1}{2}$, and with $\lambda = 2.4 \times 10^{-13}$ cm turned out to be equal to 0.5*.

Thus from the rough estimates made above, it follows that the probability observed by Alvarez for the process which involves release of energy to the meson and the probability of the process which involves emission of a gamma quantum can both be close to unity during the meson lifetime.

In a more accurate investigation, not only will it be necessary to take into account the fact that the processes is not adiabatic (thus involving terms of the order of the meson-nucleon mass ratio), but it will also be necessary to make a separate investigation of the nuclear reaction with different values of total molecular spin.

Note added in proof (February 9, 1957). The probabilities for mesomolecule production in the collisions Dμ + p = D$p\mu$ and Dμ + D = D$_2\mu$ differ not only due to the different positions of the excited vibrational levels of the molecules[4], but also because in pDμ the center of mass does not coincide with the charge center, and thus possesses a dipole moment ($\frac{1}{3}$ ea). Therefore in the collision of slow Dμ + p there is possible the dipole transition ($E1$) in the molecule into the momentum state 1, with the giving up of energy to the electron. In the case of Dμ + D, only the $E2$ transition into momentum state 2 competes with the $E0$ transition investigated[4].

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*Church and Wenezer[7] have recently drawn attention to the role of $E0$ in the case of internal conversion in the transition $J \rightarrow J \neq 0$.

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[1] L. W. Alvarez *et al.*, Lithographed document, December 1956.

[2] F. C. Frank, Nature **160**, 525 (1947).

[3] A. D. Sakharov, Report, Phys. Inst. Acad. Sci. U.S.S.R. (1948).

[4] Ia. B. Zel'dovich. Dokl. Akad. Nauk SSSR **95**, 454 (1954).

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*Choosing $\lambda = \hbar/\sqrt{2ME}$, where M is the reduced mass of p and **D**, and E is the binding energy, 5.4 Mev.

[5] G. M. Griffiths and I. B. Warren, Proc. Phys. Soc 68, 781 (1955).

[6] D. H. Wilkinson, Phil. Mag. 43, 659 (1952).

[7] E. L. Church and Wenezer, Phys. Rev. 103, 1035 (1956).

[8] N. Austern, Phys. Rev. 83, 672 (1951); 85, 147 (1952).

[9] Kaplan, Ringo and Wilzbach, Phys. Rev. 87, 785 (1952).

[10] E. E. Salpeter, Phys. Rev. 88, 547 (1952).

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Corrections to the Articles "Dispersion Formulas of the Quantum Optics of Metals in the Many-Electron Theory"

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IN PREVIOUS WORKS[1-3] the dispersion formulas of the quantum optics of metals were obtained both without taking account of electron damping and with taking it into account for the infrared, visible, and ultra-violet regions of the spectra. The present note is intended to indicate errors in the above works, as well as finally to give the correct dispersion formulas for ε and σ.

The many-electron wave function of the crystal used in the cited works was taken from Seitz[4] and is of the form

$$\psi(\mathbf{r}_1\ldots\mathbf{r}_N, s) = \chi_{\mathbf{k}_1\ldots\mathbf{k}_N}(\mathbf{r}\ldots\mathbf{r}_N, s)\exp\left(i\sum_{j=1}^{N}\mathbf{k}_j\mathbf{r}_j\right), \qquad (1)$$

where χ is a periodic function with period $\mathbf{a}$. It should be noted that the use of the wave function of Eq. (1) for a set of interacting electrons in a crystal is inconsistent, since it gives the total quasimomentum of the system (a conserved quantity) as the sum of the quasimomenta of the separate electrons; this is true, strictly speaking, only for a system of noninteracting electrons.

As was pointed out by Volz and Haken[5,6], if the independent variables are chosen as the coordinates of the center of gravity of the system $\mathbf{R}(X, Y, Z)$ and the appropriate number of relative coordinates, the wave function of a system of interacting electrons in the crystal may be written in the form

$$\psi_{\mathbf{K},l,s}(\mathbf{R}, \mathbf{r}_{jk}) = \chi_{\mathbf{K},l,s}(\mathbf{R}, \mathbf{r}_{jk})\exp(i\mathbf{K}\mathbf{R}), \qquad (2)$$

where the three quantum numbers K_x, K_y, and K_z give total quasimomentum of the system and characterize the motion of the center of mass $\mathbf{R}$ of the total electron system, and l is a continuous quantum number related to the relative motion of the electrons, that is, to the relative coordinates $\mathbf{r}_{jk} = \mathbf{r}_j - \mathbf{r}_k$, and characterizes changes in the configuration of the electron systems; s denotes the number of the band together with the other discrete quantum numbers of the system. The function χ is periodic with respect to translation of the center of mass of the system along the lattice vector $\mathbf{a}$.

The use of wave functions such as those of Eq. (2) is more correct, since in this case the total quasimomentum $\mathbf{K}$ of the system of interacting electrons uniquely characterizes the system as a whole.

If we calculate the matrix element for the probability of optical transition, using the wave function of Eq. (3), in the same way as previously[1], we obtain the energy conservation law and interference condition for the whole system of interacting electrons, namely

$$\begin{gathered} E(\mathbf{K}', l', s') = E(\mathbf{K}, l, s) \pm \hbar\omega, \\ \mathbf{K}' = \mathbf{K} + \mathbf{K}_0 + 2\pi\mathbf{g}, \end{gathered} \qquad (3)$$

where $\mathbf{K}_0$ is the wave vector of the electromagnetic wave, and g is the reciprocal lattice vector. For selection rules, see also Haken[7].

In connection with this, we note that in the previous works[1-3], essentially single-electron selection rules $\xi_i' = \xi_i$ were used. In actuality, however, the set[8] of quantum numbers $(\mathbf{K}', l')$ need not necessarily be the same as the set $(\mathbf{K}, l)$, but must merely satisfy Eq. (3). The use of the correct wave functions of Eq. (2) in the derivation of the dispersion formulas leads to the following expressions for ε and σ: