$$\operatorname{Re}\left[f_+(\omega)+f_-(\omega)-f_+(\mu)-f_-(\mu)\right]$$

$$=\frac{\omega^2-\mu^2}{2\pi^2}\int\limits_0^\infty\frac{dk'}{k'^2-k^2}\left[\sigma_+(k')+\sigma_-(k')\right] \tag{3}$$

$$+\frac{2f^2}{M}\frac{\omega^2-\mu^2}{\omega^2-(\mu^2/2M)^2}\,.$$

Using the experimental values of the phase shifts[3], we obtain

$$\operatorname{Re}\left[f_+(\mu)+f_-(\mu)\right]=-0{,}04\bar{\lambda}\,(1+\mu/M),$$

where $\bar{\lambda}$ is the Compton wavelength of the meson, and μ and M are the meson and nucleon masses, respectively. Inserting (3) into (2), after some simple operations we obtain

$$\sigma_\infty=-1{,}5+I_0+I_1; \tag{4}$$

$$I_0=\frac{1}{\pi^2}\int\limits_0^\infty\frac{dE}{\sqrt{E(E+2\mu)}}\ln\frac{E+2\mu}{E}\left[\sigma_+(E)+\sigma_-(E)\right], \tag{5}$$

$$I_1=4\bar{\lambda}^2\int\limits_0^\infty\frac{dx}{x(x+2)}\left[\frac{1}{\bar{\lambda}}\operatorname{Re}\left(f_+(x)+f_-(x)\right)+0{,}04\,(1+\mu/M)\right],\quad x=\frac{E}{\mu} \tag{6}$$

In order to calculate the integrals we make use of the experimental[4] values for $\sigma_\pm(E)$ and $\operatorname{Re} f_\pm(x)$. We obtain the following values: I_0 = 20 mb, I_1 = 11.5 mb. Inserting these values into (4), we obtain σ_∞ = 30 mb, which is in agreement with the experimental data[4],. The accuracy of σ_∞ is limited by the accuracy of the experimental data for $\sigma_\pm$ and $\operatorname{Re} f_\pm$.

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On the Second Approximation in the Problem of Slow Neutron Scattering by Bound Protons

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THE PROBLEM of scattering slow neutrons by protons bound in a molecule has been treated in the first approximation by Fermi[1]. In several other works[2-4] evaluations of the further approximations have been made. Of particular interest is the variational method developed by Schwinger and Lippmann[3], with the aid of which Lippmann calculated neutron scattering by a hydrogen molecule in the second approximation, and verified the results of Breit and Zilsel[2] who used a different model for the molecule. Soon, however, Ekstein's work[4] appeared, where it was proved that the second approximation of the Schwinger-Lippmann method for neutron scattering by bound protons always diverges, except if the proton is bound to an infinitely heavy nucleus. Ekstein commented, "whether the finite result found by Lippmann is due to the special choice of wave functions . . . or the limiting process used in the evaluation of the integral, remains undecided."

In this note we investigate the question of convergence of the second approximation in the problem of slow neutron ($E \approx 0$) scattering by a proton bound in a molecule of mass M.

In the Schwinger-Lippman method the scattering matrix T_{ba} for zero-energy neutrons from state a to state b is given in the second approximation by the equation

$$T_{ba}=-(4\pi\hbar^2a/m)\left\{\int\chi_b^*(\mathbf{r})\,\chi_a(\mathbf{r})\,d\mathbf{r}+aI\right\},$$

$$I\equiv\int\chi_b^*(\mathbf{r})\left[\sum_\gamma\chi_\gamma^*(\mathbf{r}')\,\chi_\gamma(\mathbf{r})\;\frac{(2\mu/m)\exp\left(ik_\gamma\alpha\,|\mathbf{r}-\mathbf{r}'|\right)-1}{\alpha\,|\mathbf{r}-\mathbf{r}'|}\right]\chi_a(\mathbf{r}')\,d\mathbf{r}d\mathbf{r}', \tag{1}$$

where m is the proton mass,

$$\mu = mM / (M + m);$$

$$\alpha = (M - m) / M, \quad k_\gamma = [2\mu\hbar^{-2}(E - W_\gamma)]^{1/2}, \quad E \approx 0,$$

r is the coordinate of the proton relative to the center of mass of the remainder of the molecule, $\chi_\gamma(\mathbf{r})$ is the eigenfunction of the proton in the molecule (for simplicity we shall not consider other degrees of freedom in the molecule), a is the scattering amplitude of a neutron by a free proton, and W_γ is the energy of the proton in the molecule. The summation over γ is taken over all proton states in the molecule.

Breit and Zilsel[2] replaced the molecule by an idealized system consisting of a three-dimensional oscillator. In this case

$$W_\gamma \equiv W_{n_1 n_2 n_3} = (\hbar^2 / \mu_p) \sum_{i=1}^{3} (n_i + 1/2)\, \beta_i^{-2}, \; \mu_p = m\,(M - m) / M, \; ik_\gamma \alpha = -b\,(2 \sum_i n_i / \beta_i^2)^{1/2},$$
$$\chi_\gamma \equiv \chi_{n_1 n_2 n_3} = \prod_i \frac{\exp\{-r_i^2 / 2\beta_i^2\}}{(\beta_i 2^{n_i} n_i! \sqrt{\pi})^{1/2}} H_{n_i}\left(\frac{r_i}{\beta_i}\right), \quad b = \left(\frac{M - m}{M + m}\right)^{1/2}, \tag{2}$$

where the β_i are constants which determine the potential energy of the proton for elastic scattering on the ground state. Using Eq. (2), it can be shown that in Eq. (1)

$$[\cdots] = \frac{1}{2\sqrt{\pi}} \int_0^\infty \frac{d\tau}{\tau^{3/2}} \left\{ \frac{2\mu}{m} \prod_i \frac{\exp\left(-\frac{1 + q_i^2}{2(1 - q_i^2)\,\beta_i^2} [r_i'^2 + r_i^2 + 4q_i r_i r_i']\right)}{\beta_i \sqrt{\pi} \sqrt{1 - q_i^2}} - \delta(\mathbf{r} - \mathbf{r}') \right\} e^{-|\mathbf{r} - \mathbf{r}'|/4\tau},$$

$$q_i = \exp\{-2b^2\tau / \beta_i^2\}.$$

Inserting $[\cdots]$ into (1), we obtain

$$I = \frac{8}{\sqrt{\pi}} \int_0^\infty d\tau \left\{ \frac{\mu}{m} \prod_i [4\tau + 2\beta_i^2(1 - q_i)]^{-1/2} - (16\tau^{3/2})^{-1} \right\}. \tag{3}$$

From the form of the integral of Eq. (3) it follows that it diverges only in the region of small τ. In order to investigate the integrand as $\tau \to 0$, let us expand q_i in a series, so that

$$\{\cdots\} = \frac{\mu}{m} [4\tau(1 + b^2)]^{-3/2} - 1/2(4\tau)^{-3/2}.$$

In order for the integral of Eq. (3) to converge, it is necessary that

$$2\mu / m = (1 + b^2)^{-3/2}. \tag{4}$$

Noting Eq. (2), we see that condition (4) is not satisfied, so that the integral diverges in the region $\tau \approx 0$.

Thus the second approximation in the Schwinger-Lippmann theory diverges for neutron scattering on a harmonically bound proton. Breit and Zilsel[2] obtained a finite value in solving a similar problem as a result of eliminating the divergent part of the integral by the replacement $\tau \to 2\tau$ on going over from the second integral in their expression (3.1) to their expression (3.2). This method is mathematically invalid.

It is easily shown that, however, the second approximation of Eq. (1) always converges if we use the wave functions of real molecules rather than of idealized systems. In a real molecule a highly excited state corresponds to a decaying system, when the wave functions of the proton motion relative to the rest of the molecule can be given in terms of plane waves $\chi_\gamma = \exp(i\mathbf{pr})$ with energies $W_\gamma = \hbar^2 p^2 / 2\mu_p$. In this case $ik_\gamma a = -bp$, and after replacing summation over γ by integration with respect to **p**, the integral over that part of the sum over γ in Eq. (1) which corresponds to high excitations (which are all that can cause the integral to diverge) can be written

$$I_{p_0} = \frac{2\mu}{m\alpha} \int d\mathbf{r}\, \chi_b^*(\mathbf{r}) \left\{ \int_{p_0}^\infty d\mathbf{p} e^{i\mathbf{pr}} \int d\mathbf{r}' \frac{\exp(-bp\,|\mathbf{r} - \mathbf{r}'|) - m / 2\mu}{|\mathbf{r} - \mathbf{r}'|} e^{-i\mathbf{pr}'} \chi_a(\mathbf{r}') \right\}. \tag{5}$$

For sufficiently large p_0, Eq. (5) can be calculated in the same way as was done by Ekstein[4], obtaining

$$\{\cdots\} = (4\pi)^2 \chi_a(\mathrm{r}) \left[\frac{1}{1+b^2} - \frac{m}{2\mu}\right] \int_{p_0}^{\infty} dp. \qquad (6)$$

From the definition of b and μ it follows that the expression in square brackets in Eq. (6) vanishes. It is not difficult to show that nonzero neutron energies also give a finite result.

Thus when using wave functions for real molecules in the calculation according to Eq. (1), we obtain a finite result. This conclusion contradicts that of Ekstein, since he mistakenly omitted the factor a in the exponent of Eq. (1) and stated that $k_\gamma = ip$, rather than the correct expression $k_\gamma = ip\sqrt{\mu/\mu_p}$.

The results of Breit and Zilsel are correct because an artificial (mathematically nonrigorous) method was used to eliminate the divergent part of the integral, which in turn, is due to the idealization of the problem being considered.

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Fluctuations in Gases

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THE STATE OF A GAS is completely described by the particle-distribution function in the phase space $f(\mathbf{r}, \mathbf{v}, t)$; therefore the problem of fluctuations in gases leads to the study of the correlation characteristics of the distribution function. Ordinarily one understands by f the statistically-average density of the particles in the phase space which, naturally, cannot fluctuate. Speaking of the fluctuations of the distribution function we must, however, bear in mind the "true" density

$$F(\mathbf{r}, \mathbf{v}, t) = \sum_i \delta(\mathbf{r} - \mathbf{r}_i)\, \delta(\mathbf{v} - \mathbf{v}_i),$$

where the summation is carried out with respect to all particles. We shall consider the density F to be a random quantity which only on the average coincides with f.

The function F satisfies the equation

$$\frac{\partial F}{\partial t} + (\mathbf{v}\nabla) F = \frac{1}{m} \frac{\partial F(\mathbf{r}, \mathbf{v}, t)}{\partial \mathbf{v}} \int \frac{\partial U(|\mathbf{r} - \mathbf{r}'|)}{\partial \mathbf{r}} F(\mathbf{r}', \mathbf{v}', t)\, d\mathbf{r}'\, d\mathbf{v}', \qquad (1)$$

where m is the mass of a molecule, and U is the potential energy of the interaction of the molecules among themselves. In an ideal gas we may neglect the interaction of the particles and obtain then from (1) $F(\mathbf{r}, \mathbf{v}, t) = F(\mathbf{r} - \mathbf{v}(t - t_0), \mathbf{v}, t_0)$ whence

$$\langle \varphi(\mathbf{r}, \mathbf{v}, t)\, \varphi(\mathbf{r}_0, \mathbf{v}_0, t_0) \rangle = f(\mathbf{r}, \mathbf{v}, t)\, \delta(\mathbf{r} - \mathbf{v}(t - t_0) - \mathbf{r}_0)\, \delta(\mathbf{v} - \mathbf{v}_0), \qquad (2)$$

where $\varphi = F - f$ and the angular brackets denote the averaging.

Our problem consists in finding the correlation of (2) with allowance for the collisions. If the gas is not very dense, we may confine ourselves only to the accounting of paired collisions, and then the right half of Eq. (1) may be approximately represented in the form of the collision term $S(F, F)$:

$$S(F, F) = \int \{F(\mathbf{r}, \mathbf{v}', t)\, F(\mathbf{r}_1, \mathbf{v}_1', t) - F(\mathbf{r}, \mathbf{v}, t)\, F(\mathbf{r}_1, \mathbf{v}_1, t)\}\, |\mathbf{v} - \mathbf{v}_1|\, \rho\, d\rho\, d\chi\, d\mathbf{v}_1, \qquad (3)$$

where ρ is the collision parameter, $\rho\, d\rho\, d\chi$ is an element of the surface which is perpendicular to the relative velocity $\mathbf{v} - \mathbf{v}_1$ and passes through the point $\mathbf{r}$, $\mathbf{r}_1$ is the coordinate of this element, $\mathbf{v}'$ and $\mathbf{v}_1'$ are the velocities of the particles before the impact which are transformed after the collision into $\mathbf{v}$, $\mathbf{v}_1$. It is approximately assumed here that the collision occurs at that instant when both particles intersect the surface which is perpendicular to their relative velocity.

If we average (1) using the collision term in the form of (3) and neglect the correlation of the particles before the collision and the difference between $\mathbf{r}$ and $\mathbf{r}_1$, we shall obtain the ordinary Boltzmann equation

$$\frac{\partial f}{\partial t} + (\mathbf{v}\nabla) f = \int \{f(\mathbf{r}, \mathbf{v}', t)\, f(\mathbf{r}, \mathbf{v}_1', t) - f(\mathbf{r}, \mathbf{v}, t)\, f(\mathbf{r}, \mathbf{v}_1, t)\}\, |\mathbf{v} - \mathbf{v}_1|\, \rho\, d\rho\, d\chi\, d\mathbf{v}_1. \qquad (4)$$