# Energy Loss of a Charged Particle Passing Through a Laminar Dielectric, I

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The energy losses of a uniformly moving charged particle in a laminar dielectric are considered. A general expression is obtained for the losses in the case in which the particle moves in an unbounded laminar medium or in a wave guide filled with a laminar dielectric. The polarization losses are studied in detail. An expression is derived for the spectral distribution of the parametric Cerenkov radiation. The Cerenkov radiation in the case of thin layers is studied. It is shown that division of the dielectric into layers leads to an increase of the intensity of the Cerenkov radiation. The Cerenkov radiation in a thin-layered plasma is considered.

AS IS WELL KNOWN, the Cerenkov and Doppler effects can be reduced to the ordinary resonance between the driving force caused by the uniform motion of a charged particle or dipole through a homogeneous medium and the characteristic vibrations of the electromagnetic field in the medium[1,2]. The condition for appearance of the Cerenkov effect is that the speed $v$ of the motion of the particle must exceed the phase velocity of the propagation of waves in the given medium.

In the case of uniform motion of a particle through a laminar (spatially periodic) medium one can obtain conditions in which forced parametric resonance can occur. Unlike the cases of ordinary Cerenkov or Doppler effects, the condition for resonance is here not the equality of the frequency of the driving force and the characteristic frequency of the field, but the equality of the frequency of the driving force and the frequency of vibration of the free electromagnetic oscillations in the laminar medium. It can therefore be expected that the conditions for Cerenkov radiation in the presence of forced parametric resonance will be different from the conditions for the ordinary Cerenkov effect, and the parametric Cerenkov effect will have a number of special features.

As is well known, the use of parametric resonance for the generation of electromagnetic oscillations has been the subject of a great many papers. First to be considered in this connection are the papers of Mandel'shtam and Papaleksi and their collaborators[3]. It must be pointed out, however, that in these papers the wavelengths generated were large, on account of the large inertia of the variable parameters determing the parametric resonance. In the present case the system is almost without inertia. This makes it possible to use the phenomenon of parametric resonance for the generation and amplification of very short electromagnetic waves.

It can be expected that the parametric Cerenkov effect will occur also if the speed of the particle is less than the phase velocity of the wave in the medium. Indeed, as was pointed out by Vavilov[4], it follows from the interference treatment of the Cerenkov effect that on passage of a uniformly moving particle through a layer of dielectric radiation occurs even when the speed of the particle is less than the phase velocity of the wave in the dielectric. Here, however, it is necessary that the optical thickness of the layer of dielectric be less than $\pi$. The multiple repetition of this effect in parametric resonance must lead to its amplification. It can also be expected that the parametric Cerenkov effect will occur in media with dielectric constant less than unity, and even in media in which the dielectric constant can take negative values. A special study will therefore be made of the case of the parametric Cerenkov effect in a laminar electron plasma.

When a uniformly moving particle passes through a medium a considerable part of the energy is expended in polarization losses. It is important therefore to study the peculiarities of polarization losses in laminar media.

In considering the energy losses of a particle in a laminar medium we shall start with the system of Maxwell equations describing the interaction of a uniformly moving charged particle with electromagnetic waves propagated in the medium:

$$-\frac{\partial H_\varphi}{\partial z} = \frac{\hat{\varepsilon}(z)}{c}\frac{\partial E_r}{\partial t}, \quad \frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} = -\frac{\hat{\mu}(z)}{c}\frac{\partial H_\varphi}{\partial t},$$

$$\frac{1}{r}\frac{\partial}{\partial r} r H_\varphi = \frac{\hat{\varepsilon}(z)}{c}\frac{\partial E_z}{\partial t} + \frac{4\pi}{c} ev\,\delta(vt - z)\frac{\delta(r)}{2\pi r}, \quad (1)$$

where the operators $\hat{\varepsilon}$ and $\hat{\mu}$ are defined by the relations

$$\hat{\varepsilon}(z) e^{i\omega t} = \varepsilon(\omega, z) e^{i\omega t}, \ \hat{\mu}(z) e^{i\omega t} = \mu(\omega, z) e^{i\omega t}, \quad (2)$$

and $e$ is the charge and $v$ the speed of the particle.

We shall seek solutions for the components of the electromagnetic field in the form of Fourier integrals

$$u(r, z, t) = \int_{-\infty}^{\infty} e^{i\omega t} u_\omega(r, z)\, d\omega. \quad (3)$$

Then from Eq. (1), using Eq. (2), we get the following equation for the longitudinal component of the electric induction $D_{z,\omega} = \varepsilon(\omega, z)E_{z,\omega}$:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial D_{z,\omega}}{\partial r}\right) + \varepsilon\frac{\partial}{\partial z}\left(\frac{1}{\varepsilon}\frac{\partial D_{z,\omega}}{\partial z}\right) + k^2\varepsilon\mu D_{z,\omega}$$

$$= \frac{ike\varepsilon\mu}{\pi c} e^{-i\omega z/v}\frac{\delta(r)}{r} + \frac{e\varepsilon}{\pi v}\frac{\delta(r)}{r}\frac{\partial}{\partial z}\left(\frac{1}{\varepsilon}e^{-i\omega z/v}\right), \quad (4)$$

$$k = \frac{\omega}{c}.$$

In the case in which one has to consider the energy losses of a charged particle moving along the axis of a wave guide filled with dielectric, the dependence of $D_z$ on $r$ can be written in the form

$$D_{z,\omega}(r, z) = \sum_{n=1}^{\infty}\frac{2}{R^2 J_1^2(\alpha_n)} J_0\left(\alpha_n\frac{r}{R}\right) D_{z,\omega n}(z), \quad (5)$$

where $R$ is the radius of the waveguide and $\alpha_n$ is the $n$th zero of the Bessel function of order zero. Sub-Eq. (5) into Eq. (4) and using the orthgonality contion of the Bessel functions, we obtain the following equation for $D_{z,\omega n} = X(z)$:

$$\varepsilon\frac{d}{dz}\left(\frac{1}{\varepsilon}\frac{dX}{dz}\right) + \left(k^2\varepsilon\mu - \frac{\alpha_n^2}{R^2}\right)X$$

$$= \frac{ike\varepsilon\mu}{\pi c} e^{-i\omega z/v} + \frac{\varepsilon e}{\pi v}\frac{\partial}{\partial z}\left(\frac{1}{\varepsilon}e^{-i\omega z/v}\right). \quad (6)$$

If the charged particle is moving in an unbounded laminar dielectric, perpendicular to the layers, it is natural to look for the dependence of $D_z$ on $r$ in the form

$$D_{z,\omega} = \int_0^{\infty} X(z) J_0(k_\perp r) k_\perp\, dk_\perp, \quad (7)$$

where $X = D_{z,\omega k_\perp}$ is also determined by Eq. (6), with $\alpha_n/R$ replaced by $k_\perp$. This equation is valid for an arbitrary variation of $\varepsilon(z)$ and $\mu(z)$, and is the basic equation of our problem.

In the case in which $\varepsilon(z)$ and $\mu(z)$ are periodic functions of the variable $z$, Eq. (6) is an equation with periodic coefficients, the right half of which is a known function of $z$. The general solution of the inhomogeneous equation (6) is

$$X(z) = U_1(z)[A - V_2(z)] + U_2(z)[B + V_1(z)], \quad (8)$$

where $A$ and $B$ are constants of integration and

$$V_1(z) = \frac{1}{W}\int_0^z \left[\frac{ike\mu}{\pi c} U_1(z) - \frac{e}{\pi v}\frac{1}{\varepsilon}\frac{dU_1(z)}{dz}\right] e^{-i\omega z/v}\, dz,$$

$$V_2(z) = \frac{1}{W}\int_0^z \left[\frac{ike\mu}{\pi c} U_2(z) - \frac{e}{\pi v}\frac{1}{\varepsilon}\frac{dU_2(z)}{dz}\right] e^{-i\omega z/v}\, dz,$$

$$W = U_1(z)\frac{1}{\varepsilon}\frac{dU_2(z)}{dz} - U_2(z)\frac{1}{\varepsilon}\frac{dU_1(z)}{dz},$$

$U_1(z)$ and $U_2(z)$ being linearly independent solutions of the corresponding homogeneous equations which satisfy the conditions

$$U_1(z + L) = \rho_1 U_1(z), \ U_2(z + L) = \rho_2 U_2(z), \quad (9)$$

$L$ is the period of the structure of the laminar medium, and $\rho_1$ and $\rho_2$ are constants of absolute value unity. Using Eq. (9), one readily shows that

$$V_1(z + L) = V_1(L) + \rho_1 V_1(z) e^{-i\omega L/v},$$
$$V_2(z + L) = V_2(L) + \rho_2 V_2(z) e^{-i\omega L/v}. \quad (10)$$

Since the energy losses of a uniformly moving charged particle in a laminar medium must be a periodic function of $z$ with the same period as the structure, we find from Eq. (3) ($\omega t = \omega z/v$):

$$e^{i\omega z/v} D_{z,\omega}(r, z) = e^{i\omega(z+L)/v} D_{z,\omega}(r, z + L). \quad (11)$$

This condition means that if the field at a certain point $z$ where the charge is present at a given instant is $E_z$, then this value of the field will occur at the point $z + L$ only when the charge arrives at this point. In particular, condition (11) is satisfied if we assume that a similar relation holds for the field component $X$

$$X(z) = e^{i\omega L/v} X(z + L). \quad (12)$$

Condition (12) enables us to determine the arbitrary constants $A$ and $B$, and thus to find the field produced by a uniformly moving charge in a laminar medium.

The total energy loss of the charged particle is found by the well known formula[6]

$$-d\mathcal{E}/dz = e\int E_{z,\,\omega k_\perp}\big|_{z=vt} k_\perp\, dk_\perp\, d\omega, \tag{13}$$

where $E = X/\varepsilon(\omega)$ and $X$ is the field of the charge in the medium, determined above. We calculate the loss in the case in which the medium is composed of alternate layers of two dielectrics.

We assume that the dielectric constant and permeability of the layer of dielectric of thickness $a(0 \le z \le a)$ are $\varepsilon_1$ and $\mu_1$, and those of the layer of thickness $b\,(a \le z \le L$, where $L = a + b)$ are $\varepsilon_2$ and $\mu_2$. Then the solutions of the homogeneous equation corresponding to the inhomogeneous equation (6), which satisfy the conditions (9), have the form

$$\begin{aligned} U_1(z) &= U_1(0)\left[u_1(z) - \frac{u_1(L) - \rho_1}{u_2(L)}\, u_2(z)\right],\\ U_2(z) &= U_2(0)\left[u_1(z) - \frac{u_1(L) - \rho_2}{u_2(L)}\, u_2(z)\right], \end{aligned} \tag{14}$$

where

$$\begin{gathered} u_1(z) = \cos p_1 z,\ \ u_2(z) = (\varepsilon_1/p_1)\sin p_1 z,\ \ p_1^2 = \varepsilon_1\mu_1 k^2 - k_1^2 \ \text{ for }\ 0 \leqslant z \leqslant a;\\ u_1(z) = \cos p_1 a \cos p_2(z-a) - (\varepsilon_2 p_1/\varepsilon_1 p_2)\sin p_1 a \sin p_2(z-a),\\ u_2(z) = (\varepsilon_1/p_1)\sin p_1 a \cos p_2(z-a) + (\varepsilon_2/p_2)\cos p_1 a \sin p_2(z-a),\\ p_2^2 = \varepsilon_2\mu_2 k^2 - k_1^2 \quad \text{for} \quad a \leqslant z \leqslant L, \end{gathered} \tag{15}$$

$k_1 = \alpha_n/R$ for a laminar dielectric bounded by a waveguide, and $k_1 = k_\perp$ for an unbounded laminar dielectric. The quantities $\rho_1$ and $\rho_2$ are found as the roots of the quadratic equation:

$$\rho^2 - 2A\rho + 1 = 0,\ \ A = \cos p_1 a \cos p_2 b - \frac{1}{2}\left(\frac{p_1\varepsilon_2}{p_2\varepsilon_1} + \frac{p_2\varepsilon_1}{p_1\varepsilon_2}\right)\sin p_1 a \sin p_2 b. \tag{16}$$

The relations (8) and (15) make it possible to determine the field $E_z$ produced by a uniformly moving particle in the laminar medium under consideration and, according to Eq. (13), to determine the total energy loss of the particle.

We note, however, that the greatest practical interest attaches to the energy loss averaged over the period of the structure, *i. e.*,

$$\begin{aligned} \overline{d\mathcal{E}/dz} &= e\int \overline{E}_{z,\,\omega k_\perp}\big|_{z=vt} k_\perp\, dk_\perp\, d\omega,\\ \overline{E}_{z,\,\omega k_\perp}\big|_{z=vt} &= \frac{1}{L}\int_0^L \left(\frac{1}{\varepsilon(\omega, z)} X(z)\, e^{i\omega t}\right)_{z=vt} dz. \end{aligned} \tag{17}$$

Using Eqs. (8) and (14), and also Eqs. (15) and (16), we find:

$$\begin{aligned} \overline{E}_{z,\,\omega k_\perp}\big|_{z=vt} &= \frac{ike}{\pi c}\left\{\frac{a}{L}\,\frac{(\mu_1 - 1/\varepsilon_1\beta^2)}{p_1^2 - \omega^2/v^2} + \frac{b}{L}\,\frac{(\mu_2 - 1/\varepsilon_2\beta^2)}{p_2^2 - \omega^2/v^2}\right\}\\ &+ \frac{iek_\perp^2}{\pi L v(\cos(\omega L/v) - \cos\psi)}\left\{Z_1^2\left[\frac{p_1 v}{\varepsilon_1\omega}\sin p_1 a\left(\cos p_2 b - \cos\frac{\omega}{v} b\right)\right.\right.\\ &+ \left.\frac{p_2 v}{\varepsilon_2\omega}\sin p_2 b\left(\cos p_1 a - \cos\frac{\omega}{v} a\right)\right] - 2Z_1 Z_2\left[\sin\frac{\omega}{v} a\left(\cos p_2 b\right.\right.\\ &\left.\left. - \cos\frac{\omega}{v} b\right) + \sin\frac{\omega}{v} b\left(\cos p_1 a - \cos\frac{\omega}{v} a\right)\right] +\\ &+ \left.Z_2^2\left[\frac{\varepsilon_1\omega}{p_1 v}\sin p_1 a\left(\cos p_2 b - \cos\frac{\omega}{v} b\right) + \frac{\varepsilon_2\omega}{p_2 v}\sin p_2 b\left(\cos p_1 a - \cos\frac{\omega}{v} a\right)\right]\right\},\\ Z_1 &= \left(\frac{1}{p_1^2 - \omega^2/v^2} - \frac{1}{p_2^2 - \omega^2/v^2}\right),\quad Z_2 = \left(\frac{1}{\varepsilon_1(p_1^2 - \omega^2/v^2)} - \frac{1}{\varepsilon_2(p_2^2 - \omega^2/v^2)}\right),\\ \cos\psi &= \cos p_1 a \cos p_2 b - \frac{1}{2}\left(\frac{p_1\varepsilon_2}{p_2\varepsilon_1} + \frac{p_2\varepsilon_1}{p_1/\varepsilon_2}\right)\sin p_1 a \sin p_2 b. \end{aligned} \tag{18}$$

The first two terms in (18) represent the fields produced by the uniformly moving charge in the first and second media respectively. The remaining terms describe interference effects that arise in the passage of the charged particle through the laminar medium. As can be seen from Eq. (18), these terms go to zero if $a$ or $b$ goes to zero.

Before proceeding to the study of the expression (18), we note that these formulas can also be obtained in another way.

The laminar medium under consideration consists of alternate layers of two homogeneous and isotropic dielectrics. Therefore in each of these layers Eq. (6) is an equation with constant coefficients. To obtain the solutions in the entire medium, it is necessary to satisfy the proper boundary conditions at the surfaces separating the layers.

We suppose that the parameters of the layer $-a \leq z \leq 0$ are $\varepsilon_1$ and $\mu_1$ and those of the layer $0 \leq z \leq b$, are $\varepsilon_2$ and $\mu_2$. These layers are repeated, forming an unbounded laminar dielectric with a period $L = a + b$. Then in the first region, according to Eq. (6), we have:

$$E'_{z,\omega k_\perp} = Ae^{ip_1 z} + Be^{-ip_1 z} + \frac{ike}{\pi c}\frac{(\mu_1 - 1/\varepsilon_1\beta^2)}{p_1^2 - \omega^2/v^2} e^{-i\omega z/v}, \quad (19)$$

and in the second region:

$$E''_{z,\omega k_\perp} = Ce^{ip_2 z} + De^{-ip_2 z} + \frac{ike}{\pi c}\frac{(\mu_2 - 1/\varepsilon_2\beta^2)}{p_2^2 - \omega^2/v^2} e^{-i\omega z/v}. \quad (20)$$

From Eq. (1) we find the corresponding expressions for the radial components of the field. The arbitrary constants $A$, $B$, $C$, and $D$ are determined from the boundary conditions at the surfaces of the dielectrics and the conditions of periodicity for the fields. We present here only the expressions for the coefficients $C$ and $D$, which are needed in what follows:

$$\begin{aligned} C = {} & \frac{2ie}{\pi\Delta} Z_1 \{[ip_1^2\varepsilon_2 \sin p_1 a - \varepsilon_1 p_1 p_2 (\cos p_1 a - e^{i\omega a/v})] (e^{-ip_2 b} - e^{-i\omega b/v}) e^{-i\omega L/v} \\ & + \varepsilon_1 p_1 p_2 e^{-2i\omega Lv} (1 - 2\cos p_1 a \cdot e^{i\omega a/v} + e^{2i\omega a/v})\} \\ & - \frac{2ie}{\pi\Delta} Z_2 \{[ip_2\varepsilon_1^2 \sin p_1 a - \varepsilon_1\varepsilon_2 p_1 (\cos p_1 a - e^{i\omega a/v})] (e^{-ip_2 b} - e^{-i\omega b/v}) e^{-i\omega L/v} \\ & + \varepsilon_1\varepsilon_2 p_1 e^{-2i\omega L/v} (1 - 2\cos p_1 a \cdot e^{i\omega a/v} + e^{2i\omega a/v})\}, \\ D = {} & \frac{2ie}{\pi\Delta} Z_1 \{\varepsilon_1 p_1 p_2 e^{-2i\omega L/v} (1 - 2\cos p_1 a \cdot e^{i\omega a/v} + e^{2i\omega a/v}) \\ & - [ip_1^2\varepsilon_2 \sin p_1 a + \varepsilon_1 p_1 p_2 (\cos p_1 a - e^{i\omega a/v})] (e^{ip_2 b} - e^{-i\omega b/v}) e^{-i\omega L/v}\} \\ & + \frac{2ie}{\pi\Delta} Z_2 \{\varepsilon_1\varepsilon_2 p_1 e^{-2i\omega L/v} (1 - 2\cos p_1 a \cdot e^{-i\omega a/v} + e^{2i\omega a/v}) \\ & - [i\varepsilon_1^2 p_2 \sin p_1 a + \varepsilon_1\varepsilon_2 p_1 (\cos p_1 a - e^{i\omega a/v})] (e^{ip_2 b} - e^{-i\omega b/v}) e^{-i\omega L/v}\}, \\ \Delta = {} & -4\varepsilon_1\varepsilon_2 p_1 p_2 (e^{-2i\omega L/v} - 2\cos\psi \cdot e^{-i\omega L/v} + 1); \end{aligned} \quad (21)$$

$Z_1$, $Z_2$ and $\cos\psi$ were determined previously.

Carrying out the averaging of the field (19), (20) and making a number of simplifications, one can again obtain Eq. (18).

In calculating the total losses according to Eq. (17), one must integrate Eq. (18) over all frequencies in the range $(-\infty, \infty)$. Since (18) is an odd function of $\omega$, the value of the integral is determined only by the residues of the integrand at the singularities located on the real axis. The path of integration consists therefore of the real axis and suitable detours around the singularities located on it.

By direct calculations one can convince oneself that all the singularities of (18) are simple poles. They are given by the equations

$$\text{a) } \varepsilon_1(\omega) = 0, \qquad \text{b) } \varepsilon_2(\omega) = 0,$$
$$\text{c) } \cos(\omega L/v) - \cos\psi = 0, \quad \text{d) } p_1^2 - \omega^2/v^2 = 0,$$
$$\text{e) } p_2^2 - \omega^2/v^2 = 0. \quad (22)$$

The energy losses of the charged particle that are associated with the zeroes of the dielectric constants $\varepsilon_1$ or $\varepsilon_2$ are polarization losses.

The roots of Eq. (22, c) give the radiation of the particle in the medium. In fact, the frequencies determined by these poles satisfy the relation

$$\cos\omega L/v = \cos p_1 a \cos p_2 b - \frac{1}{2}\left(\frac{p_1\varepsilon_2}{p_2\varepsilon_1} + \frac{p_2\varepsilon_1}{p_1\varepsilon_2}\right)\sin p_1 a \sin p_2 b \quad (23)$$

and correspond to waves propagated in the medium in question, since Eq. (23) is the dispersion equation of the laminar dielectric.

Radiation at a frequency satisfying Eq. (22, c) but not satisfying Eqs. (22, d or e), does not occur in a homogeneous dielectric with dielectric constants $\varepsilon_1$ or $\varepsilon_2$, and is a special Cerenkov effect due to parametric resonance. It can therefore be called Cerenkov radiation.

The connection between this radiation and the Cerenkov effect appears particularly clearly in the case of thin and closely spaced dielectric layers, when the laminar dielectric is electrodynamically equivalent to a homogeneous and anisotropic dielectric. In this case the parametric Cerenkov radiation reduces to the ordinary Cerenkov radiation in the anisotropic dielectric.

The energy losses associated with the roots of Eqs. (22, d or e) give the proper Cerenkov radiation in the first and second dielectrics, respectively. This radiation is propagated in the laminar medium if the corresponding frequencies satisfy also condition (23). The energy lost by the particle in proper Cerenkov radiation in the unbounded laminar medium is propagated also in directions perpendicular to the motion of the particle. The frequencies so radiated do not satisfy Eq. (23).

Consider now the polarization losses. Assume that if $\varepsilon_2(\omega) = 0$, then $\varepsilon_1(\omega) \neq 0$. In this case, in the integration of the (18) over the frequency it is necessary to keep only those terms for which $\varepsilon_2 = 0$ is a pole.

By several transformations of Eq. (18) we obtain:

$$\overline{E}_{z,\,\omega k_\perp\; z=vt} = -\frac{b}{L}\frac{ike}{\pi c}\frac{1}{\beta^2\varepsilon_2}\frac{1}{p_2^2-\omega^2/v^2} + \frac{2iek_\perp^2\,\omega\,(\cos p_2 b - \cos(\omega b/v))}{\pi L\varepsilon_2 v^2 p_2 \sin p_2 b\,(p_2^2-\omega^2/v^2)^2} + O(\varepsilon_2),$$

and consequently:

$$-(\overline{d\mathcal{E}}/dz)_{\text{polar}} = \frac{2e^2}{v^2}\frac{b}{L}\frac{\omega_0}{(d\varepsilon_2/d\omega)_0}\int_0^{\varkappa_m}\frac{k_\perp\,dk_\perp}{k_\perp^2+\omega_0^2/v^2} - \frac{4e^2\omega_0}{Lv^2(d\varepsilon_2/d\omega)_0}\int_0^\infty\frac{k_\perp^2\,dk_\perp}{(k_\perp^2+\omega_0^2/v^2)^2}\frac{\left(\cosh k_\perp b-\cos\frac{\omega_0}{v}b\right)}{\sinh k_\perp b}, \tag{24}$$

where $\omega_0$ is a root of the equation $\varepsilon_2(\omega) = 0$.

The first term in Eq. (24) is the ordinary polarization loss in a layer of dielectric of thickness $b$ with dielectric constant $\varepsilon_2$. The second term is due to the presence of the boundaries.

From Eq. (24) it follows that the presence of boundaries always leads to a reduction of the polarization losses. In fact, in the second term the integral is taken with an essentially positive integrand, and thus always reduces the total polarization loss.

Since the interference effects caused by the boundaries of the dielectrics are important only for wavelengths that are not very small in comparison with the structure period, and the thicknesses of the discs are at least such that a macroscopic treatment is valid, it is clear that for very high $k_\perp$ the boundary effects already do not play any important part. Therefore the second integral in Eq. (24) does not diverge at the upper limit, and the integration over $k_\perp$ is taken to infinity.

In the evaluation of this integral it is expedient to divide the region of integration into two parts: $(0, 1/b)$ and $(1/b, \infty)$. Then in the integration over the first region the integrand can be simplified by supposing that $k_\perp b \ll 1$. This is justified, since the integrand reaches its maximum for $k_\perp b \ll 1$. In the second integral the main contribution comes from the region $k_\perp b \gg 1$. Therefore:

$$-(d\mathcal{E}/dz)_{\text{polar}} = \frac{e^2}{v^2}\frac{\omega_0}{(d\varepsilon_2/d\omega)_0}\frac{b}{L}\ln\frac{\varkappa_m^2 b^2+(\omega_0 b/v)^2}{1+(\omega_0 b/v)^2} - \frac{2e^2}{v^2}\frac{\omega_0}{(d\varepsilon_2/d\omega)_0}\frac{b}{L}\left[\frac{(1+\omega_0^2b^2/2v^2)-\cos(\omega_0 b/v)}{(\omega_0 b/v)^2(1+(\omega_0 b/v)^2)} + \frac{v}{b\omega_0}\arctan\frac{b\omega_0}{v}\right]. \tag{25}$$

For sufficiently small values of $b$ we have $\omega_0 b/v \ll 1$, so that the polarization losses are given by:

$$-(d\mathcal{E}/dz)_{\text{polar}} = \frac{2e^2}{v^2}\frac{\omega_0}{(d\varepsilon_2/d\omega)_0}\frac{b}{L}\ln\frac{\varkappa_m b}{7{,}4}. \tag{26}$$

When the thickness of the dielectric discs is increased, the role of the boundary effects is diminished, and thus the second term in Eq. (25) is also decreased. For sufficiently large $b$ we have $\omega_0 b/v \gg 1$, and Eq. (25) goes over into the well known expression for the polarization losses in an unbounded dielectric with dielectric constant $\varepsilon_2$:

$$-(d\mathcal{E}/dz)_{\text{polar}} = \frac{e^2}{v^2}\frac{\omega_0}{(d\varepsilon_2/d\omega)_0}\ln\left(1+\frac{\varkappa_m^2 v^2}{\omega_0^2}\right). \tag{27}$$

From Eqs. (26) and (27) it follows that a particle loses less energy through the polarization of the medium in a laminar dielectric than in a solid dielectric, if it travels the same distance in the dielectric in the two cases. If zeroes of the dielectric constant $\varepsilon_2$ occur at not just one but at several values of the frequency, the polarization losses in the medium are given by the sum of expressions (25) calculated for each of the frequencies $\omega_i$ for which $\epsilon_2 = 0$.

In order to study in more detail the role played by transition processes in polarization losses, we consider these losses within a single dielectric layer, before averaging over the period of the structure. In doing this we shall start from the expression (20) for the fields. The zeroes of the dielectric constant $\varepsilon_2$ will be poles only for the coefficients $C$ and $D$ which determine the field in the dielectric layers with the constant $\varepsilon_2$. Since for $0 \leq z \leq b$ we have Eq. (20), where $C$ and $D$ are given by (21), we get the following expression for the polarization losses:

$$-(d\mathscr{E}/dz)_{\text{polar}} = \frac{e^2\omega_0}{v^2(d\varepsilon_2/d\omega)_0}\ln\left(1+\frac{\varkappa_m^2 v^2}{\omega_0^2}\right) - \frac{2e^2}{v(d\varepsilon_2/d\omega)_0 b}[\sin\varkappa(1-\gamma)J(\gamma)+\sin\varkappa\gamma\cdot J(1-\gamma)],$$

$$J(\gamma) = \int\limits_0^\infty \frac{x^2dx\cosh\gamma x}{x^2+\varkappa^2\sinh x}, \quad \gamma = \frac{z}{b}, \quad \varkappa = \frac{\omega_0 b}{v} \tag{28}$$

From the relation (28) it follows that in a single dielectric layer the loss of energy by the particle in polarization of the medium is not uniform. The integrals $J$ cannot be expressed in terms of known functions, so that the dependence of the energy loss on the depth can be found only by numerical calculations.

The integral $J(\gamma)$ is a monotonically increasing function of the parameter $\gamma = z/b$. Since the main contribution to the value of the integral comes from large values of $x$, and for large $x$ the integrand is proportional to $e^{-x(1-\gamma)}$, $J(\gamma)$ increases exponentially with increasing $\gamma$. The integral $J(1-\gamma)$ decreases with increase of $\gamma$. Consequently, these integrals take their greatest values at the corresponding boundaries of the dielectric. Each of the integrals is multiplied by a periodic function, so that as a whole the second term in Eq. (28) is an oscillating function with amplitude decreasing toward the center of the dielectric layer. The amplitude of the maximum variation is proportional to $1/b$, so that the part played by the second term decreases with increasing thickness of the layers.

The decrease of the polarization losses near the boundaries of the dielectric also explains the previously noted reduction of the average polarization losses in laminar dielectrics.

We now consider the parametric Cerenkov radiation. In the general expression (17) for the total losses, the parametric Cerenkov effect corresponds to the poles of the integrand (18) which give radiation and do not lead to the ordinary Cerenkov effect, *i.e.*, the roots of the equation (22, c) when $p_1^2 \neq \omega^2/v^2$ and $p_2^2 \neq \omega^2/v^2$.

For the parametric Cerenkov radiation we can find the spectral distribution of the radiation and write down in general form the expression for the energy lost by the particle to the radiation. Integrating Eq. (18) with respect to frequency [taking into account only the poles given by Eq. (22, c)], we have

$$-(d\mathscr{E}/dz)_{\text{par.}} = \frac{e^2}{L^2}\int \frac{k_\perp^3\,dk_\perp(p_1^2-p_2^2)^2}{\sin\frac{\omega}{v}L\cdot(1-v/v_{bd})\,[(p_1^2-\omega^2/v^2)(p_2^2-\omega^2/v^2)]^2}F,$$

$$F = \left\{\frac{p_1 v}{\varepsilon_1\omega}\sin p_1 a\left(\cos p_2 b - \cos\frac{\omega}{v}b\right) + \frac{p_2 v}{\varepsilon_2\omega}\sin p_2 b\left(\cos p_1 a - \cos\frac{\omega}{v}a\right)\right\} \tag{29}$$

$$\times\left\{1 - \frac{\left[\varepsilon_2\left(p_2^2-\frac{\omega^2}{v^2}\right)-\varepsilon_1\left(p_1^2-\frac{\omega^2}{v^2}\right)\right]\left[\sin\frac{\omega}{v}a\left(\cos p_2 b-\cos\frac{\omega}{v}b\right)+\sin\frac{\omega}{v}b\left(\cos p_1 a-\cos\frac{\omega}{v}a\right)\right]}{\varepsilon_1\varepsilon_2(p_1^2-p_2^2)\left[\frac{p_1 v}{\varepsilon_1\omega}\sin p_1 a\left(\cos p_2 b-\cos\frac{\omega}{v}b\right)+\frac{p_2 v}{\varepsilon_2\omega}\sin p_2 b\left(\cos p_1 a-\cos\frac{\omega}{v}a\right)\right]}\right\}^2.$$

Here $v_{bd} = L\partial\omega/\partial\psi$, while $\omega$ and $k_\perp$ are related by Eq. (23). The region of integration over $k_\perp$ is the segment of the real axis on which Eq. (23) is satisfied for real $\omega$.

Eq. (23) can be used to change variables in the integral (29). In fact, from this equation we find:

$$dk_\perp = [L(1-v/v_{bd})/(\partial\psi/\partial k_\perp)]\,d\omega/v,$$

and the integral takes the form:

$$-\overline{(d\mathscr{E}/dz)}_{\text{par.}}=\int f_\omega\, d\omega,$$

$$f_\omega = \frac{c^2}{Lv}\,\frac{k_\perp^3(\omega)\,(p_1^2-p_2^2)^2}{\sin\frac{\omega}{v}L\cdot\frac{\partial\psi}{\partial k_\perp}\left[\left(p_1^2-\frac{\omega^2}{v^2}\right)\left(p_2^2-\frac{\omega^2}{v^2}\right)\right]^2}F, \quad (30)$$

$f_\omega$ is the spectral distribution of the parametric Cerenkov radiation. The limits of the integration in Eq. (30) are determined by means of Eq. (23), and the direction of integration is chosen in such a way that it corresponds to increasing $k_\perp$.

The conditions for the parametric radiation, like those for the ordinary Cerenkov effect, restrict the frequencies emitted by the particle. Therefore in the integration of the expression (18) over $\omega$ the integral is in reality different from zero only over a certain range of frequencies. It can turn out that over this whole region or over a certain part of it the wavelengths radiated exceed considerably the period of the dielectric structure, *i.e.*,

$$\beta_\Phi\lambda \gg 1, \quad (31)$$

where $\beta_\Phi=v_\Phi/c$, $v_\Phi$ being the phase velocity of the wave.

When the condition (31) holds, the study of the expression for the energy loss is decidedly simplified. As is well known[7,8], a laminar medium in which electromagnetic waves satisfying the condition (31) are propagated is equivalent in its electrodynamic properties to an anisotropic dielectric with effective values of the dielectric constants $\varepsilon_r$ and $\varepsilon_z$ given by

$$\varepsilon_r = (a\varepsilon_1 + b\varepsilon_2)/(a+b),$$
$$\varepsilon_z = (a+b)\,\varepsilon_1\varepsilon_2/(a\varepsilon_2+b\varepsilon_1). \quad (32)$$

It is natural to expect that in this case Eq. (18) will reduce to the well known expression for the field of a charge moving uniformly in an anisotropic dielectric[9-11]. In fact, using Eq. (31), we get from Eq. (18) for $\mu_1=\mu_2=1$:

$$\bar{E}_{z,\,\omega k\perp}\big|_{z=vt} = \frac{ie\omega}{\pi c^2}\,\frac{1-1/\varepsilon_r\beta^2}{k^2\varepsilon_z - k_\perp^2 - \omega^2\varepsilon_z/v^2\varepsilon_r}. \quad (33)$$

According to Eq. (17) the total energy loss in this case is given by

$$-\overline{(d\mathscr{E}/dz)} = \frac{ie^2}{\pi c^2}\int_0^{\varkappa_m} k_\perp dk_\perp \int_{-\infty}^{\infty}\frac{\omega d\omega\,(1-1/\varepsilon_r\beta^2)}{(\omega^2\varepsilon_z/v^2\varepsilon_r)(\varepsilon_r\beta^2-1)-k_\perp^2}. \quad (34)$$

In integrating Eq. (34) over frequency we shall assume that owing to a slight attenuation in the dielectric the poles of the integrand are displaced from the real axis. These poles are given by the roots of the equation

$$k_\perp^2 = (\omega^2\varepsilon_z/v^2\varepsilon_r)(\varepsilon_r\beta^2-1). \quad (35)$$

After integration with respect to frequency the attenuation is let go to zero. Since in Eq. (34) the integral with respect to $\omega$ is equal to the sum of the residues at the points given by Eq. (35), the integration is taken only over the range of frequencies that satisfy this equation as $k_\perp$ is varied from 0 to $\kappa_m$:

$$\varkappa_m^2 > (\omega^2\varepsilon_z/v^2\varepsilon_r)(\varepsilon_r\beta^2-1) > 0. \quad (36)$$

Consequently:

$$-\overline{(d\mathscr{E}/dz)} = \frac{e^2}{c^2}\int\left(1-\frac{1}{\varepsilon_r\beta^2}\right)\omega d\omega. \quad (37)$$

The direction of integration over this range is chosen in such a way that the quantity $k_\perp$ given by the relation (35) is increasing. Then the integrand will change sign with change of direction of integration, and the necessity of taking the absolute value is avoided.

The result obtained agrees with that of Sitenko and Kaganov[12]*; *i.e.*, when the condition (31) holds one can in fact calculate the energy loss of a charged particle moving in a laminar medium from the formulas for the Cerenkov radiation in an anisotropic dielectric. The dielectric constants of the equivalent anisotropic dielectric are given by formulas (32).

Let us consider some concrete cases. Take

$$\varepsilon_1 = 1, \quad \varepsilon_2 = 1 + A/(\omega_0^2-\omega^2),$$

Then

$$\varepsilon_r = 1+\frac{bA}{L(\omega_0^2-\omega^2)}, \quad \varepsilon_z = \frac{\omega_0^2-\omega^2+A}{\omega_0^2-\omega^2+aA/L}. \quad (38)$$

In the case under consideration the range of integration over frequency specified by Eq. (36) falls into two parts. This means that the spectrum of the radiation of the particle

$$\omega_0^2 + A \geqslant \omega^2 \geqslant \omega_0^2 - R' \quad \text{for} \quad R' < \omega_0^2,$$

* For a homogeneous anisotropic medium.

or

$$\omega_0^2 + A \geqslant \omega^2 \geqslant \omega_0^2 \quad \text{for} \quad R' > \omega_0^2$$

[where $R' = bA\beta^2/L(1-\beta^2)$], does not contain the frequencies specified by the inequalities

$$\omega_0^2 + \frac{a}{L}A > \omega^2 > \omega_0^2 + bA/L \quad \text{for} \quad a > b,$$

$$\omega_0^2 + \frac{b}{L}A > \omega^2 > \omega_0^2 + aA/L \quad \text{for} \quad a < b.$$

when $a = b$ the two regions coincide.

It is convenient to represent the Cerenkov radiation loss (37) as the sum of two terms, obtained by integration over the first and second regions, respectively,

$$-\overline{d\mathcal{E}/dz} = J_1 + J_2.$$

For $a > b$:

$$J_1 = \frac{e^2 b}{2v^2 L} A\left\{(1-\beta^2) - \ln\frac{a-b}{a}\right\},$$

$$J_2 = \frac{e^2 b}{2v^2 L} A\left\{\ln\left[\frac{a-b}{a(1-\beta^2)}\frac{\varkappa_m^2 v^2}{\omega_0^2 + bA/L}\right] - 1\right\}, \quad \text{if} \quad R' < \omega_0^2, \tag{39}$$

$$J_2 = \frac{e^2}{2v^2}\left\{\frac{b}{L}A\ln\left[\frac{\varkappa_m^2 v^2}{A}\left(\frac{a}{b} - \frac{b}{a}\right)\right] - (1-\beta^2)\left(\omega_0^2 + \frac{b}{L}A\right)\right\}, \quad \text{if} \quad R' > \omega_0^2$$

For $a = b$:

$$J_1 = \frac{e^2}{4v^2}A\left\{(1-\beta^2) + \frac{1}{2}\ln\frac{\varkappa_m^2 v^2}{\omega_0^2 + A/2}\right\},$$

$$J_2 = \frac{e^2}{4v^2}A\left\{\frac{1}{2}\ln\left[\frac{\varkappa_m^2 v^2}{(1-\beta^2)^2(\omega_0^2 + A/2)}\right] - 1\right\}, \quad \text{if} \quad R' < \omega_0^2, \tag{40}$$

$$J_2 = \frac{e^2}{4v^2}\left\{\frac{A}{2}\ln\left[\frac{4\varkappa_m^2 v^2(\omega_0^2 + A/2)}{A^2}\right] - 2(1-\beta^2)\left(\omega_0^2 + \frac{1}{2}A\right)\right\}, \quad \text{if} \quad R' > \omega_0^2.$$

Finally, for $b > a$:

$$J_1 = \frac{e^2}{2v^2}A\left\{\frac{a}{L}(1-\beta^2) + \frac{b}{L}\ln\left[\frac{b-a}{b}\frac{\varkappa_m^2 v^2}{\omega_0^2 + bA/L}\right]\right\}, \tag{41}$$

$$J_2 = \frac{e^2}{2v^2}A\left\{\frac{b}{L}\ln\frac{b}{(b-a)(1-\beta^2)} - \frac{b}{L}\beta^2 - \frac{a}{L}(1-\beta^2)\right\}, \quad \text{if} \quad R' < \omega_0^2,$$

$$J_2 = \frac{e^2}{2v^2}\left\{A\frac{b}{L}\ln\left[\frac{L}{b-a}\frac{\omega_0^2 + bA/L}{A}\right] - (1-\beta^2)(\omega_0^2 + bA/L)\right\}, \quad \text{if} \quad R' > \omega_0^2.$$

From the formulas given it follows that on relative thickening of the dielectric discs the main part of the Cerenkov radiation loss is shifted from the first frequency region to the second.

Let us consider the case $b > a$, which allows the passage to the limit $a \to 0$ of the homogeneous and isotropic dielectric with dielectric constant $\varepsilon_2 = 1 + A/(\omega_0^2 - \omega^2)$. For $a \to 0$ the width of the first region goes to zero, and the integral $J_1$ over this region goes to the value

$$J_1|_{a\to 0} = \frac{e^2 A}{2v^2}\ln\frac{\varkappa_m^2 v^2}{\omega_0^2 + A}. \tag{42}$$

The radiation given by (42) corresponds to the frequency $\omega^2 = \omega_0^2 + A$ at which the dielectric constant of the medium goes to zero, and, according to Eq. (36), it cannot be propagated in the medium. Therefore Eq. (42) gives the polarization losses

The integral over the second region is

$$J_2 = \frac{e^2 A}{2v^2}\left\{\ln\frac{1}{1-\beta^2} - \beta^2\right\} \quad \text{for} \quad R' < \omega_0^2,$$

$$J_2 = \frac{e^2 A}{2v^2}\left\{\ln\frac{\omega_0^2 + A}{A} - \frac{\omega_0^2}{A}(1-\beta^2)\right\} \quad \text{for} \quad R' > \omega_0^2, \tag{43}$$

and gives the ordinary Cerenkov radiation in the dielectric in question. From Eqs. (42) and (43) it follows that in an isotropic and homogeneous dielectric the polarization losses considerably exceed the Cerenkov radiation loss.

The effective constants $\varepsilon_z$ and $\varepsilon_r$ given by (38) no common zeroes. Therefore, according to Ref. 12, it could be expected that polarization losses are absent in the laminar dielectric. But the foregoing analysis shows that in this case the polarization losses may indeed be considerably reduced, but remain finite. This seeming inconsistency is explained by the fact that in a laminated dielectric the phase velocity of electromagnetic waves is zero in the neighborhood of the frequency given by $\omega^2 = \omega_0^2 + A$, and consequently the condition for the applicability of the approximation (31) is not satisfied. Therefore, Eqs. (39)–(41) for the Cerenkov radiation do not describe the total energy loss of a charged particle in a dielectric divided into thin layers. The radiation loss is accompanied by the polarization losses given by (26).

Comparison of Eqs. (41) and (43), which give the radiation energy losses in the laminar and solid dielectrics, shows readily that in the laminar dielectric the energy losses in the second region of frequencies is smaller by about a factor $b/L$ than in the solid dielectric, *i.e.*, these are quantities of the same order. Together with these, in the laminar dielectric there are radiation losses, concentrated in the first region of frequencies. They are of the same order as the polarization losses in the solid dielectric, *i.e.*, are much greater than the radiation losses in the solid dielectric.

Thus when an isotropic dielectric is divided into layers, the frequency that previously determined the polarization losses broadens out into a band of frequencies, and an intense Cerenkov radiation arises in this band. The radiation loss increases and becomes comparable with the polarization loss. Furthermore, as $b/L$ is decreased the frequency region in which the intense Cerenkov radiation is concentrated is displaced toward longer wavelengths.

For ordinary dielectrics the effect of reduction of the polarization loss can be observed only in very thin layers of the dielectric (films), since the polarization losses in such dielectrics are determined by frequencies lying in the optical region.

It must be remarked that conditions (31) and (36) can also be simultaneously satisfied in the microwave region, if the laminar medium is formed by an electron plasma. In particular, these conditions can be satisfied in the motion of charged particles through bunches of electrons.

As is well known, Cerenkov radiation is in general absent from an unbounded plasma. The radiation energy losses arise upon the application of an external magnetic field, when the plasma becomes an optically active medium[13]. On division of the plasma into layers, according to the results presented above, the intensity of the Cerenkov radiation becomes large, since the radiation losses become comparable with the polarization losses. Moreover, the radiated frequencies are in the microwave region. Therefore the occurrence of an intense Cerenkov radiation in laminar dielectrics (plasmas) can find wide application in radio physics for the generation and amplification of ultrahigh frequencies.

In the passage of a particle through a laminar plasma (bunched electrons)

$$\varepsilon_1 = 1, \quad \varepsilon_2 = 1 - \Omega^2/\omega^2, \quad \Omega^2 = 4\pi e^2 n/m.$$

Here $n$ is the density of electrons in the bunches and $b$ is the width of a bunch. Then the condition that the wavelengths radiated in the first frequency region be large in comparison with the structure period takes the form

$$\pi m v^2 / n e^2 \gg Lb.$$

Putting $A = \Omega^2$ and $\omega_0^2 = 0$ in Eqs. (39)–(41), we get the following expression for the total Cerenkov radiation loss

$$-(\overline{d\mathcal{E}}/dz)_{\text{Cer.}} = \frac{2\pi n e^4}{m v^2}\frac{b}{L}\ln\left(\frac{L}{b}\frac{\varkappa_m^2 v^2 m}{4\pi n e^2}\right), \qquad (44)$$

and according to Eq. (26) the polarization loss is given by

$$-(\overline{d\mathcal{E}}/dz)_{\text{polar.}} = \frac{4\pi n e^4}{m v^2}\frac{b}{L}\ln\left(\frac{\varkappa_m b}{7{,}4}\right). \qquad (45)$$

Under the actual conditions the boundaries of the plasma layers are somewhat diffuse. Therefore it is of interest to take into account diffuseness of the boundaries. This problem can be solved when the condition (31) is satisfied. Indeed, it is shown in Ref. 7 that if the properties of the medium vary continuously and the period of these variations is sufficiently small, such a medium is equivalent in its electrodynamic properties to an anisotropic dielectric with effective values of its dielectric constants given by:

$$\varepsilon_r = \frac{1}{L}\int_0^L \varepsilon(z)\,dz, \qquad \frac{1}{\varepsilon_z} = \frac{1}{L}\int_0^L \frac{dz}{\varepsilon(z)}. \tag{46}$$

It is natural to expect that the energy loss of a charged particle in such a medium will also be determined by Eq. (37), with $\varepsilon_r$ and $\varepsilon_z$ given by (46).

Assuming a concrete form of the diffuse boundaries, one can solve the stated problem.

For example, if over the range of a single plasma layer the particle density increases linearly from zero to its maximum value $n_0$ in edge regions of thickness $\eta$, the effective values of the dielectric constants are given by:

$$\varepsilon_r = 1 - \frac{b}{L}\frac{(b-\eta)}{b}\frac{\Omega_0^2}{\omega^2},$$
$$\varepsilon_z = \left(1 - \frac{\Omega_0^2}{\omega^2}\right)\left[1 - \frac{a}{Z}\frac{\Omega_0^2}{\omega^2} - \frac{2\eta}{L} - \frac{\omega^2\eta}{\Omega_0^2 L}\left(1 - \frac{\Omega_0^2}{\omega^2}\right)\ln\left(1 - \frac{\Omega_0^2}{\omega^2}\right)\right]^{-1}. \tag{47}$$

On the other hand, if over the range of a single plasma layer the particle density varies continuously according to the sinusodial law

$$n(z) = n_0 \sin\frac{\pi}{b}(b - z), \tag{48}$$

then

$$\varepsilon_r = 1 - 2b\Omega_0^2/\pi L\omega^2,$$
$$\varepsilon_z = \frac{L}{a}\left(1 - \frac{b}{\pi a}\frac{\omega^2}{\sqrt{\Omega_0^4 - \omega^4}}\ln\frac{\Omega_0^2 + \sqrt{\Omega_0^4 - \omega^4}}{\Omega_0^2 - \sqrt{\Omega_0^4 - \omega^4}}\right)^{-1} \quad \text{for} \quad \Omega_0^2 > \omega^2,$$
$$\varepsilon_z = \frac{L}{a}\left(1 - \frac{4b}{\pi a}\frac{\omega^2}{\sqrt{\omega^4 - \Omega_0^4}}\arctan\sqrt{\frac{\omega^2 + \Omega_0^2}{\omega^2 - \Omega_0^2}}\right)^{-1} \quad \text{for} \quad \Omega_0^2 < \omega^2.$$

From these formulas it follows that the quantity $\varepsilon_r$ depends only slightly on the degree of diffuseness of the plasma layers, while $\varepsilon_z$ is mainly determined by the nature of the boundaries. Therefore the range of frequencies of the Cerenkov radiation arising in a laminar plasma depends essentially on the character of the boundaries [this frequency range is given by the inequalities (36)], while the spectral distribution of the radiation inside this range does not depend much on the diffuseness.

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